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TUTORIAL ARTICLE

Determinacy and indeterminacy

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The Editors are delighted to welcome this contribution from a venerable pioneer of algorithmic composition who is also a member of *Organised Sound*'s Advisory Board. In this article, edited from notes for a series of lectures delivered in Poland, and not previously published, Xenakis tackles first the questions arising from determinacy and indeterminacy, repetition and variation, symmetry and structure, and multidimensional musical space. He later describes his computer drawing interface, UPIC, and ends with a discussion of some of his statistical compositional methods employing a variety of probability distributions. Much of the article is illuminated by insights drawn from a lifetime's work in the arts and sciences.

1. INTRODUCTION

The problem encompassed by determinacy and indeterminacy is a permanent one in music, both for composition and also for performance. Two performances of the same work are never exactly the same. They hover around what we may consider an ideal mean of performance, dependent upon errors and the style of the performer. This is understandable.

Determinacy constitutes a very important and deep question, especially when considered against a background of physics and computer science. We know that since the time of Copernicus and Kepler the movements of the planets and other celestial bodies have been geometrically defined. With Newton, gravitational theory seemed to be so accurate that repetitions of planetary motion could be foreseen and predicted until the end of time. This is like saying that you have a score, you memorise it, and then all you have to do is play it.

Theories about the movements of the planets, however, make an abstraction of the influences of other celestial bodies – even remote galaxies – as a result of which the predicted movements are not exactly in conformity with theory. Actually the movements in the far future cannot be foreseen with accuracy – the seemingly predictable movements are in fact unpredictable. The whole field of science dealing with such phenomena is today known as 'chaos', and the laws which govern the variations of behaviour are caused by 'strange attractors', a term used in contemporary physics.

In performing a score or something from memory we come across surprisingly similar problems. We never hear the same performance. This fact perhaps makes the performing of traditional music interesting, because each time the skilled performer brings with him something unforeseen and interesting aesthetically. It is also a matter of interest to see how the problem of the unforeseeable – of surprise – was anticipated at the compositional level; how determinism was treated during the writing of a piece and to what extent it constituted an integral part of the composition.

In the traditional music of India, for example, there are rhythmic patterns which are very well known by both performer and audience. Nevertheless a performer tries to raise the interest of his performance by unexpected variation of the rhythmic patterns. Here the performer also becomes a kind of composer since he decides which variation to use.

A composer working in the domain of rhythmic music faces this problem of expectancy in rhythm. Through the history of the development of rhythm there has been a tendency for increasing complexity based on the degree of unexpectedness of what might follow any musical event; therefore this is an approach based upon a non-deterministic way of thinking.

2. REPETITION AND VARIATION

Let me describe for you, as an example, the treatment of a given – beautiful – melodic pattern. Suppose you have composed it, or it has been created by someone else – the writer is not important for our considerations. This fragment of music is very dense, but what will become of it afterwards? You cannot repeat it every time in the same way, even if you are very fond of it. So there has to be some change in the repetition. I suppose this was at the root of the invention of pattern transposition, from which also stems the establishment of the chromatic scale.

Therefore the point is identified: the varying manner of repetition of the same thing, i.e. of something which has been established as a kind of identity. Our musical pattern is repeated in time but in a different, unexpected way. I believe this is the meaning of transposition. So, for me, the polyphonic concept of music is born from this kind of fight between expectancy and non-expectancy.

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A secondary consideration is, perhaps, the meaning of so-called variations. A kind of identity – a very powerful idea – is presented to the listeners and afterwards changed; this idea is then repeated, but in a different way, in order to create an unexpected result. This kind of surprise represents an important factor in aesthetics, whether understood through time or outside of time. This further corroborates my statement that considerations of determinacy and indeterminacy have been deeply rooted in music from its very beginning.

Let us consider this problem in the light of serial music. Serial music is based on a string of pitches like a kind of melodic pattern. While the series of pitches is repeated over and over, by using the laws of polyphony the degree of unexpectedness is augmented. Historically, in order to derive a further layer of complexity – which in this case is something close to surprise – the so-called *klangfarbenmelodie* was introduced. Here the timbre was something that created surprise in the repetition.

I do not wish to develop further these historical facts because they are probably well known to you, but I want to stress this one important notion of repetition, i.e. the reproduction of some idea, and then the discrepancy of moving away from that idea – an antinomy to that identity – by repeating it but in a different way.

This, I think, is the core of the problem of determinism and non-determinism. Is the repetition more or less faithful to some identity or to some pattern introduced earlier? The given examples occur in time, but we can have the same principle in other domains of music, which are not dependent upon time.

These aspects of music are often referred to as 'parameters', but I prefer 'characteristics of sound'. 'Parameter' is a mathematical term, which has a very specific meaning; it has been borrowed from mathematics by composers in the wrong way, just as the term 'aleatoric' has been borrowed from physics in the wrong way. Aleatoric music simply means improvised music.

3. SYMMETRY

As an example of a domain of music which does not depend on time, consider the distribution of intervals in an arbitrary scale. We could relate this to symmetry, in the domain outside of time. What repetition is in time, symmetry would be outside of time.

Lamps in a concert hall are usually arranged symmetrically, because there is the same distance between them, the same common measure. Symmetry means common measure. One can imagine the chromatic scale like a symmetry in the pitch domain, similar to the lamps in the concert hall.

Now, with some effort, we can say the same thing about time, that since we can memorise the time intervals and compare them, then we can define a chromatic scale in time, which has the same type of symmetry as the outside-of-time characteristics.

With this kind of abstract thinking we can see the correspondence between a pitch and a time instant. Similar correspondences can be seen between pitch intervals and time intervals – i.e. durations. We can also find these correspondences in other characteristics of sound, for example in intensities.

One can define levels of intensities: after the inventor of the telephone we call them decibels (or perhaps phones!). Thanks to the telephone, musicians have a chromatic scale of intensities.

However, we cannot apply the same notion of structure to timbre, or sound colour, so we have only three domains: pitch, time, and intensity. These three realms have in common a so-called order structure, which was first identified in the field of experimental psychology by Jean Piaget, in the analysis of the growth of the child and the steps or phases of evolution of notions in the child's mind – and therefore of our minds as well. Piaget's books concerning these problems are entitled *The Child's Conception of Time* and *The Child's Conception of Space*.

We should notice the fact that things which we believe to be invented by musicians are actually not invented at all, they are deeply rooted in the structures of our minds.

4. ORDERED STRUCTURES

Jean Piaget did one particular thing – he put together the evolution of the brain of a child with the notions of mathematics. In fact mathematicians discovered the structures of the mind before the psychologists. The underlying structures that I have been discussing – the chromatic scale of pitches, time and dynamics – are called ordered structures.

