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XENAKIS

FORMALIZED MUSIC

PENDRAGON

FORMALIZED MUSIC

Thought and Mathematics in Music



HARMONOLOGIA SERIES No. 6

Formalized Music

Formalized Music

THOUGHT AND MATHEMATICS IN COMPOSITION

Revised Edition

Iannis Xenakis

Additional material compiled and edited
by Sharon Kanach

HARMONOLOGIA SERIES No. 6

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Preface

The formalization that I attempted in trying to reconstruct part of the musical edifice *ex nihilo* has not used, for want of time or of capacity, the most advanced aspects of philosophical and scientific thought. But the escalade is started and others will certainly enlarge and extend the new thesis. This book is addressed to a hybrid public, but interdisciplinary hybridization frequently produces superb specimens.

I could sum up twenty years of personal efforts by the progressive filling in of the following Table of Coherences. My musical, architectural, and visual works are the chips of this mosaic. It is like a net whose variable lattices capture fugitive virtualities and entwine them in a multitude of ways. This table, in fact, sums up the true coherences of the successive chronological chapters of this book. The chapters stemmed from monographs, which tried as much as possible to avoid overlapping.

But the profound lesson of such a table of coherences is that any theory or solution given on one level can be assigned to the solution of problems on another level. Thus the solutions in macrocomposition on the Families level (programmed stochastic mechanisms) can engender simpler and more powerful new perspectives in the shaping of microsounds than the usual trigonometric (periodic) functions can. Therefore, in considering clouds of points and their distribution over a pressure-time plane, we can bypass the heavy harmonic analyses and syntheses and create sounds that have never before existed. Only then will sound synthesis by computers and digital-to-analogue converters find its true position, free of the rooted but ineffectual tradition of electronic, concrete, and instrumental music that makes use of Fourier synthesis despite the failure of this theory. Hence, in this book, questions having to do mainly with orchestral sounds (which are more diversified and more manageable) find a rich and immediate application as soon as they are transferred to the Microsound level in the pressure-time space. All music is thus automatically homogenized and unified.

“Everything is everywhere” is the word of this book and its Table of Coherences; Herakleitos would say that the ways up and down are one.

The French edition, *Musiques Formelles*, was produced thanks to Albert Richard, director of *La Revue Musicale*. The English edition, a corrected and completed version, results from the initiative of Mr. Christopher Butchers, who translated the first six chapters. My thanks also go to Mr. G. W. Hopkins, and Mr. and Mrs. John Challifour, who translated Chapters VII and VIII, respectively; to Mr. Michael Aronson and Mr. Bernard Perry of Indiana University Press, who decided to publish it; and finally to Mrs. Natalie Wrubel, who edited this difficult book with infinite patience, correcting and rephrasing many obscure passages.

I. X.

1970

TABLE (MOSAIC) OF COHERENCES

Philosophy (in the etymological sense)

Thrust towards truth, revelation. Quest in everything, interrogation, harsh criticism, active knowledge through creativity.

Chapters (in the sense of the methods followed)

Partially inferential and experimental

Entirely inferential and experimental

Other methods
to come
?

ARTS (VISUAL, SONIC, MIXED . . .)

SCIENCES (OF MAN, NATURAL)

?

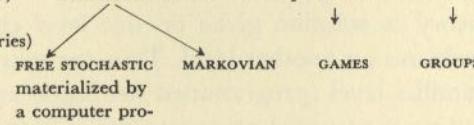
PHYSICS, MATHEMATICS, LOGIC

This is why the arts are freer, and can therefore guide the sciences, which are entirely inferential and experimental.

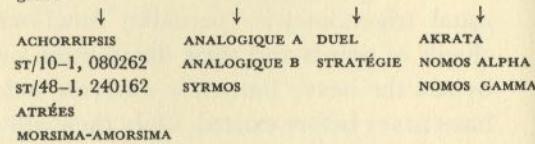
Categories of Questions (fragmentation of the directions leading to creative knowledge, to philosophy)

REALITY (EXISTENTIALITY); CAUSALITY; INFERENCE: CONNEXITY; COMPACTNESS; TEMPORAL AND SPATIAL UBIQUITY
AS A CONSEQUENCE OF NEW MENTAL STRUCTURES; INDETERMINISM . . . ← bi-pole → . . . DETERMINISM; . . .

Families of Solutions or Procedures (of the above categories)



Pieces (examples of particular realization)



Classes of Sonic Elements (sounds that are heard and recognized as a whole, and classified with respect to their sources)

ORCHESTRAL, ELECTRONIC (produced by analogue devices), CONCRETE (microphone collected), DIGITAL (realized with computers and digital-to-analogue converters), . . .

Microsounds

Forms and structures in the pressure-time space, recognition of the classes to which microsounds belong or which microstructures produce.

Microsound types result from questions and solutions that were adopted at the CATEGORIES, FAMILIES, and PIECES levels.

Preface to *Musiques Formelles*

This book is a collection of explorations in musical composition pursued in several directions. The effort to reduce certain sound sensations, to understand their logical causes, to dominate them, and then to use them in wanted constructions; the effort to materialize movements of thought through sounds, then to test them in compositions; the effort to understand better the pieces of the past, by searching for an underlying unit which would be identical with that of the scientific thought of our time; the effort to make “art” while “geometrizing,” that is, by giving it a reasoned support less perishable than the impulse of the moment, and hence more serious, more worthy of the fierce fight which the human intelligence wages in all the other domains—all these efforts have led to a sort of abstraction and formalization of the musical compositional act. This abstraction and formalization has found, as have so many other sciences, an unexpected and, I think, fertile support in certain areas of mathematics. It is not so much the inevitable use of mathematics that characterizes the attitude of these experiments, as the overriding need to consider sound and music as a vast potential reservoir in which a knowledge of the laws of thought and the structured creations of thought may find a completely new medium of materialization, i.e., of communication.

For this purpose the qualification “beautiful” or “ugly” makes no sense for sound, nor for the music that derives from it; the quantity of intelligence carried by the sounds must be the true criterion of the validity of a particular music.

This does not prevent the utilization of sounds defined as pleasant or beautiful according to the fashion of the moment, nor even their study in their own right, which may enrich symbolization and algebration. *Efficacy* is in itself a sign of intelligence. We are so convinced of the historical necessity of this step, that we should like to see the visual arts take an

analogous path—unless, that is, “artists” of a new type have not already done it in laboratories, sheltered from noisy publicity.

These studies have always been matched by actual works which mark out the various stages. My compositions constitute the experimental dossier of this undertaking. In the beginning my compositions and research were recognized and published, thanks to the friendship and moral and material support of Prof. Hermann Scherchen. Certain chapters in the present work reflect the results of the teaching of certain masters, such as H. Scherchen and Olivier Messiaen in music, and Prof. G. Th. Guilbaud in mathematics, who, through the virtuosity and liberality of his thought, has given me a clearer view of the algebras which constitute the fabric of the chapter devoted to Symbolic Music.

I. X.

1962

Preface to the Pendragon Edition

Here is a new expanded edition of *Formalized Music*. It invites two fundamental questions:

Have the theoretical propositions which I have made over the past thirty-five years

- a) survived in my music ?
- b) been aesthetically efficient ?

To the first question, I will answer a general “yes.” The theories which I have presented in the various chapters preceding this new edition have always been present in my music, even if some theories have been mingled with others in a same work. The exploration of the conceptual and sound world in which I have been involved necessitated an harmonious or even conflicting synthesis of earlier theses. It necessitated a more global architectural view than a mere comparative confrontation of the various procedures. But the supreme criterion always remained the validation, the aesthetic efficiency of the music which resulted.

Naturally, it was up to me and to me alone to determine the aesthetic criteria, consciously or not, in virtue of the first principle which one can not get around. The artist (man) has the duty *and* the privilege to decide, radically alone, his choices and the value of the results. By no means should he choose any other means; those of power, glory, money, ...

Each time, he must throw himself and his chosen criteria into question all while striving to start from scratch yet not forget. We should not “monkey” ourselves by virtue of the habits we so easily acquire due to our own “echolalic” properties. But to be reborn at each and every instant, like a child with a new and “independent” view of things.

All of this is part of a second principle: It is absolutely necessary to free oneself, as much as possible, from any and all contingencies.

This may be considered man's destiny in particular, and the universe's in general. Indeed, the Being's constant dislocations, be they continuous or not, deterministic or chaotic (or both simultaneously) are manifestations of the vital and incessant drive towards change, towards freedom without return.

An artist can not remain isolated in the universal ocean of forms and their changes. His interest lies in embracing the most vast horizon of knowledge and problematics, all in accordance with the two principles presented above. From hence comes the new chapter in this edition entitled "Concerning Time, Space and Music."

Finally, to finish with the first question, I have all along continued to develop certain theses and to open up some new ones. The new chapter on "Sieves" is an example of this along with the computer program presented in Appendix III which represents a long aesthetic and theoretical search. This research was developed as well as its application in sound synthesis on UPIC.*

Another approach to the mystery of sounds is the use of cellular automata which I have employed in several instrumental compositions these past few years. This can be explained by an observation which I made: scales of pitch (sieves) automatically establish a kind of global musical style, a sort of macroscopic "synthesis" of musical works, much like a "spectrum of frequencies, or iterations," of the physics of particles. Internal symmetries or their dissymmetries are the reason behind this. Therefore, through a discerning logico-aesthetic choice of "non-octave" scales, we can obtain very rich simultaneities (chords) or linear successions which revive and generalize tonal, modal or serial aspects. It is on this basis of sieves that cellular automata can be useful in harmonic progressions which create new and rich timbric fusions with orchestral instruments. Examples of this can be found in works of mine such as *Ata*, *Horos*, etc.

Today, there is a whole new field of investigation called "Experimental Mathematics," that gives fascinating insights especially in automatic dynamic systems, by the use of math and computer graphics. Thus, many structures such as the already-mentioned cellular automata or those which possess self-

*UPIC—Unité Polygogique Informatique du CEMAMu. A sort of musical drawing board which, through the digitalization of a drawing, enables one to compose music, teach acoustics, engage in musical pedagogy at any age. This machine was developed at the Centre d'Etudes de Mathématiques et Automatiques Musicales de Paris.

similarities such as Julia or Mandelbrot sets, are studied and visualized. These studies lead one right into the frontiers of determinism and indeterminism. Chaos to symmetry and the reverse orientation are once again being studied and are even quite fashionable! They open up new horizons, although for me, the results are novel aspects of the equivalent compositional problems I started dealing with about thirty-five years ago. The theses presented in the earlier editions of this book bear witness to this fact although the dynamic of musical works depends on several levels simultaneously and not only on the calculus level.

An important task of the research program at CEMAMu is to develop synthesis through quantified sounds but with up-to-date tools capable of involving autosimilitudes, symmetries or deterministic chaos, or stochastics within a dynamic evolution of amplitude frequency frames where each pixel corresponds to a sound quantum or "phonon," as already imagined by Einstein in the 1910s. This research, which I started in 1958 and wrongly attributed to Gabor, can now be pursued with much more powerful and modern means. Some surprises can be expected!

In Appendix IV of this edition, a new, more precise formulation of stochastic sound synthesis can be found as a follow-up of the last chapter of the preceding edition of Formalized Music (presented here as Chapter IX). In the interim, this approach has been tested and used in my work *La Légende d'Eer* for seven-track tape. This approach was developed at the CEMAMu in Paris and worked out at the WDR, the West-German National Radio studio in Cologne. This work was part of the Diatope which was installed for the inauguration of the Pompidou/Beaubourg Center in Paris. The event was entirely automated with a complete laser installation and 1600 electronic flashes. This synthesis is part of CEMAMu's permanent research program.

In this same spirit, random walks or Brownian movements have been the basis for several of my works, especially instrumental pieces such as *N'Shima*, which means "breath" or "spirit" in Hebrew; for 2 female voices, 2 French Horns, 2 trombones and 1 'cello. This piece was written at the request of Recha Freier, founder of the "Aliya movement" and premiered at the Testimonium Festival in Jerusalem.

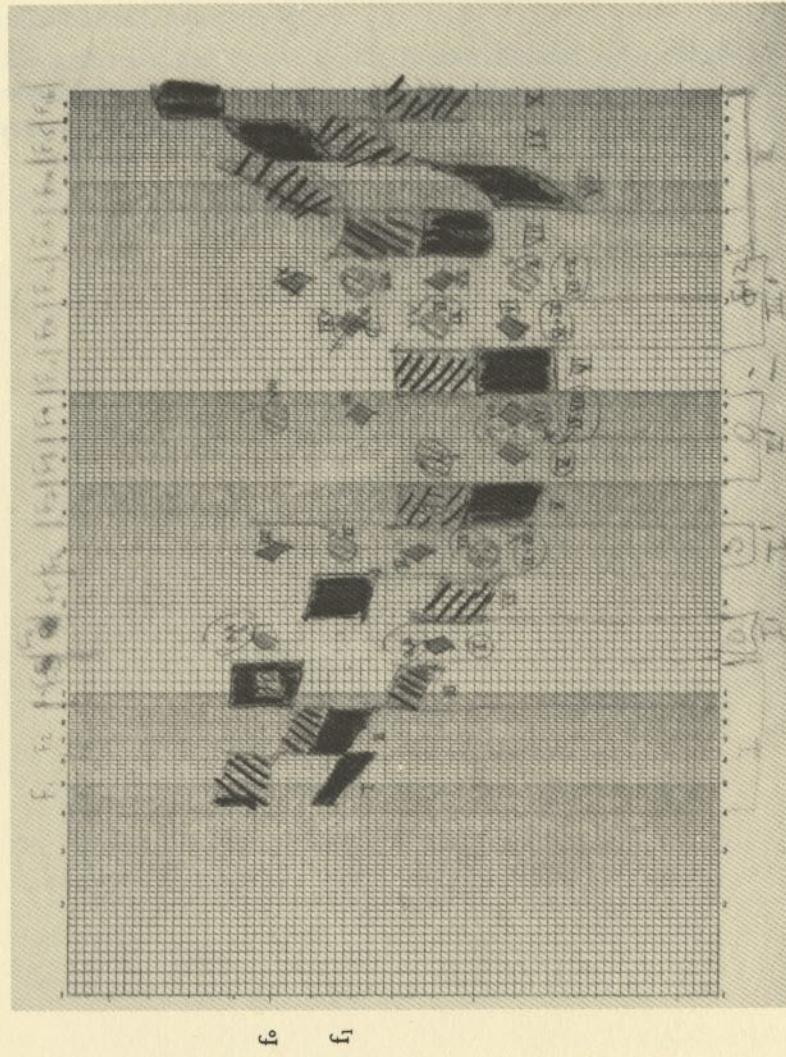
The answer to the second question posed at the beginning of this Preface is not up to me. In absolute terms, the artisan musician (not to say creator) must remain doubtful of the decisions he has made, doubtful, however subtly, of the result. The percentage of doubt should not exist in virtue of the principles elaborated above. But in relative terms, the public, or connoisseurs (either synchronic or diachronic), alone decide upon a work's

efficiency. However, any culture's validation follows "seasonal" rules, varying between periods of a few years to centuries or even millennia. We must never forget the nearly-total lack of consideration Egyptian art suffered for over 2000 years, or Meso-American art.