The definition of an ordered structure is as follows. Given three elements of a set, they can be ordered in just one way by saying that one of the three is between the other two. In other words a set of elements has an ordered structure if you can put them in a string, placing each between two others and completing the set by applying this rule.

We can say this about pitches, about dynamics, instants in time, about sizes of objects, sizes of people – but we cannot say this about timbre.

For instance, if we wish to go from the timbre of a noise to that of a pleasant soprano voice, there are many ways, timbrally, in which this can be done. However, in order to go from a low pitch to a high one we always traverse the same set of pitches, even if we do not play them. We can apply this test to a set in music or elsewhere to see if it is ordered or not.

Figure 1

This leads us to the following: since we can order elements of some musical characteristics, we can assign them to points on a line – a straight line, or a curved one which does not cross itself. (A straight line is topologically the same as a curved line which does not cross itself; see figure 1.) The points on the line also have an ordered structure, because any point can be placed in between any other two points. Perhaps this is why musical notation has developed since the time of Guido d'Arezzo – because of this community of structures with the graphic, spatial, straight line.

As an example, notes on a stave are ordered in two ways – horizontally (in time) and vertically (in the pitch domain; see figure 2). Given the notes 'e' and 'g', you can place the third, 'f', in between them vertically and also horizontally. The stave is a traditional musical version of a more general, more universal notation of points on a straight line.

In this way we can imagine pitches, instants in time, levels of intensity and also other characteristics from different points of view, like density (the number of events per time unit) and degree of order, or – better still – disorder. I do not, however, intend that only instrumental sounds be considered in this discussion. We can, for instance, conceive of instantaneous sounds without pitches, i.e. purely percussive. There are also sounds without a definite attack time, sounds that might start at a very low level: we do not know when these happen, but we suddenly realise the sounds are there! In fact, we make a kind of strong abstraction when we define pitches, dynamics, or instants with any degree of precision, but it is necessary to attempt this in order to be able to go forward and manipulate more complex phenomena.

Now I shall return to what was said before about repetition. Here, the translation of repetition into the representation of the domains of characteristics of sound can be treated as the problem of distributing points on a line. The latter is very general and much easier to manipulate and to think about. So, a chromatic scale can be pictured as points with equal distance between them (figure 3). It can be formed using semitones, quartertones, commas, octaves, or indeed

Figure 2

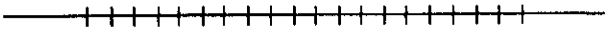

Figure 3. A chromatic scale as points on a line.

any equal interval. The same principle can be applied to time, levels of intensity, density, degree of disorder, and so on.

So the problem of determinacy and indeterminacy is the problem of distribution of points on a line in such a way that a very strong symmetry or repetition will not occur. It is easy to imagine or design patterns which are more or less repetitive or absolutely non-repetitive. The latter can be created by hand or by other means to ensure that there is no repetition at all.

What means might we use?

5. MULTIDIMENSIONAL SPACE

It is possible to introduce graphical means to represent the dimensions taken into consideration. We can define several straight lines with the same starting point, as shown in figure 4: they serve to describe time, pitch, dynamics, disorder, density, etc. A point in such a multidimensional domain is a sound, very short in time, whose characteristics are described by coordinates plotted on all the axes (points on lines). In the simplest case we can take only two dimensions – pitch and time; if a sound does evolve in these five dimensions it can be represented by a more complex shape.

The concept of multidimensional space needs some explanation. In order to define one-dimensional (1D) 'space' one has to take a reference point and a unit measure (figure 5). Any point P placed on such a line has logical validity because one can measure its distance from a reference point R. In two dimensions we can take two lines and fix the space by relating each point to two points on the axes. In other words we have a plane surface and every point on it can be

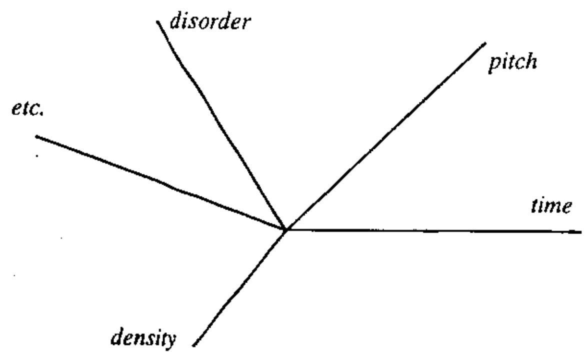

Figure 4. A multidimensional space of sounds.

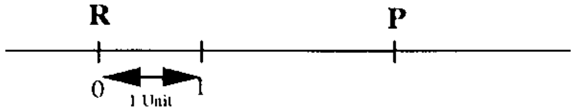

Figure 5. One-dimensional 'space'.

Level	Complexes
Two	Form (a movement)
One	Phrases, Chords
Zero	Pitch, Intensity, Timbre

Figure 7. Levels of a musical work.

related to two lines. This is a 2D space (figure 6). By adding a third line in the same way we can obtain a 3D space.

It was discovered historically how to represent 3D objects on a 2D surface by means of perspective. In ancient paintings there was a kind of perspective. This was rediscovered and fully developed during the Renaissance, when on a picture a horizon was used as a reference line. This art formed a kind of realism (even a social realism) from which paintings later escaped. In modern times we no longer try to treat the geometry of a picture with this principle of geometric perspective. The representation of 3D objects in a space of two dimensions – as on canvas – is a convention developed over quite a long period. When one looks at the paintings in the caves of Altamira in France one does not see perspective at all. Perspective in the fine arts is a representation which enables one to understand the relationship of 3D objects.

As for more than three dimensions, we do not know their reality, but we can imagine them. The theory of relativity uses time as a fourth dimension, and time is treated geometrically as an extra spatial dimension. In physics and mathematics we can use an infinity of dimensions. To define one point on one line we need one number; to define a point in a multidimensional space we need as many numbers as there are dimensions. Each point in an n-dimensional space is defined by an ordered set of n values. In the case of musical multidimensional space these values represent the characteristics of a sound.

So indeed music has aspects of multidimensional space.

Traditionally music has two established dimensions – time and pitch. Historically others were then added, e.g. dynamics, although these were rather vaguely represented in instrumental music. With the development of electronic and computer music, multidimensionality of sound representation turned out to be both natural and useful. But music goes beyond multidimensionality – it is even more complex. In music we have several layers which can be described as shown in figure 7. At the bottom, at level zero, are the elements discussed earlier – easy to describe and well understood from traditional music. Here are the elementary things: pitches, time instants, levels of intensity. At the next level we have aggregates of these elements – for example, in instrumental music we might have melodic patterning, as in a theme. At an even higher level we have interrelationships of a more complex character.