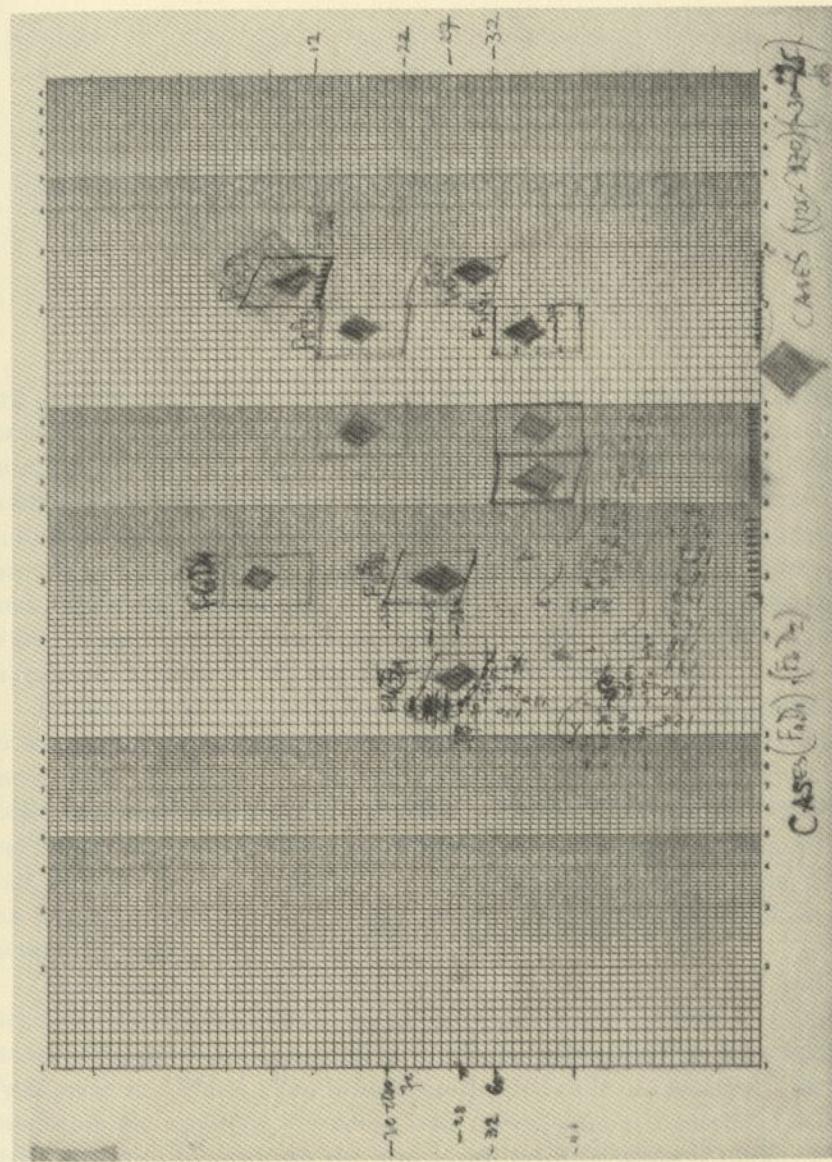
One can assimilate a work of art, or, let us say, just a work, to the information we can put on a document, seal in a bottle which we will throw into the middle of the ocean. Will it ever be found? When and by whom and how will it be read, interpreted?

My gratitude and thanks go to Sharon Kanach, who translated and supervised the new material in this updated edition of *Formalized Music* and to Robert Kessler, the courageous publisher.

Formalized Music



Preliminary sketch *Analogique B* (1959). See Chapter III, pp. 103-09.



Preliminary sketch *Analogue B* (1959). See Chapter III, pp. 103-09.

Free Stochastic Music

Art, and above all, music has a fundamental function, which is to catalyze the sublimation that it can bring about through all means of expression. It must aim through fixations which are landmarks to draw towards a total exaltation in which the individual mingles, losing his consciousness in a truth immediate, rare, enormous, and perfect. If a work of art succeeds in this undertaking even for a single moment, it attains its goal. This tremendous truth is not made of objects, emotions, or sensations; it is beyond these, as Beethoven's Seventh Symphony is beyond music. This is why art can lead to realms that religion still occupies for some people.

But this transmutation of every-day artistic material which transforms trivial products into meta-art is a secret. The "possessed" reach it without knowing its "mechanisms." The others struggle in the ideological and technical mainstream of their epoch which constitutes the perishable "climate" and the stylistic fashion. Keeping our eyes fixed on this supreme meta-artistic goal, we shall attempt to define in a more modest manner the paths which can lead to it from our point of departure, which is the magma of contradictions in present music.

There exists a historical parallel between European music and the successive attempts to explain the world by reason. The music of antiquity, causal and deterministic, was already strongly influenced by the schools of Pythagoras and Plato. Plato insisted on the principle of causality, "for it is impossible for anything, to come into being without cause" (*Timaeus*). Strict causality lasted until the nineteenth century when it underwent a

The English translation of Chaps. I–VI is by Christopher A. Butchers.

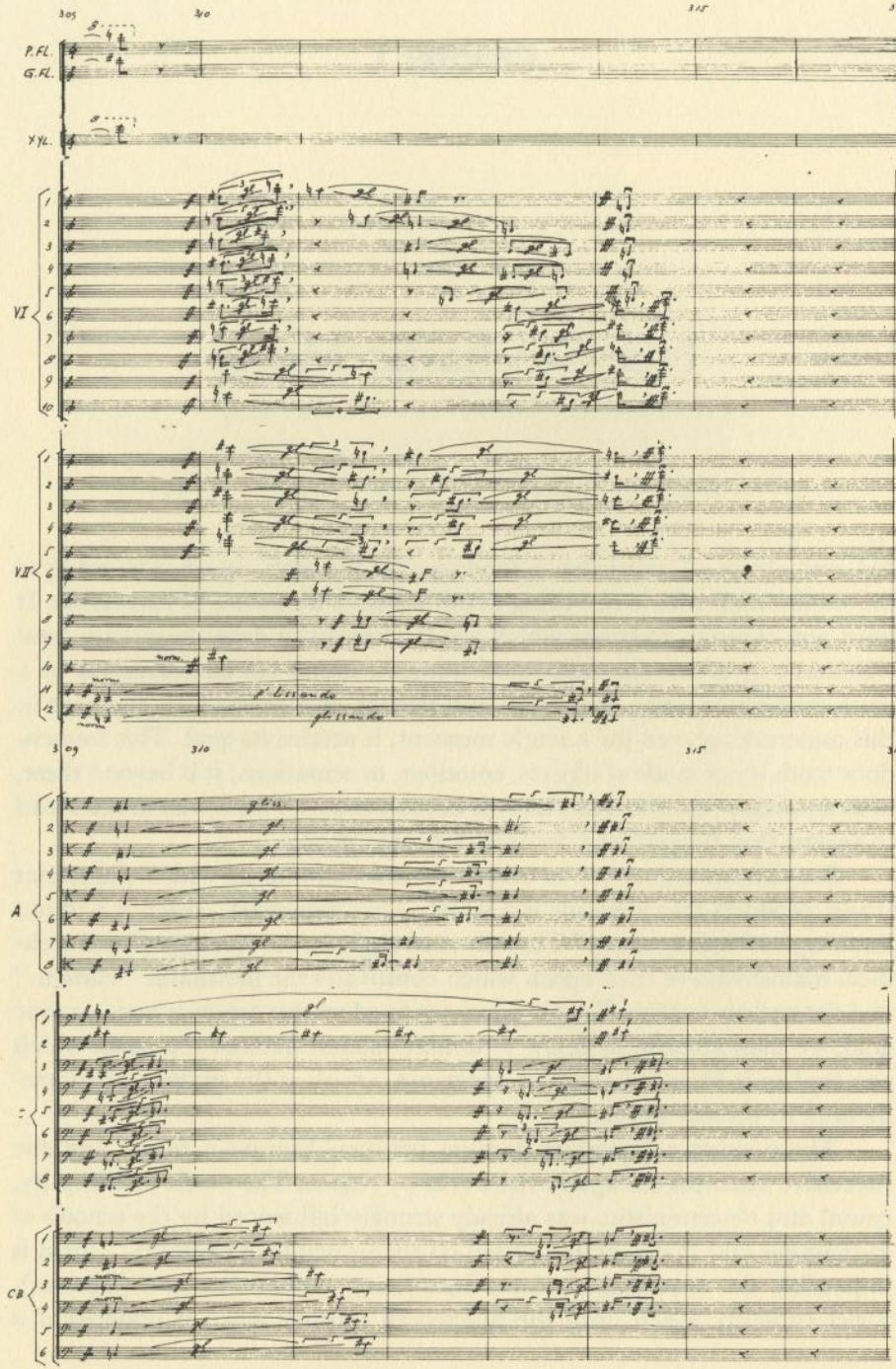


Fig. I-1. Score of *Metastasis*, 1953/54, Bars 309–17

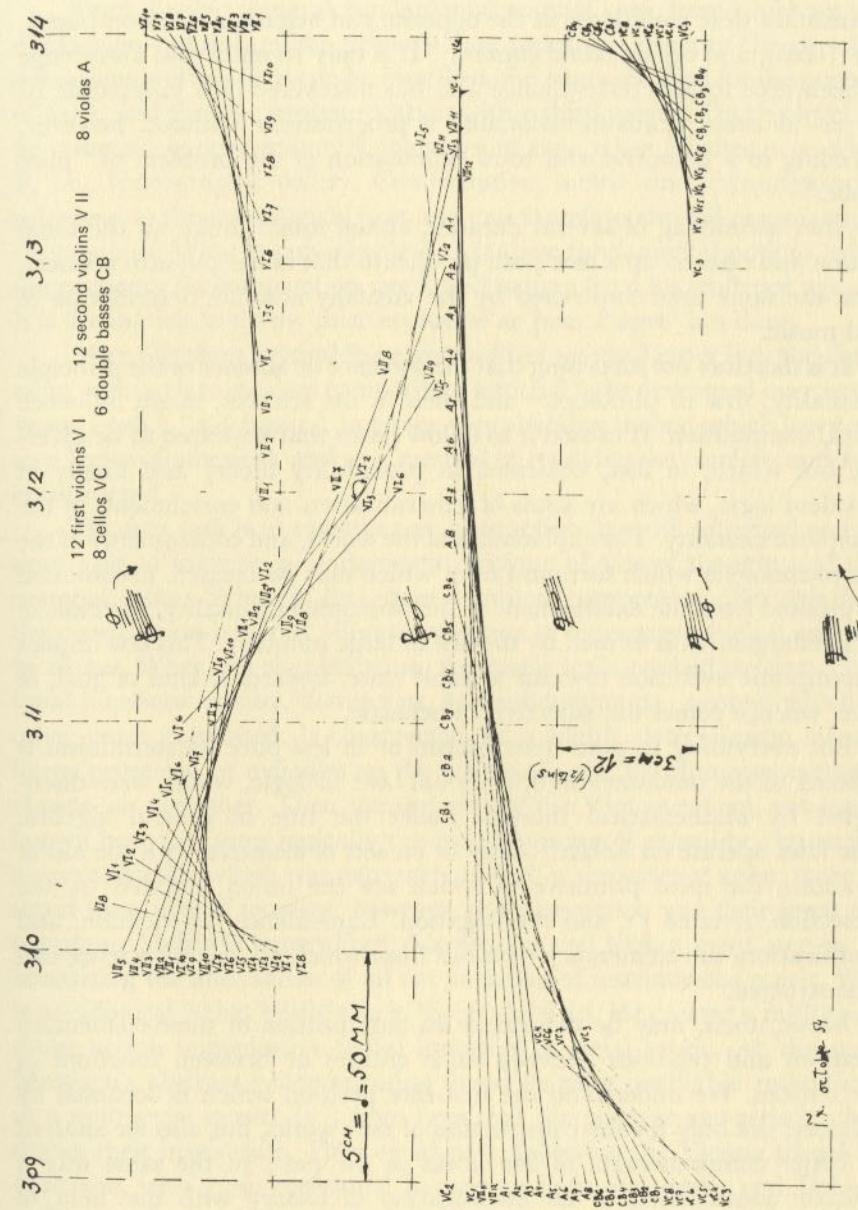


Fig. I-2. String Glissandi, Bars 309–14 of *Metastasis*

brutal and fertile transformation as a result of statistical theories in physics. Since antiquity the concepts of chance (*tyche*), disorder (*ataxia*), and disorganization were considered as the opposite and negation of reason (*logos*), order (*taxis*), and organization (*systasis*). It is only recently that knowledge has been able to penetrate chance and has discovered how to separate its degrees—in other words to rationalize it progressively, without, however, succeeding in a definitive and total explanation of the problem of “pure chance.”

After a time lag of several decades, atonal music broke up the tonal function and opened up a new path parallel to that of the physical sciences, but at the same time constricted by the virtually absolute determinism of serial music.

It is therefore not surprising that the presence or absence of the principle of causality, first in philosophy and then in the sciences, might influence musical composition. It caused it to follow paths that appeared to be divergent, but which, in fact, coalesced in probability theory and finally in polyvalent logic, which are kinds of generalization and enrichments of the principle of causality. The explanation of the world, and consequently of the sonic phenomena which surround us or which may be created, necessitated and profited from the enlargement of the principle of causality, the basis of which enlargement is formed by the law of large numbers. This law implies an asymptotic evolution towards a stable state, towards a kind of goal, of *stochos*, whence comes the adjective “stochastic.”

But everything in pure determinism or in less pure indeterminism is subjected to the fundamental operational laws of logic, which were disentangled by mathematical thought under the title of general algebra. These laws operate on isolated states or on sets of elements with the aid of operations, the most primitive of which are the union, notated \cup , the intersection, notated \cap , and the negation. Equivalence, implication, and quantifications are elementary relations from which all current science can be constructed.

Music, then, may be defined as an organization of these elementary operations and relations between sonic entities or between functions of sonic entities. We understand the first-rate position which is occupied by set theory, not only for the construction of new works, but also for analysis and better comprehension of the works of the past. In the same way a stochastic construction or an investigation of history with the help of stochastics cannot be carried through without the help of logic—the queen of the sciences, and I would even venture to suggest, of the arts—or its mathematical form algebra. For everything that is said here on the subject

is also valid for all forms of art (painting, sculpture, architecture, films, etc.).

From this very general, fundamental point of view, from which we wish to examine and *make* music, primary time appears as a wax or clay on which operations and relations can be inscribed and engraved, first for the purposes of work, and then for communication with a third person. On this level, the asymmetric, noncommutative character of time is use (B after $A \neq A$ after B , i.e., lexicographic order). Commutative, metric time (symmetrical) is subjected to the same logical laws and can therefore also aid organizational speculations. What is remarkable is that these fundamental notions, which are necessary for construction, are found in man from his tenderest age, and it is fascinating to follow their evolution as Jean Piaget¹ has done.

After this short preamble on generalities we shall enter into the details of an approach to musical composition which I have developed over several years. I call it “stochastic,” in honor of probability theory, which has served as a logical framework and as a method of resolving the conflicts and knots encountered.

The first task is to construct an abstraction from all inherited conventions and to exercise a fundamental critique of acts of thought and their materialization. What, in fact, does a musical composition offer strictly on the construction level? It offers a collection of sequences which it wishes to be causal. When, for simplification, the major scale implied its hierarchy of tonal functions—tonics, dominants, and subdominants—around which the other notes gravitated, it constructed, in a highly deterministic manner, linear processes, or melodies on the one hand, and simultaneous events, or chords, on the other. Then the serialists of the Vienna school, not having known how to master logically the indeterminism of atonality, returned to an organization which was extremely causal in the strictest sense, more abstract than that of tonality; however, this abstraction was their great contribution. Messiaen generalized this process and took a great step in systematizing the abstraction of all the variables of instrumental music. What is paradoxical is that he did this in the modal field. He created a multimodal music which immediately found imitators in serial music. At the outset Messiaen’s abstract systematization found its most justifiable embodiment in a multiserial music. It is from here that the postwar neo-serialists have drawn their inspiration. They could now, following the Vienna school and Messiaen, with some occasional borrowing from Stravinsky and Debussy, walk on with ears shut and proclaim a truth greater than the others. Other movements were growing stronger; chief among them was the systematic exploration of sonic entities, new instruments, and “noises.” Varèse was the

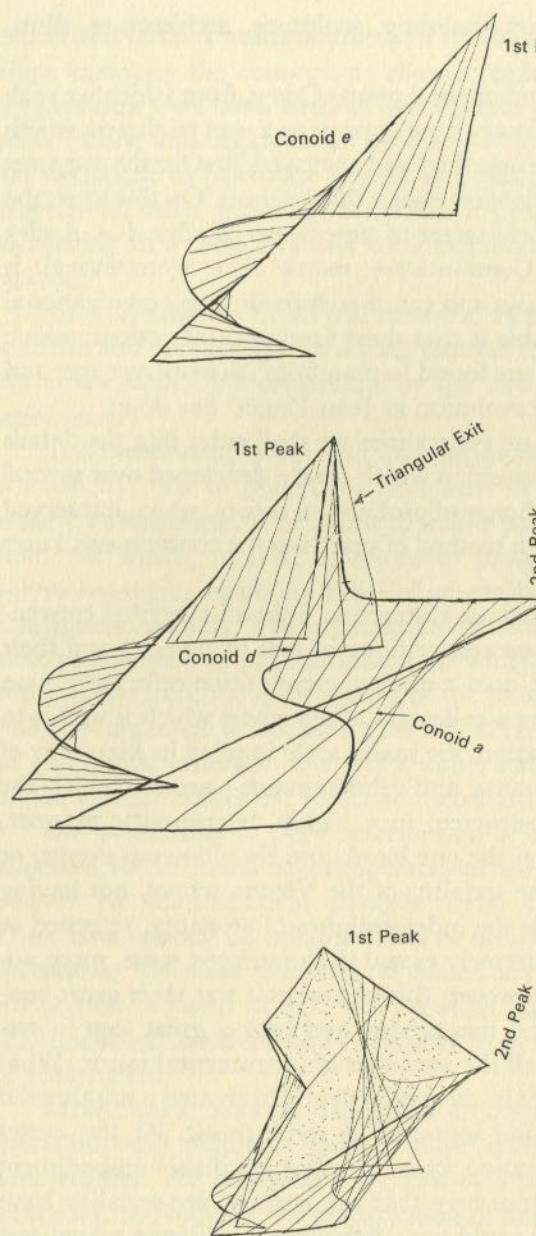


Fig. I-3. Stages in the Development of the First Design of the Philips Pavilion

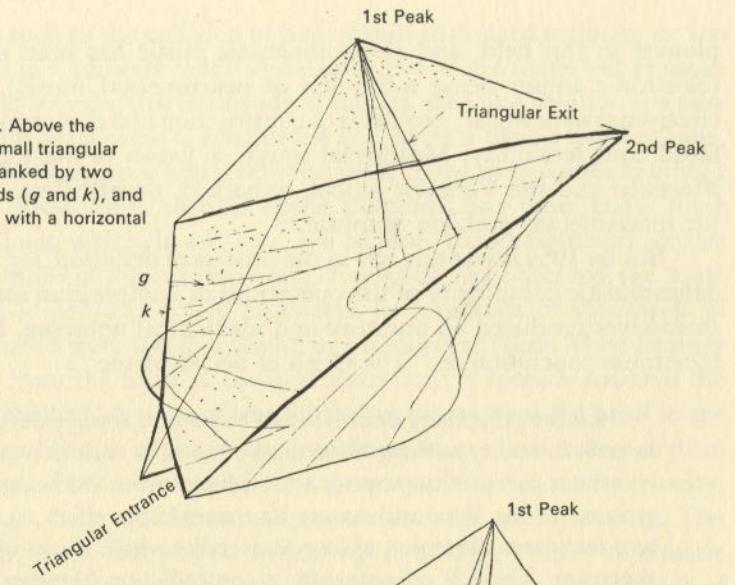
A. Ground profile of the *left half* of the "stomach." The intention was to build a shell, composed of as few ruled surfaces as possible, over the ground plan. A conoid (*e*) is constructed through the ground profile curve; this wall is bounded by two straight directrixes: the straight directrix (rising from the left extremity of the ground profile), and the outermost generatrix (passing through the right extremity of the ground profile). This produces the first "peak" of the pavilion.

B. A ruled surface consisting of two conoids, *a* and *d*, is laid through the curve bounding the *right half* of the "stomach." The straight directrix of *d* passes through the first peak, and the outermost generatrix at this side forms a triangular exit with the generatrix of *e*. The straight directrix of *a* passes through a second peak and is joined by an arc to the directrix of *d*.

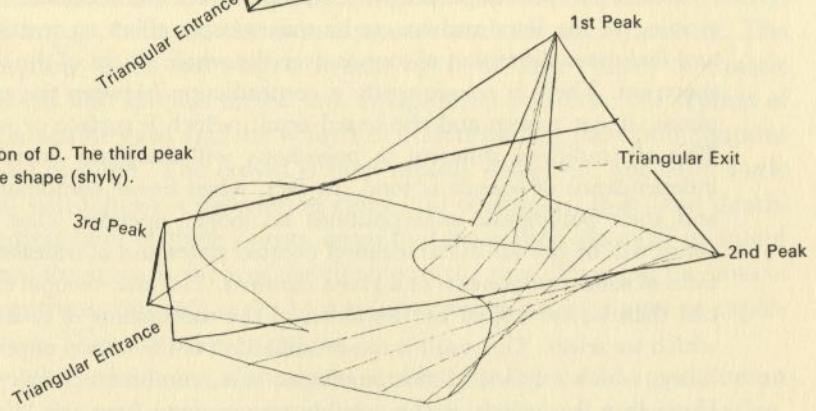
This basic form is the one used in the first design and was retained, with some modifications, in the final structure. The main problem of the design was to establish an aesthetic balance between the two peaks.

C. Attempt to close the space between the two ruled surfaces of the first design by flat surfaces (which might serve as projection walls).

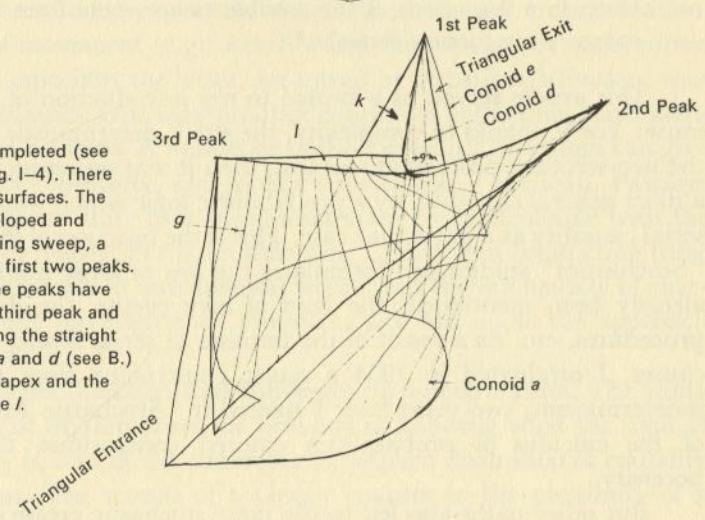
D. Another attempt. Above the entrance channel a small triangular opening is formed, flanked by two hyperbolic paraboloids (*g* and *k*), and the whole is covered with a horizontal top surface.



E. Elaboration of D. The third peak begins to take shape (shyly).



F. The first design completed (see also the first model, Fig. I-4). There are no longer any flat surfaces. The third peak is fully developed and creates, with its opposing sweep, a counterbalance for the first two peaks. The heights of the three peaks have been established. The third peak and the small arc connecting the straight directrices of conoids *a* and *d* (see B) form, respectively, the apex and the base of a part of a cone *l*.



pioneer in this field, and electromagnetic music has been the beneficiary (electronic music being a branch of instrumental music). However, in electromagnetic music, problems of construction and of morphology were not faced conscientiously. Multiserial music, a fusion of the multimodality of Messiaen and the Viennese school, remained, nevertheless, at the heart of the fundamental problem of music.

But by 1954 it was already in the process of deflation, for the completely deterministic complexity of the operations of composition and of the works themselves produced an auditory and ideological nonsense. I described the inevitable conclusion in "The Crisis of Serial Music":

Linear polyphony destroys itself by its very complexity; what one hears is in reality nothing but a mass of notes in various registers. The enormous complexity prevents the audience from following the intertwining of the lines and has as its macroscopic effect an irrational and fortuitous dispersion of sounds over the whole extent of the sonic spectrum. There is consequently a contradiction between the polyphonic linear system and the heard result, which is surface or mass. This contradiction inherent in polyphony will disappear when the independence of sounds is total. In fact, when linear combinations and their polyphonic superpositions no longer operate, what will count will be the statistical mean of isolated states and of transformations of sonic components at a given moment. The macroscopic effect can then be controlled by the mean of the movements of elements which we select. The result is the introduction of the notion of probability, which implies, in this particular case, combinatory calculus. Here, in a few words, is the possible escape route from the "linear category" in musical thought.²

This article served as a bridge to my introduction of mathematics in music. For if, thanks to complexity, the strict, deterministic causality which the neo-serialists postulated was lost, then it was necessary to replace it by a more general causality, by a probabilistic logic which would contain strict serial causality as a particular case. This is the function of stochastic science. "Stochastics" studies and formulates the law of large numbers, which has already been mentioned, the laws of rare events, the different aleatory procedures, etc. As a result of the impasse in serial music, as well as other causes, I originated in 1954 a music constructed from the principle of indeterminism; two years later I named it "Stochastic Music." The laws of the calculus of probabilities entered composition through musical necessity.

But other paths also led to the same stochastic crossroads—first of all,

natural events such as the collision of hail or rain with hard surfaces, or the song of cicadas in a summer field. These sonic events are made out of thousands of isolated sounds; this multitude of sounds, seen as a totality, is a new sonic event. This mass event is articulated and forms a plastic mold of time, which itself follows aleatory and stochastic laws. If one then wishes to form a large mass of point-notes, such as string pizzicati, one must know these mathematical laws, which, in any case, are no more than a tight and concise expression of chain of logical reasoning. Everyone has observed the sonic phenomena of a political crowd of dozens or hundreds of thousands of people. The human river shouts a slogan in a uniform rhythm. Then another slogan springs from the head of the demonstration; it spreads towards the tail, replacing the first. A wave of transition thus passes from the head to the tail. The clamor fills the city, and the inhibiting force of voice and rhythm reaches a climax. It is an event of great power and beauty in its ferocity. Then the impact between the demonstrators and the enemy occurs. The perfect rhythm of the last slogan breaks up in a huge cluster of chaotic shouts, which also spreads to the tail. Imagine, in addition, the reports of dozens of machine guns and the whistle of bullets adding their punctuations to this total disorder. The crowd is then rapidly dispersed, and after sonic and visual hell follows a detonating calm, full of despair, dust, and death. The statistical laws of these events, separated from their political or moral context, are the same as those of the cicadas or the rain. They are the laws of the passage from complete order to total disorder in a continuous or explosive manner. They are stochastic laws.

Here we touch on one of the great problems that have haunted human intelligence since antiquity: continuous or discontinuous transformation. The sophisms of movement (e.g., Achilles and the tortoise) or of definition (e.g., baldness), especially the latter, are solved by statistical definition; that is to say, by stochastics. One may produce continuity with either continuous or discontinuous elements. A multitude of short glissandi on strings can give the impression of continuity, and so can a multitude of pizzicati. Passages from a discontinuous state to a continuous state are controllable with the aid of probability theory. For some time now I have been conducting these fascinating experiments in instrumental works; but the mathematical character of this music has frightened musicians and has made the approach especially difficult.

Here is another direction that converges on indeterminism. The study of the variation of rhythm poses the problem of knowing what the limit of total asymmetry is, and of the consequent complete disruption of causality among durations. The sounds of a Geiger counter in the proximity of a

radioactive source give an impressive idea of this. Stochastics provides the necessary laws.

Before ending this short inspection tour of events rich in the new logic, which were closed to the understanding until recently, I would like to include a short parenthesis. If glissandi are long and sufficiently interlaced, we obtain sonic spaces of continuous evolution. It is possible to produce ruled surfaces by drawing the glissandi as straight lines. I performed this experiment with *Metastasis* (this work had its premiere in 1955 at Donaueschingen). Several years later, when the architect Le Corbusier, whose collaborator I was, asked me to suggest a design for the architecture of the Philips Pavilion in Brussels, my inspiration was pin-pointed by the experiment with *Metastasis*. Thus I believe that on this occasion music and architecture found an intimate connection.³ Figs. I-1-5 indicate the causal chain of ideas which led me to formulate the architecture of the Philips Pavilion from the score of *Metastasis*.