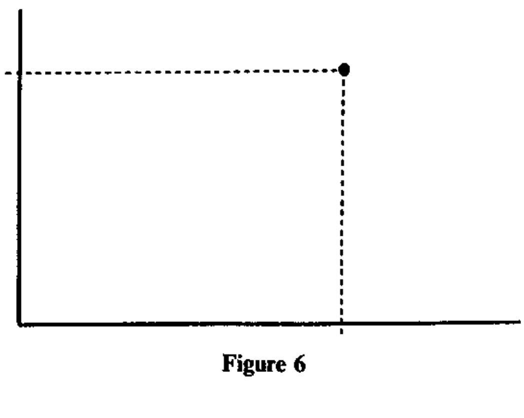

Figure 6

We may continue this argument by regarding even the whole movement of a sonata or symphony as a yet higher-order structure again. I think this explains why traditional classical music up to the end of the nineteenth century or even into the twentieth century seems more complex than contemporary, avant-garde music. It has a structure of many simultaneous layers, which surround the listener. One has to listen to pitches, time instants and durations, dynamics, phrases, themes, structures of movements and so on simultaneously, even if one is not entirely conscious of it. While perceiving music one is in all the domains, on all levels at the same time.

In music we have not only the multidimensionality of space (five or even seven dimensions), but a much more complex way of thinking, perhaps the most complex in the whole of human creation. Music is like a multiple sandwich, but a transparent one. Whilst in the middle of it, one can see at the same time lower or higher layers everywhere.

Contemporary music and, even more so, electronic and computer music, is not as rich in structure; it does not have as many layers as instrumental music of the past. This is probably because of the departure from what we call a classic phase in the evolution of music in the West. During classic phases of architecture, sculpture, painting and music, there are many layers to be found in a work of art. Following these phases, when things start to change in a more fundamental or 'barbarian' way, they are simplified, sometimes perhaps over-simplified, and the artist has to add other elements in order to make the art complex again. This is not yet the case with contemporary music.

6. SYMMETRY AND DETERMINACY

Let us return to the subject introduced at the beginning of this article. The problem at the root of all these layers, of the ways in which they are constructed, is again the problem of repetitions, of symmetries and the problem of the destruction and change of these symmetries in the flow of musical movement. It is like being in the flow of a stream or river, where everything is either expected or happens unexpectedly. Therefore our problem is linked with the question of determinacy and indeterminacy in the widest sense and with so-called causality in physics, which is an aspect of determinacy.

Below the zero-level of instrumental music that I have discussed, there are deeper levels which are dealt with by experimental and theoretical acoustics.

The first of these lower levels is described in the language of sound synthesis, which is based on Fourier analysis of sound structure. This is the level of the analysis or synthesis of harmonics. Below this level is – due to computers – the level of the individual sound samples, up to 50,000 samples per second, representing a very high fidelity in the analysis of a sound. Probably one can discover even lower levels; I am personally convinced that sampling rates should go higher than 50,000 per second, because even at that level there is already quite a distortion.

We have identified the problem of the discovery of determinacy and indeterminacy, the two poles between which music goes back and forth, and the first suggestion of a solution comes from distributing points on a line.

7. NOTATION

The image of a line with points on it, which is close to the musician and to the tradition of music, is very useful. The representation of characteristics of sound by points and lines is a tradition which started long ago.

It may have begun as long ago as the Babylonians, but certainly the musical accents used in Hellenistic Alexandria may represent the origins of this way of thinking. In this notation the movement of a pitch line going upwards or downwards was represented by a graphic sign – an accent. This was a spatial representation of a phenomenon that had nothing to do with space. Later on in music this kind of representation was linked with cheironomy, the movement of the hand – e.g. the gestures of a person conducting a group of singers, which is at the root of neumatic notation.

Now, the fact that we can use a line with points on it, a geometric representation of the characteristics of sound, enables us to make a further translation which is essential for work with computers and as a means of manipulating the characteristics of sounds. It is the translation to real numbers.

Both graphical and digital forms may represent characteristics of sound, and in the computer either may be converted into the other. Later I will discuss a computer system developed in Paris called UPIC, which is based on a graphical notation and which is easy to use.

8. DISCRETE AND CONTINUOUS FORMS

Everything considered so far has originated from a discrete way of considering music, discrete in the sense that we have well-defined objects which can be associated in a combinatorial manner. Another, perhaps more general, way of thinking is to consider the aspects of sound as continuous. Let us become more abstract and regard the sound as a form in which we are able to distinguish some features. Our ability to discern these features stems from the evolution of the human mind and culture. We should talk about the social and cultural aspects of musical sound, some of which are perhaps rooted in our brain structure, which is hereditary. The ordered structures previously discussed are very likely to have a hereditary origin rather than a socio-cultural one.

Here we distinguish some features of a sound that can be called characteristic at the elementary level and also at higher-order levels. We can perceive these characteristics because of our training or because of our cultural conditioning, and they are considered to be discrete – the easiest way to grasp and deal with things in the objective world.

The continuous changing of these forms is more difficult to comprehend because they are in a process of evolution all the time. This fact occurs not only in music but also in different domains of sciences like physics and mathematics. The continuity is more difficult to observe and to deal with, and also to theorise about, but it is a basic aspect that has always existed in music, although in a less developed way than the discrete one. The cultures of Asia provide us with examples of continuous change in pitch and intensity. We may very easily observe continuous variation in the pitch and intensity domains employed by performers in Japan or China. Whilst playing the *shamisen* or the *biwa* the performers continuously change the pitch subtly, increasing the musical interest through added expression.

Glissandi represent only one aspect of continuity in the musical domain, which has always existed but in the West came to the fore only after the Second World War. Examples from the intensity domain in the West are older. The continuous flow of intensity with crescendo and decrescendo was first introduced some centuries ago. And in the domain of time we may also find aspects of continuous change such as accelerando and decelerando.

With the assistance of computer synthesis it is possible to imagine changes in many other domains of musical structure taking place in a continuous way. This makes the computer a most powerful tool to generalise the flow of sounds, but it is based on the same principles as before: the continuity that can be foreseen. Accidents or surprises – i.e. indeterminacy – should rule the continuity domain in much the same way as they rule the discrete domain.

9. AN ILLUSTRATIVE EXAMPLE – *JONCHAIES* (1977) – WORK FOR LARGE ORCHESTRA

The title of my work *Jonchaies* means a kind of marsh-reed. We have fields of such plants in France. Some of the reeds are bent in one direction, others in different directions, so there is a multiplicity of directions in patches and paths.