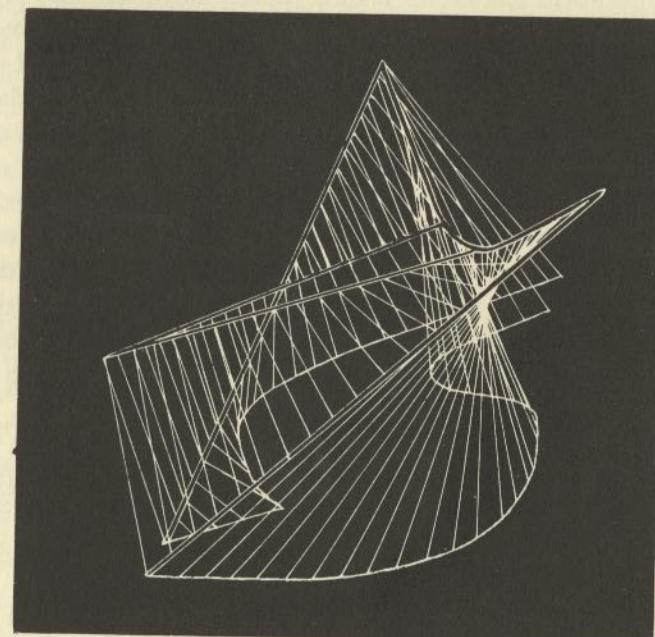


Fig. I-4. First Model of Philips Pavilion

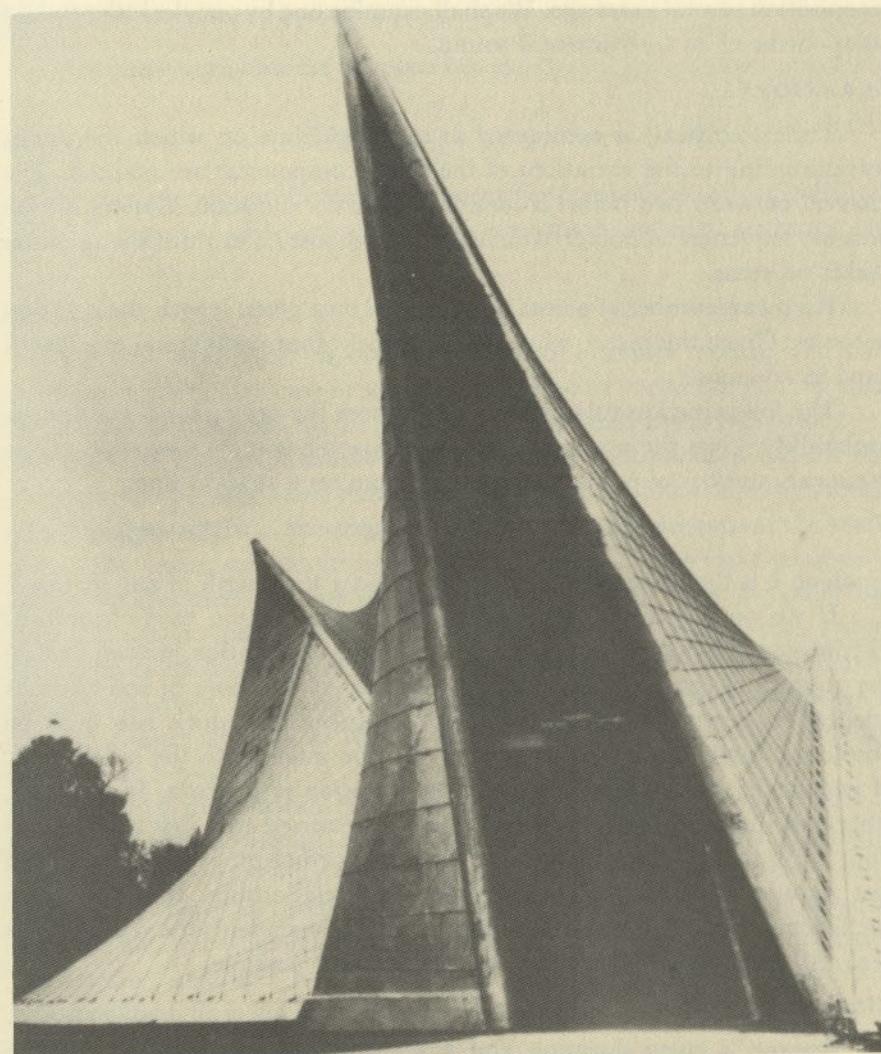


Fig. I-5. Philips Pavilion, Brussels World's Fair, 1958

STOCHASTIC LAWS AND INCARNATIONS

I shall give quickly some of the stochastic laws which I introduced into composition several years ago. We shall examine one by one the independent components of an instrumental sound.

DURATIONS

Time (metrical) is considered as a straight line on which the points corresponding to the variations of the other components are marked. The interval between two points is identical with the duration. Among all the possible sequences of points, which shall we choose? Put thus, the question makes no sense.

If a mean number of points is designated on a given length the question becomes: Given this mean, what is the number of segments equal to a length fixed in advance?

The following formula, which derives from the principles of continuous probability, gives the probabilities for all possible lengths when one knows the mean number of points placed at random on a straight line.

$$P_x = \delta e^{-\delta x} dx, \quad (\text{See Appendix I.})$$

in which δ is the linear density of points, and x the length of any segment.

If we now choose some points and compare them to a theoretical distribution obeying the above law or any other distribution, we can deduce the amount of chance included in our choice, or the more or less rigorous adaptation of our choice to the law of distribution, which can even be absolutely functional. The comparison can be made with the aid of tests, of which the most widely used is the χ^2 criterion of Pearson. In our case, where all the components of sound can be measured to a first approximation, we shall use in addition the correlation coefficient. It is known that if two populations are in a linear functional relationship, the correlation coefficient is one. If the two populations are independent, the coefficient is zero. All intermediate degrees of relationship are possible.

Clouds of Sounds

Assume a given duration and a set of sound-points defined in the intensity-pitch space realized during this duration. Given the mean superficial density of this tone cluster, what is the probability of a particular density occurring in a given region of the intensity-pitch space? Poisson's Law answers this question:

$$P_\mu = \frac{\mu_0^\mu}{\mu!} e^{-\mu_0},$$

where μ_0 is the mean density and μ is any particular density. As with durations, comparisons with other distributions of sound-points can fashion the law which we wish our cluster to obey.

INTERVALS OF INTENSITY, PITCH, ETC.

For these variables the simplest law is

$$\theta(\gamma) d\gamma = \frac{2}{a} \left(1 - \frac{\gamma}{a}\right) d\gamma, \quad (\text{See Appendix I.})$$

which gives the probability that a segment (interval of intensity, pitch, etc.) within a segment of length a , will have a length included within γ and $\gamma + d\gamma$, for $0 \leq \gamma \leq a$.

SPEEDS

We have been speaking of sound-points, or granular sounds, which are in reality a particular case of sounds of continuous variation. Among these let us consider glissandi. Of all the possible forms that a glissando sound can take, we shall choose the simplest—the uniformly continuous glissando. This glissando can be assimilated sensorially and physically into the mathematical concept of speed. In a one-dimensional vectorial representation, the scalar size of the vector can be given by the hypotenuse of the right triangle in which the duration and the melodic interval covered form the other two sides. Certain mathematical operations on the continuously variable sounds thus defined are then permitted. The traditional sounds of wind instruments are, for example, particular cases where the speed is zero. A glissando towards higher frequencies can be defined as positive, towards lower frequencies as negative.

We shall demonstrate the simplest logical hypotheses which lead us to a mathematical formula for the distribution of speeds. The arguments which follow are in reality one of those "logical poems" which the human intelligence creates in order to trap the superficial incoherencies of physical phenomena, and which can serve, on the rebound, as a point of departure for building abstract entities, and then incarnations of these entities in sound or light. It is for these reasons that I offer them as examples:

Homogeneity hypotheses [11]*

1. The density of speed-animated sounds is constant; i.e., two regions of equal extent on the pitch range contain the same average number of mobile sounds (glissandi).

* The numbers in brackets correspond to the numbers in the Bibliography at the end of the book.

2. The absolute value of speeds (ascending or descending glissandi) is spread uniformly; i.e., the mean quadratic speed of mobile sounds is the same in different registers.

3. There is isotropy; that is, there is no privileged direction for the movements of mobile sounds in any register. There is an equal number of sounds ascending and descending.

From these three hypotheses of symmetry, we can define the function $f(v)$ of the probability of the absolute speed v . ($f(v)$ is the relative frequency of occurrence of v .)

Let n be the number of glissandi per unit of the pitch range (density of mobile sounds), and r any portion taken from the range. Then the number of speed-animated sounds between v and $v + dv$ and positive, is, from hypotheses 1 and 3:

$$n r \frac{1}{2} f(v) dv \quad (\text{the probability that the sign is } + \text{ is } \frac{1}{2}).$$

From hypothesis 2 the number of animated sounds with speed of absolute value $|v|$ is a function which depends on v^2 only. Let this function be $g(v^2)$. We then have the equation

$$n r \frac{1}{2} f(v) dv = n r g(v^2) dv.$$

Moreover if $x = v$, the probability function $g(v^2)$ will be equal to the law of probability H of x , whence $g(v^2) = H(x)$, or $\log g(v^2) = h(x)$.

In order that $h(x)$ may depend only on $x^2 = v^2$, it is necessary and sufficient that the differentials $d \log g(v^2) = h'(x) dx$ and $v dv = x dx$ have a constant ratio:

$$\frac{d \log g(v^2)}{v dv} = \frac{h'(x) dx}{x dx} = \text{constant} = -2j,$$

whence $h'(x) = -2jx$, $h(x) = -jx^2 + c$, and $H(x) = ke^{-jx^2}$.

But $H(x)$ is a function of elementary probabilities; therefore its integral from $-\infty$ to $+\infty$ must be equal to 1. j is positive and $k = \sqrt{j}/\sqrt{\pi}$. If $j = 1/a^2$, it follows that

$$\frac{1}{2} f(v) = g(v^2) = H(x) = \frac{1}{a\sqrt{\pi}} e^{-v^2/a^2}$$

and

$$f(v) = \frac{2}{a\sqrt{\pi}} e^{-v^2/a^2}$$

for $v = x$, which is a Gaussian distribution.

This chain of reasoning borrowed from Paul Lévy was established after Maxwell, who, with Boltzmann, was responsible for the kinetic theory of gases. The function $f(v)$ gives the probability of the speed v ; the constant a defines the "temperature" of this sonic atmosphere. The arithmetic mean of v is equal to $a/\sqrt{\pi}$, and the standard deviation is $a/\sqrt{2}$.

We offer as an example several bars from the work *Pithoprakta* for string orchestra (Fig. I-6), written in 1955-56, and performed by Prof. Hermann Scherchen in Munich in March 1957.⁴ The graph (Fig. I-7) represents a set of speeds of temperature proportional to $a = 35$. The abscissa represents time in units of 5 cm = 26 MM (Mälzel Metronome). This unit is subdivided into three, four, and five equal parts, which allow very slight differences of duration. The pitches are drawn as the ordinates, with the unit 1 semitone = 0.25 cm. 1 cm on the vertical scale corresponds to a major third. There are 46 stringed instruments, each represented by a jagged line. Each of the lines represents a speed taken from the table of probabilities calculated with the formula

$$f(v) = \frac{2}{a\sqrt{\pi}} e^{-v^2/a^2}.$$

A total of 1148 speeds, distributed in 58 distinct values according to Gauss's law, have been calculated and traced for this passage (measures 52-60, with a duration of 18.5 sec.). The distribution being Gaussian, the macroscopic configuration is a plastic modulation of the sonic material. The same passage was transcribed into traditional notation. To sum up we have a sonic compound in which:

1. The durations do not vary.
2. The mass of pitches is freely modulated.
3. The density of sounds at each moment is constant.
4. The dynamic is *ff* without variation.
5. The timbre is constant.
6. The speeds determine a "temperature" which is subject to local fluctuations. Their distribution is Gaussian.

As we have already had occasion to remark, we can establish more or less strict relationships between the component parts of sounds.⁵ The most useful coefficient which measures the degree of correlation between two variables x and y is

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2} \sqrt{\sum (y - \bar{y})^2}},$$

where \bar{x} and \bar{y} are the arithmetic means of the two variables.

Here then, is the technical aspect of the starting point for a utilization of the theory and calculus of probabilities in musical composition. With the above, we already know that:

1. We can control continuous transformations of large sets of granular and/or continuous sounds. In fact, densities, durations, registers, speeds, etc., can all be subjected to the law of large numbers with the necessary approximations. We can therefore with the aid of means and deviations shape these sets and make them evolve in different directions. The best known is that which goes from order to disorder, or vice versa, and which introduces the concept of entropy. We can conceive of other continuous transformations: for example, a set of plucked sounds transforming continuously into a set of arco sounds, or in electromagnetic music, the passage from one sonic substance to another, assuring thus an organic connection between the two substances. To illustrate this idea, I recall the Greek sophism about baldness: "How many hairs must one remove from a hairy skull in order to make it bald?" It is a problem resolved by the theory of probability with the standard deviation, and known by the term *statistical definition*.

2. A transformation may be explosive when deviations from the mean suddenly become exceptional.

3. We can likewise confront highly improbable events with average events.

4. Very rarified sonic atmospheres may be fashioned and controlled with the aid of formulae such as Poisson's. Thus, even music for a solo instrument can be composed with stochastic methods.

These laws, which we have met before in a multitude of fields, are veritable diamonds of contemporary thought. They govern the laws of the advent of being and becoming. However, it must be well understood that they are not an end in themselves, but marvelous tools of construction and logical lifelines. Here a backfire is to be found. This time it is these stochastic tools that pose a fundamental question: "What is the minimum of logical constraints necessary for the construction of a musical process?" But before answering this we shall sketch briefly the basic phases in the construction of a musical work.

The image shows a page from a musical score for 'Pithoprakta'. The score is divided into six systems, each with multiple staves. The systems are labeled on the left: W.B., V.I, V.II, A., Vc., and C.B. Each system contains several staves, each with a unique set of rhythmic patterns and dynamic markings. The first system (W.B.) starts with 'pizz. gliss.' and 'ff'. The second system (V.I) starts with 'pizz. gliss.' and 'ff'. The third system (V.II) starts with 'pizz. gliss.' and 'ff'. The fourth system (A.) starts with 'pizz. gliss.' and 'ff'. The fifth system (Vc.) starts with 'pizz. gliss.' and 'ff'. The sixth system (C.B.) starts with 'pizz. gliss.' and 'mf'. The score is dated 'B. & H. 19583' at the bottom right.