In *Jonchaies* there are very rhymthical parts, rhythms like a chromatic scale in time, and examining the score one would notice the progressions from rhythmical, foreseeable patterns into more complex ones which finally break into a vast number of scattered rhythms.

The rhythmic structures progress from regularity to irregularity. The regular principle is easy to achieve. In the case of the irregular progression, there are two methods available. One is to construct irregularity by means of some probability distribution. The other way is based on the inability of the human brain to follow wide complexity. Let me give an example. We will take again the image of a line with points on it. We can illustrate regular events by points an equal distance apart. On a second, lower parallel line, more points represent other regular patterns with a different time unit, so they are shifted with respect to the first line's points even if they start together. This procedure can be repeated with regular points on other lines. When we hear all these lines together, we obtain a flow of events which consist of a regular intervallic series, but which as a whole is impossible to grasp (figure 8). Our brain is totally unable to follow such a complicated, multilayered flow. This is a basic rhythmic principle which was used in the composition of *Jonchaies*.

The same principle was also applied to the domains of pitch and time. Many lines were created which were supported by various groups of instruments. As a result there was a mass of events, in which – because of the complexity – separate occurrences could not be distinguished. From time to time different fragments of these lines could be heard, but this depended on their intensity.

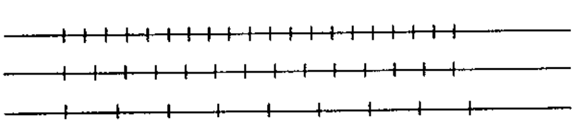

Figure 8

For the listener it is perhaps like being in a river, drowning, and holding on to one log or another, depending on whether you can catch them or not. So there is a flow of expectancy and denial. Of course, this is an external metaphor. Within the music there are other elements which cannot be so easily illustrated. When one writes a piece one may think one consciously controls everything, but in fact this cannot be the case.

People tend to overestimate the importance of consciousness. Consciousness is like the tip of an iceberg. It is like saying that we can control something by the rational means of physics, mathematics or the rules of music – such rules that we have learnt or that we have created. Actually, all this is a matter of decisions made without one being aware of them. In art, as in science, creation is based on intuition much more than on rational elements, which come afterwards.

When Newton posed the principles of his mechanics it was because he felt them first. Suddenly there was a kind of conscious recognition of what he felt and then he was obliged to express it in a more rigorous way, by rationalising and inventing a new mathematics in order to express clearly what he had felt. This relationship between rationality, consciousness, control and that which is not controlled exists in other domains as well as in music. In mathematics, it is interesting to read how Cantor proposed his set theory. All the anxiety of the discovery that he felt is written in his letters and even in his ways of demonstrating the theory. It is interesting because an artist behaves in the same way. Cantor had problems in his mind, but in order to show what he felt he had to find a language for expressing his thoughts. His theories were at first very hotly debated and opposed by other mathematicians who did not feel the way he felt, despite his attempts to prove his discoveries rationally.

I think that the antinomy between rationality and irrationality is false. The contradiction is so much a matter of discussion among critics and lay people, especially in art, who say: 'It is not music, it is not art. It is just a matter of computation, therefore this music is not valid.'

There is no such thing as creation by rationality. The computer, which has arisen through the wealth of achievements of the human mind through the millennia, cannot create anything new. Several mathematicians, for instance the Nobel prize-winner

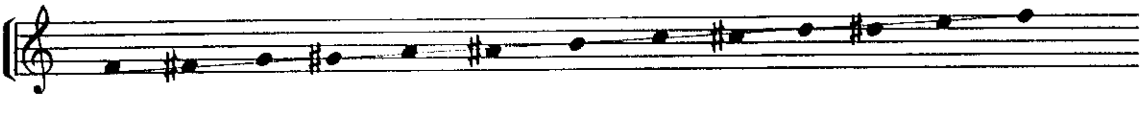

Figure 9. A chromatic scale of semitones as points on a line.

Simon Newell, tried to create theorems with the computer. Recently it was demonstrated that this is not possible. Beneath the level of consciousness there lies all this fantastic amount of intuition that ultimately leads to a rational expression, but without this intuition it is impossible to create anything.

This is one reason for the failure of Newell's experiments with the computer. The other reason was proved in logic and general mathematics by Gödel. Inside a logical system, undecidable oppositions exist. One has to go outside the system in order to prove things occurring within the system.

This is particularly important for the artistic domain, where rationality seems to be something marginal. It is not marginal, but basic; however, the importance of intuition in artistic creativity overshadows rational thinking.

Back to *Jonchaies*. There was the problem of the flow of the music based on continuity. We had transformations which were continuous, with glissandi or the quantity of information which could not be grasped as discrete. The principles of this piece were based both on physiological limits of the mind and also conceptual trends, such as going from complexity to simplicity and *vice versa*.

For me it is always important to go to the limits, to push them, as it were, and to explore these domains which, in a sense, are beyond the aesthetical concerns of art. What is interesting to notice is that in other artistic domains the same sorts of things happen.

The listener may notice that at the beginning of *Jonchaies* in the string parts and in the percussion there are some problems of scales, proposed and solved. The question of scales underlies at least all instrumental music. When writing for instruments the composer uses a scale which is assumed to be there – i.e. the chromatic well-tempered scale. It is a set of pitches, and one allows oneself to choose any of the pitches from the set, adopting a structure which is ready-made. It is again like choosing points on a line (figure 9). However, one can also provide oneself with a different set of pitches, another structure, giving a different possibility of choice. One can have, for instance, the diatonic scale, which has a more complex structure than the chromatic scale. In the diatonic scale we have: tone, tone, semitone, tone, tone, tone, semitone (2, 2, 1, 2, 2, 2, 1). This structure, although it uses fewer elements than the chromatic scale, is more complex because of its internal asymmetries. The same can also happen in time, in the domain of rhythm. If one takes the pattern 2, 2, 1, 2, 2, 2, 1 and repeats it in time, one obtains a pattern more complex than a repeating pulse.

The evolution of rhythm in the work of composers like Stravinsky or Bartók was actually based on the use of new symmetries and repeating patterns. The general question is whether one can produce patterns different from those adopted previously, and in what way can one achieve it? The solution lies in creating different scales in time, pitch, and other characteristics of sound which in experiments prove to be musically interesting. Several examples of such scales have been tested and assimilated by cultures of South-Eastern Asia – in Japan, Bali, India and China, although in the pitch domain all of their scales are very symmetric, octaviating. Rhythmic passages also occur as repetitious ones, as in Indian music.

The theoretical problem is the following: Is there any way to create a new type of scale in relation to the inner symmetries produced by the intervals? This again can be related to the problem of constructing the positions of time points on a straight line. Again the basic principle is to go from symmetry to asymmetry or *vice versa*. The main activity of our minds is to compare things and find similarities, or the lack of them, between phenomena. We say that things are similar or are not similar, or they are distant, and how far distant, and we express judgements based on these comparisons.

So how can we originate systems of points that are symmetric or asymmetric, and how can we initiate movement from one to the other? This problem is very important in computer music.

Suppose we have a 2D space of amplitude and time (see figure 10). Let us assume a numerical amplitude space of 16 bits, i.e. 65,536 possible points, while

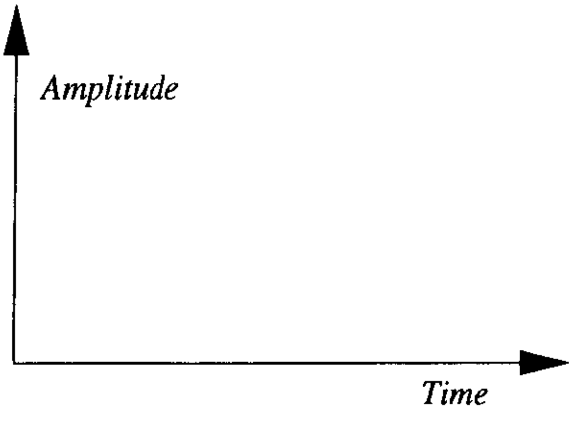

Figure 10

in time we use a sampling rate of, say, 50,000 samples per second. This is like using a chromatic scale in two directions, in which complex shapes correspond to the music we hear, i.e. the variation of the atmospheric pressure with time. The basic problem for the originator of computer music is how to distribute points on a line.

If a composer has a strong inner necessity, then a musical organism, which is a kind of universe, evolves. Either things have to be, or they do not exist.

10. UPIC

The UPIC system consists of a large board, on which one can draw different lines with a ball-point pen, and a computer, to which both the board and the pen are linked. If one designs a line on the board it is interpreted by the computer, at first in 2D space – pitch and time. Therefore, if one draws a line parallel to the time axis, the corresponding sound has a steady pitch and a given duration in time (figure 11). If one draws a curve it means that pitch is changing in a continuous manner (figure 12(a)). If one draws many lines a kind of polyphony is created (figure 12(b)).

Now, we can ask a question about these lines. What intensity do they have? So, for each line one has to design intensity envelopes on the board. Let us suppose the sound we have drawn is a sustained pitch – a horizontal line. If one creates an intensity curve starting from zero, going up in amplitude sharply and then slowly decaying, the result will be a kind of percussive sound (figure 13(a)). If one designs a square shape the result is a sound held without any variation in intensity (figure 13(b)), or if one is feeling very imaginative, a complex curve representing continuous changes in amplitude, can be drawn (figure 13(c)). Any curve can be considered when designing the envelope and instructing the computer in how to interpret this curve.

For the time being let us consider a note that is at a constant pitch, with a certain duration, and also having an intensity envelope. Now we may ask what is the timbre of the sound? One has to design it.