Fig. I-6. Bars 52-57 of *Pithoprakta*

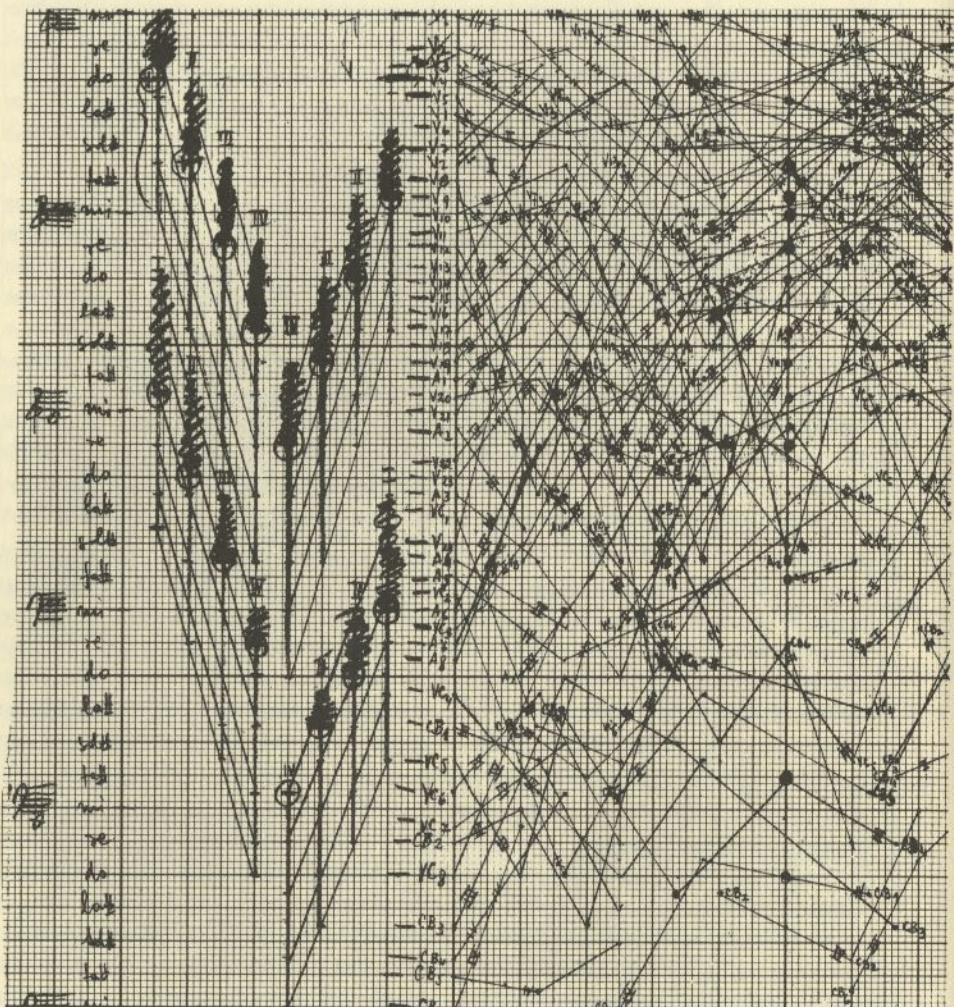
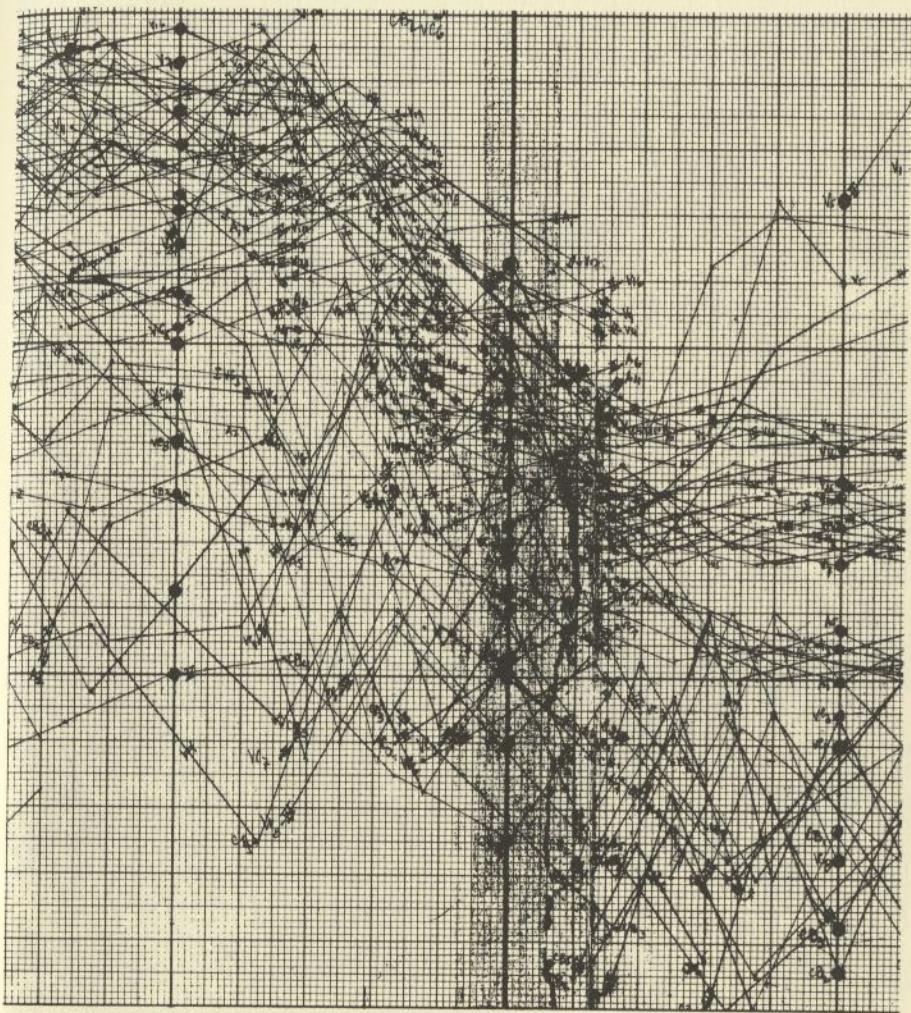


Fig. I-7. Graph of Bars 52–57 of *Pithoprakta*



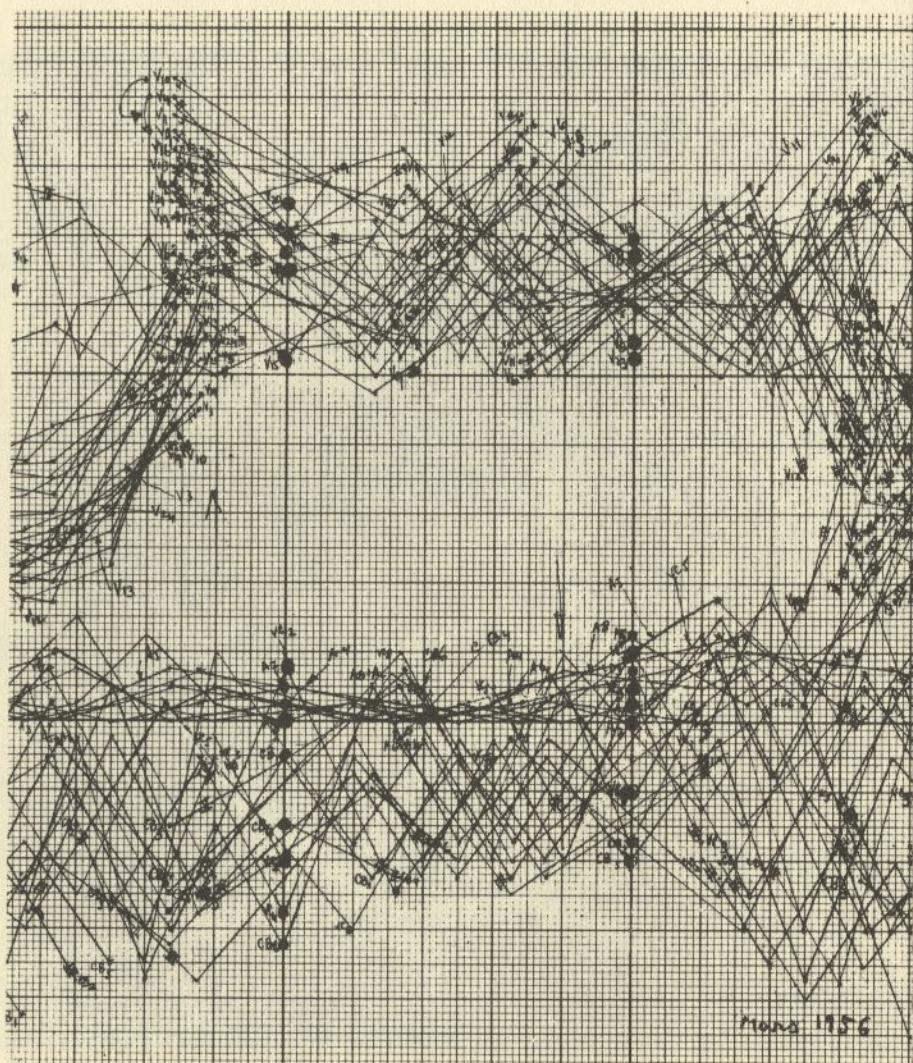
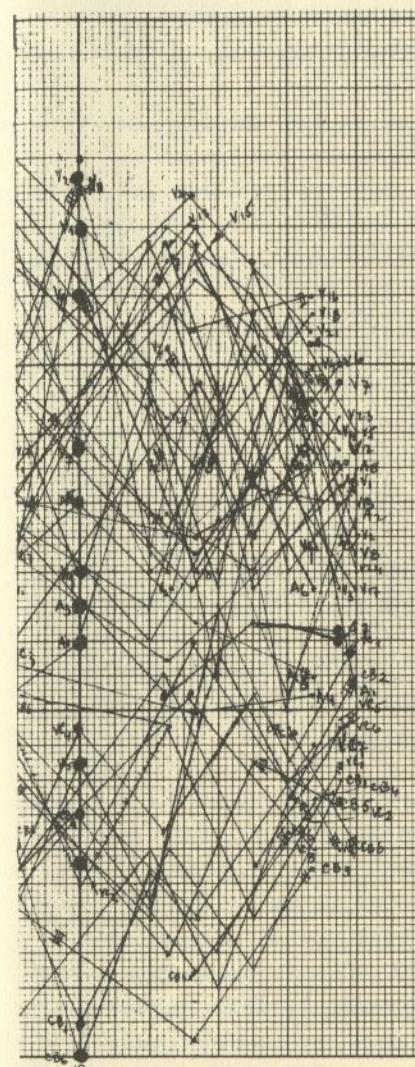


Fig. I-7 (continued)



FUNDAMENTAL PHASES OF A MUSICAL WORK

1. *Initial conceptions* (intuitions, provisional or definitive data);
2. *Definition of the sonic entities* and of their symbolism communicable with the limits of possible means (sounds of musical instruments, electronic sounds, noises, sets of ordered sonic elements, granular or continuous formations, etc.);
3. *Definition of the transformations* which these sonic entities must undergo in the course of the composition (macrocomposition: general choice of logical framework, i.e., of the elementary algebraic operations and the setting up of relations between entities, sets, and their symbols as defined in 2.); and the arrangement of these operations in lexicographic time with the aid of succession and simultaneity);
4. *Microcomposition* (choice and detailed fixing of the functional or stochastic relations of the elements of 2.), i.e., algebra outside-time, and algebra in-time;
5. *Sequential programming* of 3. and 4. (the schema and *pattern* of the work in its entirety);
6. *Implementation of calculations*, verifications, feedbacks, and definitive modifications of the sequential program;
7. *Final symbolic result* of the programming (setting out the music on paper in traditional notation, numerical expressions, graphs, or other means of solfeggio);
8. *Sonic realization* of the program (direct orchestral performance, manipulations of the type of electromagnetic music, computerized construction of the sonic entities and their transformations).

The order of this list is not really rigid. Permutations are possible in the course of the working out of a composition. Most of the time these phases are unconscious and defective. However, this list does establish ideas and allows speculation about the future. In fact, computers can take in hand phases 6. and 7., and even 8. But as a first approach, it seems that only phases 6. and 7. are immediately accessible. That is to say, that the final symbolic result, at least in France, may be realized only by an orchestra or by manipulations of electroacoustic music on tape recorders, emitted by the existing electroacoustic channels; and not, as would be desirable in the very near future, by an elaborate mechanization which would omit orchestral or tape interpreters, and which would assume the computerized fabrication of the sonic entities and of their transformations.

Here now is an answer to the question put above, an answer that is true for instrumental music, but which can be applied as well to all kinds of

sound production. For this we shall again take up the phases described:

2. Definition of sonic entities. The sonic entities of the classical orchestra can be represented in a first approximation by vectors of four usually independent variables, $E_r(c, h, g, u)$:

c_a = timbre or instrumental family

h_i = pitch of the sound

g_j = intensity of the sound, or dynamic form

u_k = duration of the sound.

The vector E_r defines a point M in the *multidimensional space* provided by a *base* (c, h, g, u). This point M will have as *coordinates* the numbers c_a, h_i, g_j, u_k . For example: c_3 played arco and forte on a violin, one eighth note in length, at one eighth note = 240 MM, can be represented as $c_{\text{viol. arco}}, h_{39}$ (= c_3), g_4 (= forte), u_5 (= $\frac{1}{8}$ sec.). Suppose that these points M are plotted on an axis which we shall call E_r , and that through its origin we draw another axis t , at right angles to axis E_r . We shall represent on this axis, called the *axis of lexicographic time*, the lexicographic-temporal succession of the points M . Thus we have defined and conveniently represented a two-dimensional space (E_r, t). This will allow us to pass to phase 3., definition of transformation, and 4., microcomposition, which must contain the answer to the problem posed concerning the minimum of constraints.

To this end, suppose that the points M defined above can appear with no necessary condition other than that of obeying an aleatory law without memory. This hypothesis is equivalent to saying that we admit a stochastic distribution of the events E_r in the space (E_r, t). Admitting a sufficiently weak superficial distribution n , we enter a region where the law of Poisson is applicable:

$$P_k = \frac{n^k}{K!} e^{-n}.$$

Incidentally we can consider this problem as a synthesis of several conveniently chosen linear stochastic processes (law of radiation from radioactive bodies). (The second method is perhaps more favorable for a mechanization of the transformations.)

A sufficiently long fragment of this distribution constitutes the musical work. The basic law defined above generates a whole family of compositions as a function of the superficial density. So we have a formal archetype of composition in which the basic aim is to attain the greatest possible *asymmetry* (in the etymological sense) and the *minimum of constraints, causalities, and rules*. We think that from this archetype, which is perhaps the most

general one, we can redescend the ladder of forms by introducing progressively more numerous constraints, i.e., choices, restrictions, and negations. In the analysis in several linear processes we can also introduce other processes: those of Wiener-Lévy, P. Lévy's infinitely divisible, Markov chains, etc., or mixtures of several. It is this which makes this second method the more fertile.

The exploration of the limits a and b of this archetype $a \leq n \leq b$ is equally interesting, but on another level—that of the mutual comparison of samples. This implies, in effect, a gradation of the increments of n in order that the differences between the families n_i may be recognizable. Analogous remarks are valid in the case of other linear processes.

If we opt for a Poisson process, there are two necessary hypotheses which answer the question of the minimum of constraints: 1. there exists in a given space musical instruments and men; and 2. there exist means of contact between these men and these instruments which permit the emission of rare sonic events.

This is the only hypothesis (cf. the *ekklisis* of Epicurus). From these two constraints and with the aid of stochastics, I built an entire composition without admitting any other restrictions. *Achorripsis* for 21 instruments was composed in 1956–57, and had its first performance in Buenos Aires in 1958 under Prof. Hermann Scherchen. (See Fig. I-8.)

At that time I wrote:*

$\tau\ddot{o}\gamma\ddot{a}\rho\alpha\ddot{u}\tau\ddot{o}\nu\ddot{o}\epsilon\ddot{u}\nu\ddot{\epsilon}\sigma\ddot{t}\nu\tau\epsilon\kappa\ddot{a}\epsilon\ddot{u}\nu\ddot{a}$
 $\tau\ddot{o}\gamma\ddot{a}\rho\alpha\ddot{u}\tau\ddot{o}\epsilon\ddot{u}\nu\ddot{\epsilon}\sigma\ddot{t}\nu\tau\epsilon\kappa\ddot{a}\nu\ddot{u}\kappa\ddot{e}\ddot{u}\nu\ddot{a}\dagger$

ONTOLOGY

In a universe of nothingness. A brief train of waves, so brief that its end and beginning coincide (negative time) disengaging itself endlessly.

Nothingness resorbs, creates.

It engenders being.

Time, Causality.

These rare sonic events can be something more than isolated sounds. They can be melodic figures, cell structures, or agglomerations whose

* The following excerpt (through p. 37) is from "In Search of a Stochastic Music," *Gravesaner Blätter*, no. 11/12.

† "For it is the same to think as to be" (*Poem* by Parmenides); and my paraphrase, "For it is the same to be as not to be."

characteristics are also ruled by the laws of chance, for example, clouds of sound-points or speed-temperatures.⁶ In each case they form a sample of a succession of rare sonic events.

This sample may be represented by either a simple table of probabilities or a double-entry table, a matrix, in which the cells are filled by the frequencies of events. The rows represent the particular qualifications of the events, and the columns the dates (see Matrix *M*, Fig. I-9). The frequencies in this matrix are distributed according to Poisson's formula, which is the law for the appearances of rare random events.

We should further define the sense of such a distribution and the manner in which we realize it. There is an advantage in defining chance as an aesthetic law, as a normal philosophy. Chance is the limit of the notion of evolving symmetry. Symmetry tends to asymmetry, which in this sense is equivalent to the negation of traditionally inherited behavioral frameworks. This negation not only operates on details, but most importantly on the composition of structures, hence tendencies in painting, sculpture, architecture, and other realms of thought. For example, in architecture, plans worked out with the aid of regulating diagrams are rendered more complex and dynamic by exceptional events. Everything happens as if there were one-to-one oscillations between symmetry, order, rationality, and asymmetry, disorder, irrationality in the reactions between the epochs of civilizations.

At the beginning of a transformation towards asymmetry, exceptional events are introduced into symmetry and act as aesthetic stimuli. When these exceptional events multiply and become the general case, a jump to a higher level occurs. The level is one of disorder, which, at least in the arts and in the expressions of artists, proclaims itself as engendered by the complex, vast, and rich vision of the brutal encounters of modern life. Forms such as abstract and decorative art and action painting bear witness to this fact. Consequently chance, by whose side we walk in all our daily occupations, is nothing but an extreme case of this controlled disorder (that which signifies the richness or poverty of the connections between events and which engenders the dependence or independence of transformations); and by virtue of the negation, it conversely enjoys all the benevolent characteristics of an artistic regulator. It is a regulator also of sonic events, their appearance, and their life. But it is here that the iron logic of the laws of chance intervenes; this chance cannot be created without total submission to its own laws. On this condition, chance checked by its own force becomes a hydroelectric torrent.

Musical score for orchestra and woodwind quintet, showing bars 104-111. The score includes parts for Picc., Klar. Es, Baßkl. B, Xyl., H.-Bl., gr.Tr., Vl. 2, Vcl. 2, Kb. 2, and Trpt. 7. The music features complex stochastic patterns with various dynamics and performance instructions like 'f' and 'pizz.'

Fig. I-8. Bars 104-111 of *Achorripsis*

Continuation of the musical score for orchestra and woodwind quintet, starting at bar 110. The score includes parts for Picc., Ob., Klar. Es, Baßkl. B, Trpt. 7, Xyl., H.-Bl., gr.Tr., Vl. 2, Vcl. 2, Kb. 2, and Trpt. 7. The score continues the stochastic patterns established in the previous section.

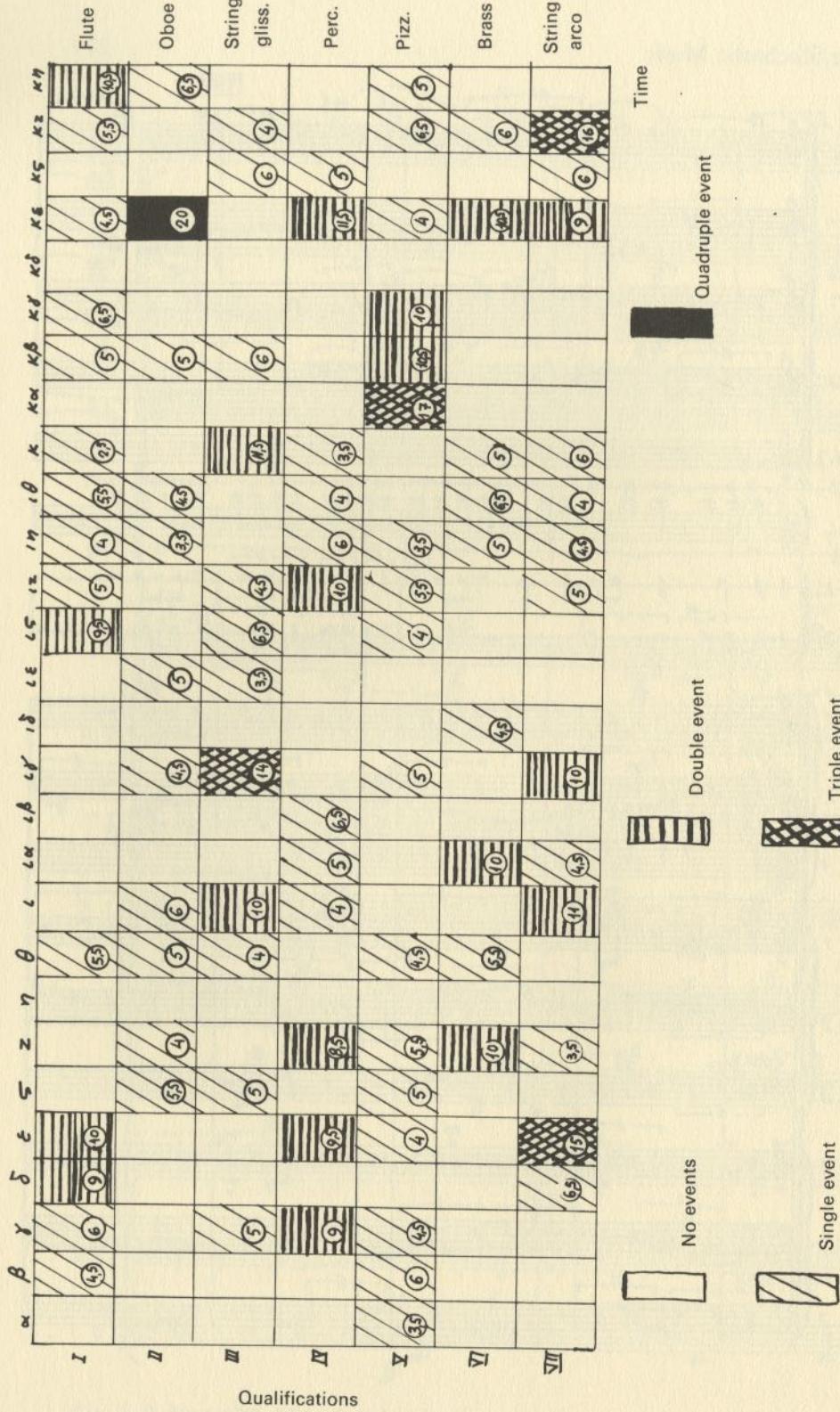


Fig. 1-9. Vector Matrix M , Matrix of *Achorripsis*

Free Stochastic Music

29

However, we are not speaking here of cases where one merely plays heads and tails in order to choose a particular alternative in some trivial circumstance. The problem is much more serious than that. It is a matter here of a philosophic and aesthetic concept ruled by the laws of probability and by the mathematical functions that formulate that theory, of a coherent concept in a new region of coherence.

The analysis that follows is taken from *Achorripsis*.

For convenience in calculation we shall choose a priori a mean density of events

$$\lambda = 0.6 \text{ events/unit.}$$

Applying Poisson's formula,

$$P_k = \frac{\lambda^k}{K!} e^{-\lambda}$$

we obtain the table of probabilities:

$$\begin{aligned}
 P_0 &= 0.5488 \\
 P_1 &= 0.3293 \\
 P_2 &= 0.0988 \\
 P_3 &= 0.0198 \\
 P_4 &= 0.0030 \\
 P_5 &= 0.0004.
 \end{aligned} \tag{1}$$

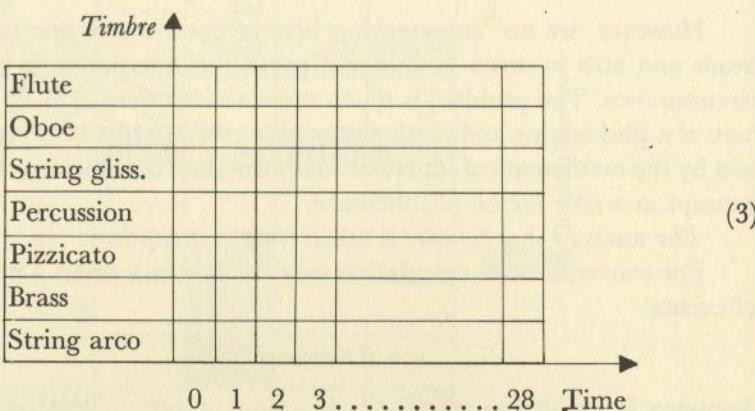
P_i is the probability that the event will occur i times in the unit of volume, time, etc. In choosing a priori 196 units or cells, the distribution of the frequencies among the cells is obtained by multiplying the values of P_i by 196.

Number of cells

i	196 P_i
0	107
1	65
2	19
3	4
4	1

(2)

The 196 cells may be arranged in one or several groups of cells, qualified as to timbre and time, so that the number of groups of timbres times the number of groups of durations = 196 cells. Let there be 7 distinct timbres; then $196/7 = 28$ units of time. Thus the 196 cells are distributed over a two-dimensional space as shown in (3).



If the musical sample is to last 7 minutes (a subjective choice) the unit of time U_t will equal 15 sec., and each U_t will contain 6.5 measures at MM = 26.

How shall we distribute the frequencies of zero, single, double, triple, and quadruple events per cell in the two-dimensional space of Matrix (3)? Consider the 28 columns as cells and distribute the zero, single, double, triple, and quadruple events from table (2) in these 28 new cells. Take as an example the single event; from table (2) it must occur 65 times. Everything happens as if one were to distribute events in the cells with a mean density $\lambda = 65/28 = 2.32$ single events per cell (here cell = column).

In applying anew Poisson's formula with the mean density $\lambda = 2.32$ ($2.32 \ll 30$) we obtain table (4).

Poisson Distribution			Arbitrary Distribution		
Frequency K	No. of Columns	Product $col \times K$	Frequency K	No. of Columns	Product $col \times K$
0	3	0	0	10	0
1	6	6	1	3	3
2	8	16	2	0	0
3	5	15	3	9	27
4	3	12	4	0	0
5	2	10	5	1	5
6	1	6	6	5	30
7	0	0	7	0	0
Totals	28	65	Totals	28	65

(4) (5)

One could choose any other distribution on condition that the sum of single events equals 65. Table (5) shows such a distribution.

But in this axiomatic research, where chance must bathe all of sonic space, we must reject every distribution which departs from Poisson's law. And the Poisson distribution must be effective not only for the columns but also for the rows of the matrix. The same reasoning holds true for the diagonals, etc.

Contenting ourselves just with rows and columns, we obtain a homogeneous distribution which follows Poisson. It was in this way that the distributions in rows and columns of Matrix (M) (Fig. I-9) were calculated.

So a *unique* law of chance, the law of Poisson (for rare events) through the medium of the arbitrary mean λ is capable of conditioning, on the one hand, a whole sample matrix, and on the other, the partial distributions following the rows and columns. The a priori, arbitrary choice admitted at the beginning therefore concerns the variables of the "vector-matrix."

Variables or entries of the "vector-matrix"

1. Poisson's Law
2. The mean λ
3. The number of cells, rows, and columns

The distributions entered in this matrix are not always rigorously defined. They really depend, for a given λ , on the number of rows or columns. The greater the number of rows or columns, the more rigorous is the definition. This is the law of large numbers. But this indeterminism allows free will if the artistic inspiration wishes it. It is a second door that is open to the subjectivism of the composer, the first being the "state of entry" of the "Vector-Matrix" defined above.

Now we must specify the unit-events, whose frequencies were adjusted in the standard matrix (M). We shall take as a single event a cloud of sounds with linear density δ sounds/sec. Ten sounds/sec is about the limit that a normal orchestra can play. We shall choose $\delta = 5$ sounds/measure at MM 26, so that $\delta = 2.2$ sounds/sec ($\approx 10/4$).

We shall now set out the following correspondence:

Event	Cloud of density $\delta =$		Mean number of sounds/cell (15 sec)
	Sounds/ measure 26MM	Sounds/ sec	
zero	0	0	0
single	5	2.2	32.5
double	10	4.4	65
triple	15	6.6	97.5
quadruple	20	8.8	130

The hatchings in matrix (M) show a Poisson distribution of frequencies, homogeneous and verified in terms of rows and columns. We notice that the rows are interchangeable (= interchangeable timbres). So are the columns. This leads us to admit that the determinism of this matrix is weak in part, and that it serves chiefly as a basis for thought—for thought which manipulates frequencies of events of all kinds. The true work of molding sound consists of distributing the clouds in the two-dimensional space of the matrix, and of anticipating a priori all the sonic encounters before the calculation of details, eliminating prejudicial positions. It is a work of patient research which exploits all the creative faculties instantaneously. This matrix is like a game of chess for a single player who must follow certain rules of the game for a prize for which he himself is the judge. This game matrix has no unique strategy. It is not even possible to disentangle any balanced goals. It is very general and incalculable by pure reason.

Up to this point we have placed the cloud densities in the matrix. Now with the aid of calculation we must proceed to the coordination of the aleatory sonic elements.