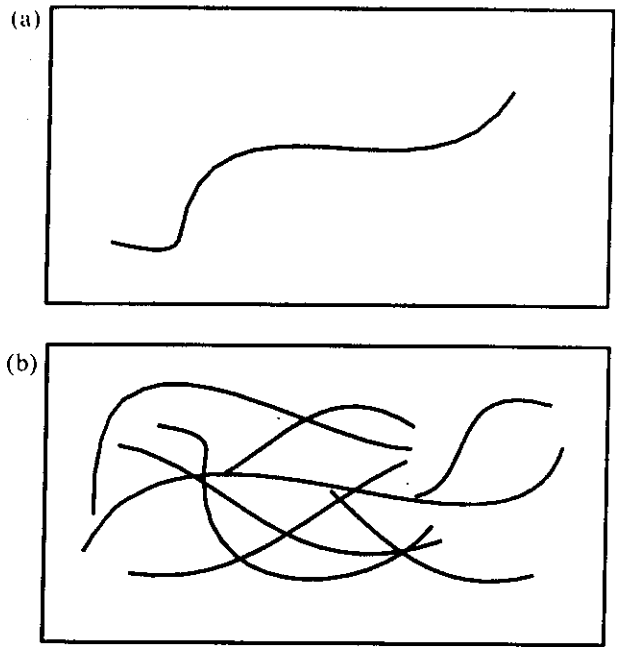

Figure 12

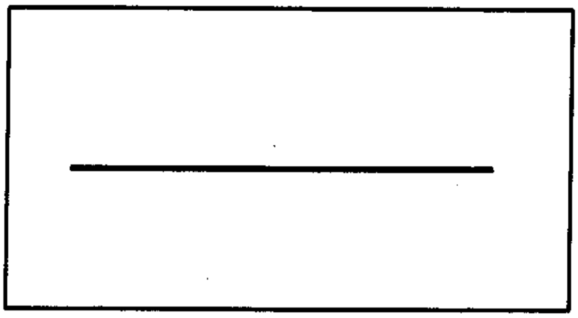

Figure 11

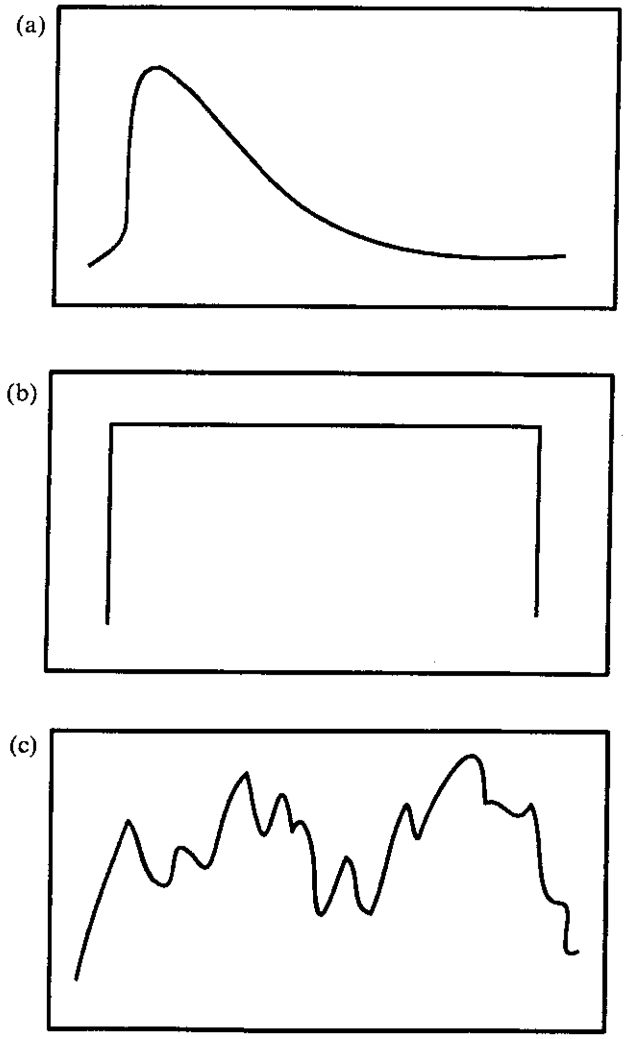

Figure 13

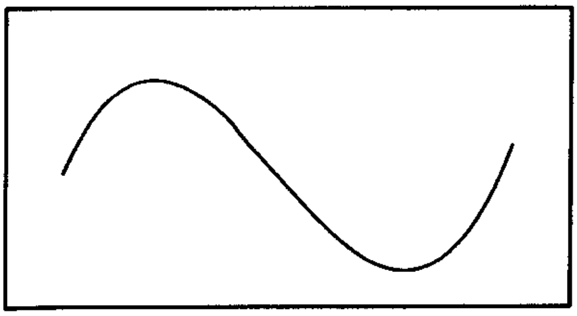

Figure 14

If one now draws the curve shown in figure 14, representing a single period of a waveform, we obtain an electronic sine wave sound. Feeling more imaginative, one may design one period of any waveform, and this will result in a particular timbre. Now we have defined a note which is a pitch in time, with its envelope and timbre. We may design many notes in the same way, creating for each its own envelope and timbre – it is like creating a full orchestra.

This is the principle of UPIC, at the first level. UPIC stands for *Unité Polyagogique Informatique du CeMAMu*, a neologism which I invented. It was developed in Paris in 1976 at the Centre d'Études de Mathematique et Automatique Musicale.

The board is akin to the page of a score. The user draws lines representing the notes, and each line is associated with an envelope and an elementary waveform. When one asks the computer to calculate the sounds, one needs to specify the duration of the page – whether it is one minute long, or two seconds. If there is a complex page and we calculate it to be played in a short span of time, then the aural result is very different from that produced by the same page executed slowly, over a long duration. This of course depends on the physiology of our ears and the integration capacity of our hearing.

Let us take a simple example. If one has slow, rhythmically repeated beats one can appreciate them as a rhythm, but when the repetition is very fast, 20 or 25 beats per second, then the ear cannot follow the rhythm and transforms it into a pitch and a timbre. It is interesting to see that when the beats are slow one can almost count them, as when listening to music or at the dentist counting the seconds until the treatment is finished. But when the beats become too fast, our counting changes into a kind of feeling for the pitches. This is also a kind of macroscopic counting, but a very precise one. It would be interesting to look into the processing of this information by the ear and brain, to determine the way in which our hearing system is constructed. The counting used in our ear is in certain respects not regular, it is a statistical counting of the firings of electric impulses by the auditory nerve cells.

Coming back to UPIC, on which we have designed several pages in the way described earlier, we should now be able to make an interconnection between these pages, or carry out a certain kind of mixing between them.

Suppose we want to incorporate one of these written pages into some other design. This is possible by using computer memory. At one side of the board there are black spots or cells which one can press with the pen; they correspond to specific commands for the computer. If something designed on the board should be considered as a waveform, one presses a specific spot which corresponds to the instruction 'This is a waveform'. Another spot means 'Page terminated' – press it and the computer considers all the lines drawn on the board as notes in the pitch-versus-time domain.

On the left-hand side of the board there is a kind of keyboard which corresponds to five octaves of pitch. The accuracy or precision of pitch is about one twentieth of a comma, i.e. about one eightieth of a semitone. Beneath the board there are cells which connect with a storage system. If one designs a curve and likes it, one can store it. There are banks of storage for the waveforms, envelopes and pages.

There is also a screen on which one can see the response of the computer to what has been designed, so that one can check what to do. Some of the instructions on the right-hand side of the board are, for example, 'Show the page on the screen', 'Show the waveform', and so on, or 'Listen to such and such a page'. When one prepares a score for the UPIC one draws with a normal pencil and then traces over the lines with the special ball-point pen, which is connected to the computer.

One does need a little practice drawing the lines. When the system is used by musicians or students of music, they tend to apply what they know. However, it is best to go further than one knows – even making mistakes with the hand one can find new and interesting sounds. Often children begin by drawing houses, but soon learn how to coordinate the design with the sound result. I have discovered there are two categories of people who tend to produce interesting results. The first category is children between ten and twelve years of age. They are revolutionary – they do not have preconceptions, and at the age of twelve they have alert mental capabilities that have not yet been stultified by school and family. The second category

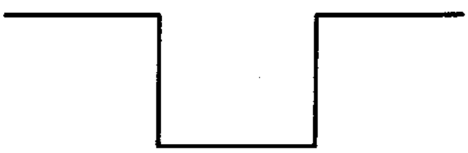

Figure 15

is those adults who have never played a musical instrument.

11. SOUND STRUCTURE AND SOUND SYNTHESIS

I should now like to discuss sound structure and sound synthesis in relation to my work *Légende d'Ér*, which was produced on the UPIC system.

When we listen to music we actually hear variations of atmospheric pressure with time. The ear recognises a Beethoven symphony as music, but if we look at the screen of an oscilloscope showing us the continuous changes of air pressure caused by the sound of this symphony, then the eye cannot distinguish what is happening there.

The purest form of sound for the ear is a sine wave, which mathematically is a continuously varying trigonometrical function. Mathematically speaking the simplest type of waveform would be the square wave, with alternating presence and absence of a signal (figure 15). However, the ear does not agree with this mathematical simplicity. To the ear this is much more noisy than the smooth sine wave. Those who are involved in electronic music know this very well.

Now, what is noise? So-called white noise can be represented by a curve with no smoothness at all, and no periodicity (figure 16). So noise could be created by a random walk in the computer using probabilities. There are many probabilities available as functions, many with their own 'personality' which one can perceive from the different kinds of noise that they produce. An example is the logistic distribution.

A given distribution can be regarded as channelled between two elastic mirrors, or borders. I should explain this. When a ball is thrown at a wall, it hits the wall; let us suppose that there is no loss of energy and the ball rebounds according to the rules of reflection. This means that the ball bounces with the same energy – a kind of elastic rebound (figure 17(a)). If we throw the ball in a direction perpendicular to the wall, it is like looking into a mirror (figure 17(b)). If we regard the wall as a mirror, then an image appears behind the wall at the same distance as the rebounded ball in front of the wall (figures 17(c) and

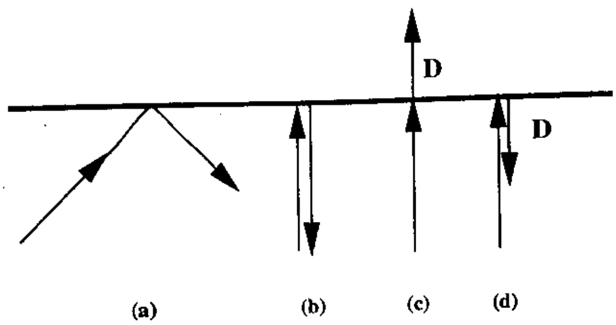

Figure 17

(d)). If there were some absorption of energy, the distance of the image to the wall would be smaller than that of the origin of the ball to the wall; with an increase of energy the distance would be greater. The symmetric bouncing of the ball is the simplest case. I introduce this idea because of the need to discuss the use of probability distributions.

When we use a probability distribution we may obtain values greater than a certain value designated as our upper limit. If we design a complex curve, each point on the curve will have a certain coordinate y; when this is calculated we will obtain the probability distribution as a function of time: $y=f(t)$ (figure 18). However, the amplitude of y may be very large and sometimes go beyond the acceptable limit. In a computer, we might use 16 bits to express our maximum amplitude, and the calculated value of y may exceed this. In order to keep the values inside the limit we have to do something, so we take this limit as an elastic one, and if a value exceeds it by a certain amount we take the symmetric point below the limit. It is like taking the magnitude of the overshoot reflected by the same length.

Such procedures can be applied with distributions other than the logistic, for example with exponential and hyperbolic functions, also using the elastic barriers. All of these produce complex sounds in a computer very easily. This contrasts with traditional sound synthesis which starts from pure sounds like the sine tone and superposes them to create richer

Figure 16

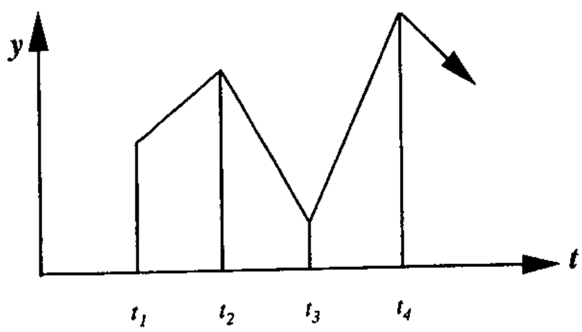

Figure 18

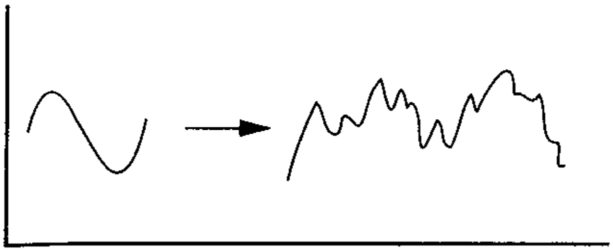

Figure 19

sounds, even up to the complexity of noise. Using the principle of harmonic analysis, it is possible to create a route from simplicity to higher complexity (figure 19). How might one reverse this progression? By starting from random walks produced by stochastic functions, and by injecting symmetries, regularities and periodicities, even up to the point of repeated waveforms.

I used such sounds in the Diatope called *Légende d'Ér*. In another piece I used the Cauchy distribution, which has its own characteristic features. I also created sounds with the macroscopic evolution produced by the combination of a random distribution with a symmetric function. These ideas were originally proposed in the last chapter of my book *Formalized Music*. At that time they were not proved – one might have bright ideas but if they do not work they are not bright after all!

So eventually I did demonstrate the possibility of going from complexity to simplicity in sound synthesis, of a new method of sound production not related to harmonic analysis. There is still much to be done in this domain; the results could be very interesting if it were further explored. This exploration need have nothing to do with the UPIC system; it is a problem of programming and research. However, I am convinced that for the discovery of yet-unknown sound qualities a higher sampling rate is necessary, as well as the use of more than 16 bits.

My pocket computer has a simple language in which there are only 87 commands. The aim of the program I use is to create a series of points in the pitch–time domain; this could produce a melodic pattern, for instance a kind of Brownian movement. For each point we need the pitch and the instant of its occurrence. The computer is too small to calculate more characteristics of the sound, so what the machine calculates each time is a couple of values.

Let us number the keys of a piano keyboard from 0 to 86, so that these numbers can represent the pitches of the sounds. For time values we take whatever approximation we need: crotchets, quavers, and so on, depending on what limits we choose.

In order to calculate the time we can use a special sub-program called EXPON, based on the exponential distribution. Let us say that we have two notes separated by a certain time interval t. The probability for the next t is given by the formula

$$(1) \qquad f(t) = \delta e^{-\delta_t} \, dt.$$

The time instants give just the attacks of the sounds; there is no duration, no intensity, no timbre. δ is the linear density of the attacks, i.e. the average number of points per time unit. For example, δ could be an average of one sound per second or one sound per half-second. In such a way we can calculate the t values.

The next step is to calculate the features of another function – say the Cauchy function. If we denote the pitch by x, the probability of x in the Cauchy distribution is given by the formula

$$(2) \qquad f(x) = \frac{\alpha}{\pi(\alpha^2 + x^2)},$$

where α is a scaling factor. This gives the values of t: t_1, t_2, and so on. Now we have to cumulate the t's by adding each time the calculated t value in order to get a progression. In this way we can arrive at the cumulation of the Cauchy distribution, the sigma of the x's. Why? Because this distribution gives negative and positive values ranging from minus infinity to plus infinity, while the t in the formula gives only values from zero to infinity, and the infinite values are very rare. When t becomes very large the likelihood of its occurrence predicted by the formula approaches zero.

In the Cauchy distribution there is no mean value, no curve representing the expected values, and since it contains values from minus infinity to plus infinity we need to put it within limits.

A problem arises: How can we construct any function of a probability distribution? We can solve it by using a theorem, which says that if we have a random number (uniform distribution) between 0 and 1 – let us call it y – and if we take the sum of all values between minus infinity and the point x of the probability distribution (i.e. if we take the integral of the sum of all probability values from minus infinity up to a point which we call $Q(x)$), then we can take x out of the equation, which simplifies matters greatly.

For instance, if we plot the Q function up to a point T, then $Q(T)$ is equal to this area, and we have y expressed as a function of t (figure 20):

$$(3) \qquad y = \int_0^T f(t) \, dt.$$

A similar method can be adopted for almost all probability distributions, and in this way the intervals or values of the probability may be generated. There is a special name for this exponential formula – the Poisson distribution.