HYPOTHESES OF CALCULATION

Let us analyze as an example cell III, ιz of the matrix: third row, sounds of continuous variation (string glissandi), seventeenth unit of time (measures 103–11). The density of the sounds is 4.5 sounds/measure at MM 26 ($\delta = 4.5$); so that 4.5 sounds/measure times 6.5 measures = 29 sounds for this cell. How shall we place the 29 glissando sounds in this cell?

Hypothesis 1. The acoustic characteristic of the glissando sound is assimilated to the speed $v = df/dt$ of a uniformly continuous movement. (See Fig. I-10.)

Hypothesis 2. The quadratic mean α of all the possible values of v is proportional to the sonic density δ . In this case $\alpha = 3.38$ (temperature).

Hypothesis 3. The values of these speeds are distributed according to the most complete asymmetry (chance). This distribution follows the law of

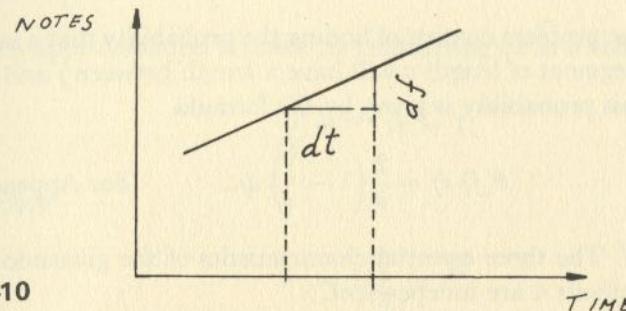


Fig. I-10

Gauss. The probability $f(v)$ for the existence of the speed v is given by the function

$$f(v) = \frac{2}{a\sqrt{\pi}} e^{-v^2/a^2};$$

and the probability $P(\lambda)$ that v will lie between v_1 and v_2 , by the function

$$P(\lambda) = \theta(\lambda_2) - \theta(\lambda_1),$$

in which $\lambda_1 = v_1/a$ and

$$\theta(\lambda) = \frac{2}{\sqrt{\pi}} \int_0^\lambda e^{-\lambda^2} d\lambda \quad (\text{normal distribution}).$$

Hypothesis 4. A glissando sound is essentially characterized by *a.* the moment of its departure; *b.* its speed $v_m = df/dt$, ($v_1 < v_m < v_2$); and *c.* its register.

Hypothesis 5. Assimilate time to a line and make each moment of departure a point on that line. It is as if one were to distribute a number of points on a line with a linear density $\delta = 4.5$ points at MM = 26. This, then, is a problem of continuous probabilities. These points define segments and the probability that the i -th segment will have a length x_i between x and $x + dx$ is

$$P_x = \delta e^{-\delta x} dx.$$

Hypothesis 6. The moment of departure corresponds to a sound. We shall attempt to define its pitch. The strings have a range of about 80 semitones, which may be represented by a line of length $a = 80$ semitones. Since between two successive or simultaneous glissandi there exists an interval between the pitches at the moments of departure, we can define not only the note of attack for the first glissando, but also the melodic interval which separates the two origins.

Put thus, the problem consists of finding the probability that a segment s within a line segment of length a will have a length between j and $j + dj$ ($0 \leq j \leq a$). This probability is given by the formula

$$\theta(j) dj = \frac{2}{a} \left(1 - \frac{j}{a}\right) dj. \quad (\text{See Appendix I.})$$

Hypothesis 7. The three essential characteristics of the glissando sound defined in Hypothesis 4 are independent.

From these hypotheses we can draw up the three tables of probability: a table of durations, a table of speeds, and a table of intervals.

All these tables furnish us with the elements which materialize in cell III, *i.e.* The reader is encouraged to examine the score to see how the results of the calculations have been used. Here also, may we emphasize, a great liberty of choice is given the composer. The restrictions are more of a general canalizing kind, rather than peremptory. The theory and the calculation define the tendencies of the sonic entity, but they do not constitute a slavery. Mathematical formulae are thus tamed and subjugated by musical thought. We have given this example of glissando sounds because it contains all the problems of stochastic music, controlled, up to a certain point, by calculation.

Table of Durations

$\delta = 4.5$ sounds/measure at MM 26

Unit $x = 0.10$ of the measure at 26 MM

$4.5 \cdot 6.5 = 29$ sounds/cell, *i.e.*, 28 durations

x	δx	$e^{-\delta x}$	$\delta e^{-\delta x}$	$\delta e^{-\delta x} dx$	$28P_x$
0.00	0.00	1.000	4.500	0.362	10
0.10	0.45	0.638	2.870	0.231	7
0.20	0.90	0.407	1.830	0.148	4
0.30	1.35	0.259	1.165	0.094	3
0.40	1.80	0.165	0.743	0.060	2
0.50	2.25	0.105	0.473	0.038	1
0.60	2.70	0.067	0.302	0.024	1
0.70	3.15	0.043	0.194	0.016	0
<i>Totals</i>		12.415	0.973	28	

An approximation is made by considering dx as a constant factor.

$$\sum_0^{\infty} \delta e^{-\delta x} dx = 1.$$

Therefore

$$dx = 1 / \sum_0^{\infty} \delta e^{-\delta x}.$$

In this case $dx = 1/12.415 = 0.0805$.

Table of Speeds

$\delta = 4.5$ glissando sounds/measure at 26 MM

$\alpha = 3.88$, quadratic mean of the speeds

v is expressed in semitones/measure at 26 MM

v_m is the mean speed $(v_1 + v_2)/2$

$4.5 \cdot 6.5 = 29$ glissando sounds/cell.

v	$\lambda = v/\alpha$	$\theta(\lambda)$	$P(\lambda) = \theta(\lambda_2) - \theta(\lambda_1)$	29 $P(\lambda)$	v_m
0	0.000	0.0000			
1	0.258	0.2869	0.2869	9	0.5
2	0.516	0.5379	0.2510	7	1.5
3	0.773	0.7238	0.1859	5	2.5
4	1.032	0.8548	0.1310	4	3.5
5	1.228	0.9319	0.0771	2	4.5
6	1.545	0.9716	0.0397	1	5.5
7	1.805	0.9895	0.0179	1	6.5
			0.0071	0	7.5

Table of Intervals

$\delta = 4.5$ glissandi/measure at 26 MM.

$a = 80$ semitones, or 18 times the arbitrary unit of 4.5 semitones.

j is expressed in multiples of 4.5 semitones.

dj is considered to be constant. Therefore $dj = 1/\sum\theta(j)$ or $dj = a/(m + 1)$, and we obtain a step function. For $j = 0$, $\theta(j) dj = 2/(m + 1) = 0.105$; for $j = 18$, $\theta(j) dj = 0$.

$4.5 \cdot 6.5 = 29$ glissando sounds per cell.

We can construct the table of probabilities by means of a straight line.

j	$\theta(j) dj = P(j)$	$29 P(j)$
0	0.105	3
1		3
2		3
3		3
4		2
5		2
6		2
7		2
8		2
9		2
10		1
11		1
12		1
13		1
14		1
15		0
16		0
17		0
18		0

We shall not speak of the means of verification of liaisons and correlations between the various values used. It would be too long, complex, and tedious. For the moment let us affirm that the basic matrix was verified by the two formulae:

$$r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2} \sqrt{\sum (y - \bar{y})^2}},$$

and

$$z = \frac{1}{2} \log \frac{1 + r}{1 - r}.$$

Let us now imagine music composed with the aid of matrix (M). An observer who perceived the frequencies of events of the musical sample would deduce a distribution due to chance and following the laws of probability. Now the question is, when heard a number of times, will this music keep its surprise effect? Will it not change into a set of *foreseeable* phenomena through the existence of memory, despite the fact that the law of frequencies has been derived from the laws of chance?

In fact, the data will appear aleatory only at the first hearing. Then, during successive rehearsals the relations between the events of the sample ordained by "chance" will form a network, which will take on a definite meaning in the mind of the listener, and will initiate a special "logic," a new cohesion capable of satisfying his intellect as well as his aesthetic sense; that is, if the artist has a certain flair.

If, on the other hand, we wish the sample to be unforeseeable at all times, it is possible to conceive that at each repetition certain data might be transformed in such a way that their deviations from theoretical frequencies would not be significant. Perhaps a programming useful for a first, second, third, etc., performance will give aleatory samples that are not identical in an absolute sense, whose deviations will also be distributed by chance.

Or again a system with electronic computers might permit variations of the parameters of entrance to the matrix and of the clouds, under certain conditions. There would thus arise a music which can be distorted in the course of time, giving the same observer n results apparently due to chance for n performances. In the long run the music will follow the laws of probability and the performances will be *statistically* identical with each other, the identity being defined once for all by the "vector-matrix."

The sonic scheme defined under this form of vector-matrix is consequently capable of establishing a more or less self-determined regulation of the rare sonic events contained in a musical composition sample. It represents a compositional attitude, a fundamentally stochastic behavior, a unity of superior order.

[1956-57].

If the first steps may be summarized by the process vision \rightarrow rules \rightarrow works of art, the question concerning the minimum has produced an inverse

path: rules → vision. In fact stochastics permits a philosophic vision, as the example of *Achorripsis* bears witness.

CHANCE—IMPROVISATION

Before generalizing further on the essence of musical composition, we must speak of the principle of improvisation which caused a furore among the neo-serialists, and which gives them the right, or so they think, to speak of chance, of the aleatory, which they thus introduce into music. They write scores in which certain combinations of sounds may be freely chosen by the interpreter. It is evident that these composers consider the various possible circuits as equivalent. Two logical infirmities are apparent which deny them the right to speak of chance on the one hand and "composition" on the other (composition in the broad sense, that is):

1. The interpreter is a highly conditioned being, so that it is not possible to accept the thesis of unconditioned choice, of an interpreter acting like a roulette game. The martingale betting at Monte Carlo and the procession of suicides should convince anyone of this. We shall return to this.

2. The composer commits an act of resignation when he admits several possible and equivalent circuits. In the name of a "scheme" the problem of choice is betrayed, and it is the interpreter who is promoted to the rank of composer by the composer himself. There is thus a substitution of authors.

The extremist extension of this attitude is one which uses graphical signs on a piece of paper which the interpreter reads while improvising the whole. The two infirmities mentioned above are terribly aggravated here. I would like to pose a question: If this sheet of paper is put before an interpreter who is an incomparable expert on Chopin, will the result not be modulated by the style and writing of Chopin in the same way that a performer who is immersed in this style might improvise a Chopin-like cadenza to another composer's concerto? From the point of view of the composer there is no interest.

On the contrary, two conclusions may be drawn: first, that serial composition has become so banal that it can be improvised like Chopin's, which confirms the general impression; and second, that the composer resigns his function altogether, that he has nothing to say, and that his function can be taken over by paintings or by cuneiform glyphs.

Chance needs to be calculated

To finish with the thesis of the roulette-musician, I shall add this: Chance is a rare thing and a snare. It can be constructed up to a certain

point with great difficulty, by means of complex reasoning which is summarized in mathematical formulae; it can be constructed a little, but never improvised or intellectually imitated. I refer to the demonstration of the impossibility of imitating chance which was made by the great mathematician Emile Borel, who was one of the specialists in the calculus of probabilities. In any case—to play with sounds like dice—what a truly simplistic activity! But once one has emerged from this primary field of chance worthless to a musician, the calculation of the aleatory, that is to say stochastics, guarantees first that in a region of precise definition slips will not be made, and then furnishes a powerful method of reasoning and enrichment of sonic processes.

STOCHASTIC PAINTING?

In line with these ideas, Michel Philippot introduced the calculus of probabilities into his painting several years ago, thus opening new directions for investigation in this artistic realm. In music he recently endeavored to analyze the act of composition in the form of a *flow chart* for an *imaginary machine*. It is a fundamental analysis of voluntary choice, which leads to a chain of aleatory or deterministic events, and is based on the work *Composition pour double orchestre* (1960). The term *imaginary machine* means that the composer may rigorously define the entities and operating methods, just as on an electronic computer. In 1960 Philippot commented on his *Composition pour double orchestre*:

If, in connection with this work, I happened to use the term "experimental music," I should specify in what sense it was meant in this particular case. It has nothing to do with concrete or electronic music, but with a very banal score written on the usual ruled paper and requiring none but the most traditional orchestral instruments. However, the experiment of which this composition was in some sense a by-product does exist (and one can think of many industries that survive only through the exploitation of their by-products).

The end sought was merely to effect, in the context of a work which I would have written independent of all experimental ambitions, an exploration of the processes followed by my own cerebral mechanism as it arranged the sonic elements. I therefore devised the following steps:

1. Make the most complete inventory possible of the set of my gestures, ideas, mannerisms, decisions, and choices, etc., which were mine when I wrote the music.

2. Reduce this set to a succession of simple decisions, binary, if possible; i.e., accept or refuse a particular note, duration, or silence in a situation determined and defined by the context on one hand, and by the conditioning to which I had been subjected and my personal tastes on the other.

3. Establish, if possible, from this sequence of simple decisions, a scheme ordered according to the following two considerations (which were sometimes contradictory): the manner in which these decisions emerged from my imagination in the course of the work, and the manner in which they would have to emerge in order to be most useful.

4. Present this scheme in the form of a flow chart containing the logical chain of these decisions, the operation of which could easily be controlled.

5. Set in motion a mechanism of simulation respecting the rules of the game in the flow chart and note the result.

6. Compare this result with my *musical* intentions.

7. Check the differences between result and intentions, detect their causes, and correct the operating rules.

8. Refer these corrections back to the sequence of experimental phases, i.e., start again at 1. until a satisfactory result has been obtained.

If we confine ourselves to the most general considerations, it would simply be a matter of proceeding to an analysis of the complexity, considered as an accumulation, in a certain order, of single events, and then of reconstructing this complexity, at the same time verifying the nature of the elements and their rules of combination. A cursory look at the flow chart of the first movement specifies quite well by a mere glance the method I used. But to confine oneself to this first movement would be to misunderstand the essentials of musical composition.

In fact the "preludial" character which emerges from this combination of notes (elementary constituents of the orchestra) should remind us of the fact that composition in its ultimate stage is also an assembly of groups of notes, motifs, or themes and their transformations. Consequently the task revealed by the flow charts of the following movements ought to make conspicuous a grouping of a higher order, in which the data of the first movement were used as a sort of "prefabricated" material. Thus appeared the phenomenon, a rather banal one, of autogeneration of complexity by juxtaposition and combination of a large number of single events and operations.