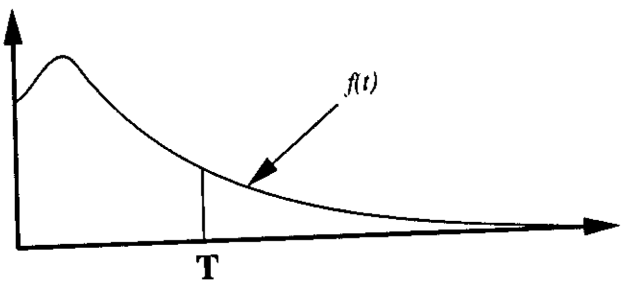

Figure 20

In physics a special physical model exists for this distribution. If we have a radioactive element and approach it with a Geiger tube, and convert the scintillation of the particles in the Geiger tube into sound, we have a special random rhythm of noises which follows exactly the Poisson law. As the 'Geiger counter' approaches the radioactive source, the density of the scintillation increases, and as it is taken away the average density decreases. The average density of noise clicks heard corresponds at all times with the parameters set by the Poisson distribution.

Instead of having a radioactive element in one's pocket, one can simulate the probability distributions with the computer, using the equations given above. We need to choose a random number y between zero and one and insert it into the formula

$$(4) \qquad y = \int_0^T \delta e^{-\delta_t} \, dt,$$

then do the cumulation, and end up with result T.

We start with a generator of random numbers such as a computer, although this may seem paradoxical, the computer being a deterministic machine. The formula used to do this is

$$(5) \qquad x_i = (x_{i-1} a + c) \bmod M.$$

We obtain a number x_i from a preceding result x_{i-1} by multiplying it by a factor a plus another term c, modulo another given number M. (What is a modulo? It is very simple: suppose we have the number 73 modulo 7. Divide 73 by 7 and the remainder is 3: so 73 modulo 7 = 3.)

If we take the number M we will have all the possible numbers x_i between zero and M, and if we divide this number by M we have a number between zero and one, which leads us to the Q function in both distributions; we obtain each time a 'gun' – like the radioactive gun – which tells us the value of T in every instance. Since the Cauchy distribution can include values from minus infinity to plus infinity, we have to specify limits, to reflect back into our numerical space the values which are outside the specified limits. As the outcome of the whole program we obtain pairs of numbers, for instance (1, 39), which specify the time and the corresponding pitch.

The data that we have to enter into the formula are the parameters of the probability distributions: M, α, δ, a, c; I use a value of $M = 2^{32}$. Of course we need to know in advance what range is required, what the density of events is, and so on. The placement of the limits depends on the nature of the music intended; for example, if one is writing for violin, the limits should be adapted to the violin range.

I have referred many times to 'domains'; while it is possible to transform one domain according to the laws of another, this can never be done without penalty. There are many things specific to one domain which do not occur in other domains. Many mistakes result from this, especially in computer music. The ideas should stem from problems in one's own domain, despite the fact that there are chances of connection with other domains. One has to be sure that one uses tools in a way appropriate to the field in question.

I shall give an example, but not from music. For many years the Poisson distribution constituted a mathematical problem, and only at the beginning of this century did a retired Prussian General named Bortkiewicz prove that Poisson's law actually describes events from physical reality. Bortkiewicz counted the occurrence of events without causality – the death of soldiers looking after horses in peacetime – over a period of ten years. He proved that the frequency of deaths followed the Poisson distribution.

There are many examples of similar happenings in the history of science. For instance, non-Euclidean geometries were thought of in mathematics as bizarre and abstract, until after Einstein they became a reality. This involved a transfer from one totally abstract domain into another physical and astrophysical domain.

Here is an example from music: musical notation was invented a long time ago when Guido d'Arezzo proposed staves for writing down pitches, with the direction from left to right corresponding to time, as in writing. This connection of two dimensions, i.e. pitch and time forming a 'space', was achieved long before analytical geometry, which came centuries later.

Again, in the fine arts, we can appreciate that mosaics, which produce shapes – visible, understandable, recognisable patterns – by using stones of different colours, represent a kind of statistical approach. This involves the issue of pattern recognition so popular in contemporary psychology. Similar issues emerged in the pointillistic school of French painting at the end of the nineteenth century; these issues are close to those of ancient mosaics in a way, but freer in style.

The ideas that emerge in fine art somehow overlap the same ideas in science or psychology. So it is possible to use in one domain something drawn from another, more advanced domain, providing it has its own necessity.

Another example in music is the group structure. Musicians discovered and employed such ideas much earlier than mathematicians dealt with group theory. Mathematicians like Klein and Galois worked in the nineteenth century, while musicians used inverted forms of melodies in the fifteenth century. If the musicians were more theoretically inclined they would have discovered group theory themselves! There are many examples of group thinking in music, not only the four forms of a melodic pattern.

In my book *Formalized Music* I tried to show that if musicians were aware of the problems in their own domain, we would have a 'kinetic music theory' analogous to the 'kinetic gas theory' in physics. The basic principles of kinetic gas theory, which are described by statistical mechanics, are very simple and very general. They can be found in music as well.

The same can be said about symmetry, an important principle in music. So if one follows a similar thread of thought from another domain, one arrives at the same results but from a musical point of view.

12. CONCLUDING REMARKS

In the fifth century BC the Greek philosopher Parmenides wrote, 'To think and to be is the same'. This was modified by later philosophers. Descartes said, 'Cogito ergo sum' – 'I think therefore I am', and Berkeley stated that there is no objective world, because there is no proof of its existence.

In Parmenides' statement we have the equation: thought = reality. In the idealistic version there is an arrow: thought → reality. In materialism, which says that thought is a reflection of reality, the arrow is reversed: thought ← reality.

My paraphrase of the verse of Parmenides would be: 'To be and not to be is the same'. I first expressed this thought in an article in 1958, published in the *Gravesaner Blätter*, No. 11. There I continued:

> Ontology. In a Universe of emptiness. A brief train of waves, of which end and beginning coincide. Time is triggered, again and again. The Nothingness resolves, creates. It is the generator of being, of time, of causality.

Fifteen years later I found the same idea being expressed in the domain of astrophysics. It was speculated that the entire universe evolved from literally nothing. Yet my thoughts came out of purely musical considerations.

These issues are interesting for musicians, whether composers, interpreters or listeners, because they represent fundamental questions for music. The problem of nothingness is identical to the problem of originality. A composer should be original, should create his music uninfluenced by the past. In a way he should act as the whole universe does: Nothingness creating ....