At the end of this experiment I possessed at most some insight into my own musical tastes, but to me, the obviously interesting aspect of

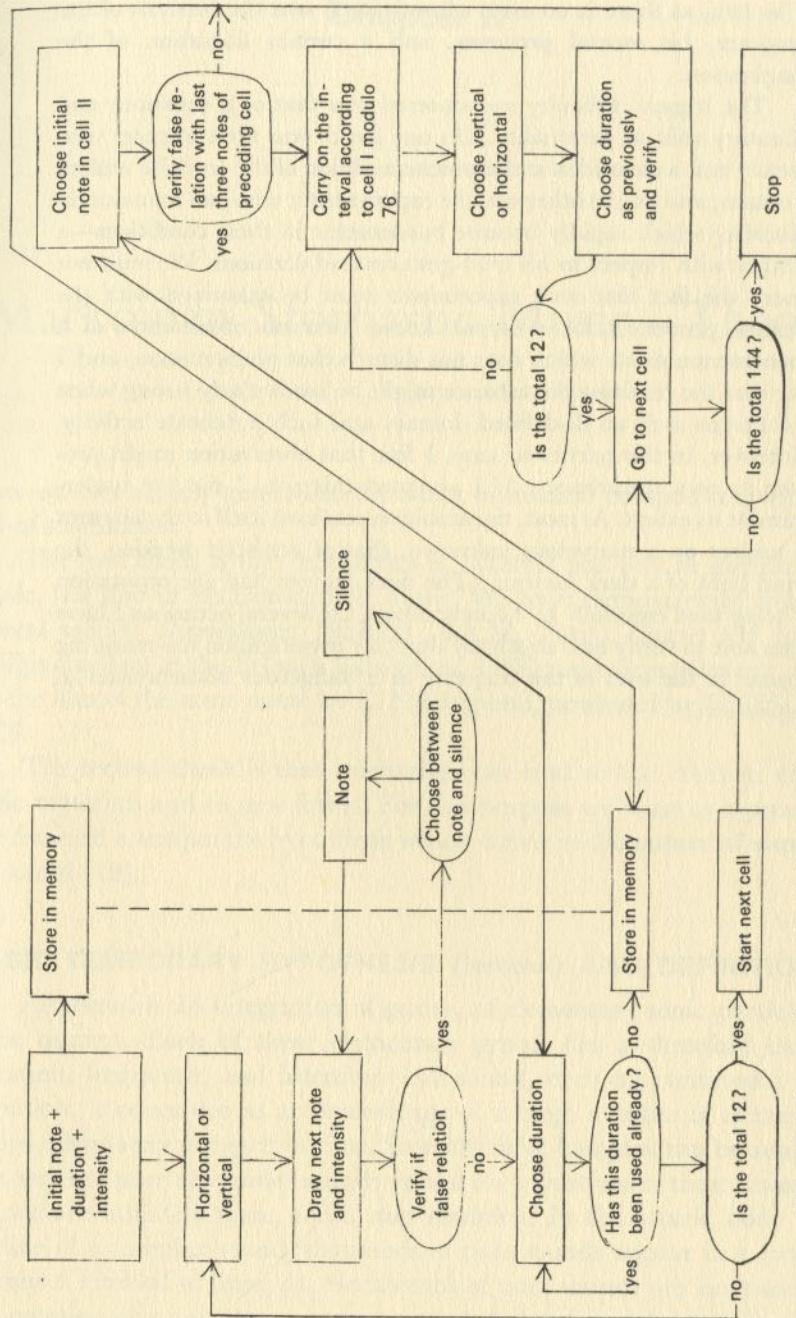


Fig. I-11. *Composition for Double Orchestra*, by Michel Philippot, 1959
Flow Chart of the First Movement

it (as long as there is no error of omission!) was the analysis of the composer, his mental processes, and a certain liberation of the imagination.

The biggest difficulty encountered was that of a conscious and voluntary split in personality. On one hand, was the composer who already had a clear idea and a precise audition of the work he wished to obtain; and on the other was the experimenter who had to maintain a lucidity which rapidly became burdensome in these conditions—a lucidity with respect to his own gestures and decisions. We must not ignore the fact that such experiments must be examined with the greatest prudence, for everyone knows that no observation of a phenomenon exists which does not disturb that phenomenon, and I fear that the resulting disturbance might be particularly strong when it concerns such an ill-defined domain and such a delicate activity. Moreover, in this particular case, I fear that observation might provoke its own disturbance. If I accepted this risk, I did not underestimate its extent. At most, my ambition confined itself to the attempt to project on a marvelous unknown, that of aesthetic creation, the timid light of a dark lantern. (The dark lantern had the reputation of being used especially by housebreakers. On several occasions I have been able to verify how much my thirst for investigation has made me appear in the eyes of the majority as a dangerous housebreaker of inspiration.)

Chapter II

Markovian Stochastic Music—Theory

Now we can rapidly generalize the study of musical composition with the aid of stochastics.

The first thesis is that stochastics is valuable not only in instrumental music, but also in electromagnetic music. We have demonstrated this with several works: *Diamorphoses* 1957–58 (B.A.M. Paris), *Concret PH* (in the Philips Pavilion at the Brussels Exhibition, 1958); and *Orient-Occident*, music for the film of the same name by E. Fulchignoni, produced by UNESCO in 1960.

The second thesis is that stochastics can lead to the creation of new sonic materials and to new forms. For this purpose we must as a preamble put forward a temporary hypothesis which concerns the nature of sound, of all sound [19].

BASIC TEMPORARY HYPOTHESIS (lemma) AND DEFINITIONS

All sound is an integration of grains, of elementary sonic particles, of sonic quanta. Each of these elementary grains has a threefold nature: duration, frequency, and intensity.¹ All sound, even all continuous sonic variation, is conceived as an assemblage of a large number of elementary grains adequately disposed in time. So every sonic complex can be analyzed as a series of pure sinusoidal sounds even if the variations of these sinusoidal sounds are infinitely close, short, and complex. In the attack, body, and decline of a complex sound, thousands of pure sounds appear in a more or less short interval of time, Δt . Hecatombs of pure sounds are necessary for the creation of a complex sound. A complex sound may be imagined as a multi-colored firework in which each point of light appears and instant-

taneously disappears against a black sky. But in this firework there would be such a quantity of points of light organized in such a way that their rapid and teeming succession would create forms and spirals, slowly unfolding, or conversely, brief explosions setting the whole sky aflame. A line of light would be created by a sufficiently large multitude of points appearing and disappearing instantaneously.

If we consider the duration Δt of the grain as quite small but invariable, we can ignore it in what follows and consider frequency and intensity only. The two physical substances of a sound are frequency and intensity in association. They constitute two sets, F and G , independent by their nature. They have a set product $F \times G$, which is the elementary grain of sound. Set F can be put in any kind of correspondence with G : many-valued, single-valued, one-to-one mapping, The correspondence can be given by an extensive representation, a matrix representation, or a canonical representation.

EXAMPLES OF REPRESENTATIONS

Extensive (term by term):

Frequencies	f_1	f_2	f_3	f_4	\dots
Intensities	$\downarrow g_3$	g_n	g_3	g_n	\dots

Matrix (in the form of a table):

Canonical (in the form of a function):

$$\sqrt{f} = Kg$$

f = frequency
 g = intensity
 K = coefficient.

The correspondence may also be indeterminate (stochastic), and here the most convenient representation is the matrical one, which gives the transition probabilities.

Example:

\downarrow	f_1	f_2	f_3	f_4	\dots
g_1	0.5	0	0.2	0	\dots
g_2	0	0.3	0.3	1	\dots
g_3	0.5	0.7	0.5	0	\dots

The table should be interpreted as follows: for each value f_i of f there are one or several corresponding intensity values g_i , defined by a probability. For example, the two intensities g_2 and g_3 correspond to the frequency f_2 , with 30% and 70% chance of occurrence, respectively. On the other hand, each of the two sets F and G can be furnished with a structure—that is to say, internal relations and laws of composition.

Time t is considered as a totally ordered set mapped onto F or G in a lexicographic form.

Examples:

$$a. \begin{matrix} f_1 & f_2 & f_3 & \dots \\ t = 1, 2, \dots \end{matrix} \qquad b. \begin{matrix} f_{0.5} & f_3 & f_{\sqrt{11}} & f_x & \dots \\ t = 0.5, 3, \sqrt{11}, x, \dots \end{matrix}$$

$$t = \begin{array}{|c|c|c|c|c|c|c|c|c|c|c|c|c|c|} \hline & f_1 & f_1 & f_2 & f_1 & f_2 & f_2 & f_2 & f_n & f_3 & \cdots & \cdots & \cdots \\ \hline c. & A & B & C & D & E & \cdots \\ \hline & \Delta t & \cdots & \cdots & \cdots \\ \hline \end{array}$$

$$\Delta t = \Delta t$$

Example *c.* is the most general since continuous evolution is sectioned into slices of a single thickness Δt , which transforms it in discontinuity; this makes it much easier to isolate and examine under the magnifying glass.

GRAPHICAL REPRESENTATIONS

We can plot the values of pure frequencies in units of octaves or semitones on the abscissa axis, and the intensity values in decibels on the ordinate axis, using logarithmic scales (see Fig. II-1). This cloud of points is the cylindrical projection on the plane (*FG*) of the grains contained in a thin slice Δt (see Fig. II-2). The graphical representations Figs. II-2 and II-3 make more tangible the abstract possibilities raised up to this point.

Psychophysiology

We are confronted with a cloud of evolving points. This cloud is the product of the two sets F and G in the slice of time Δt . What are the possible

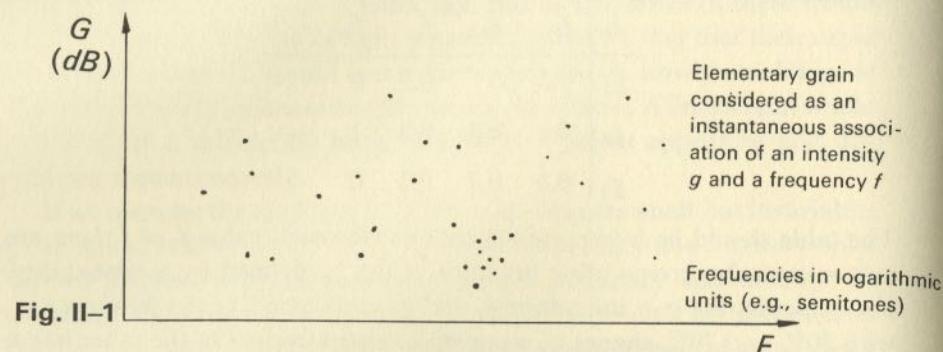


Fig. II-1

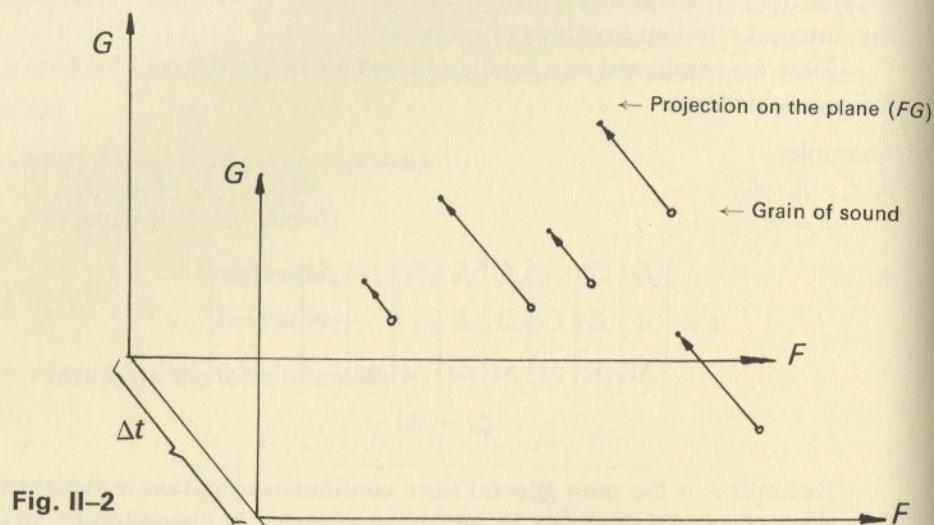


Fig. II-2

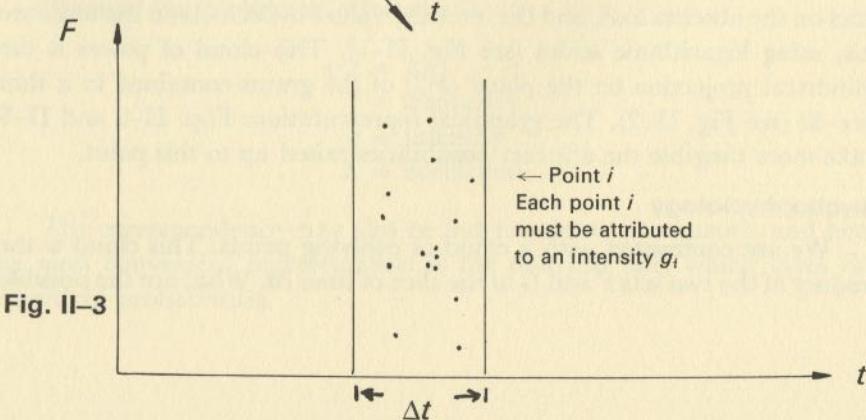


Fig. II-3

restrictive limits of human psychophysiology? What are the most general manipulations which may be imposed on the clouds and their transformations within psychophysiological limits?

The basic abstract hypothesis, which is the granular construction of all possible sounds, gives a very profound meaning to these two questions. In fact within human limits, using all sorts of manipulations with these grain clusters, we can hope to produce not only the sounds of classical instruments and elastic bodies, and those sounds generally preferred in concrete music, but also sonic perturbations with evolutions, unparalleled and unimaginable until now. The basis of the timbre structures and transformations will have nothing in common with what has been known until now.

We can even express a more general supposition. Suppose that each point of these clusters represents not only a pure frequency and its satellite intensity, but an already present structure of elementary grains, ordered *a priori*. We believe that in this way a sonority of a second, third, or higher order can be produced.

Recent work on hearing has given satisfactory answers to certain problems of perception. The basic problems which concern us and which we shall suppose to be resolved, even if some of the solutions are in part lacking, are [2, 3]: 1. What is the minimum perceptible duration (in comfort) of a sinusoidal sound, as a function of its frequency and its intensity? 2. What are the minimum values of intensities in decibels compatible with minimum frequencies and durations of sinusoidal sounds? 3. What are the minimum melodic interval thresholds, as a function of register, intensity, and duration? A good approximation is the Fletcher-Munson diagram of equal loudness contours (see Fig. II-4).

The total number of elementary audible grains is about 340,000. The ear is more sensitive at the center of the audible area. At the extremities it perceives less amplitude and fewer melodic intervals, so that if one wished to represent the audible area in a homogeneous manner using the coordinates F and G , i.e., with each surface element $\Delta F \Delta G$ containing the same density of grains of perceptible sounds, one would obtain a sort of *mappa mundi* (Fig. II-5).

In order to simplify the reasoning which will follow without altering it, we shall base our argument on Fletcher's diagram and suppose that an appropriate one-to-one transformation applied to this group of coordinates will change this curved space into an ordinary rectangle (Fig. II-6).

All the above experimental results were established in ideal conditions and without reference to the actual complexity of the natural sounds of the orchestra and of elastic bodies in general, not to mention the more complex

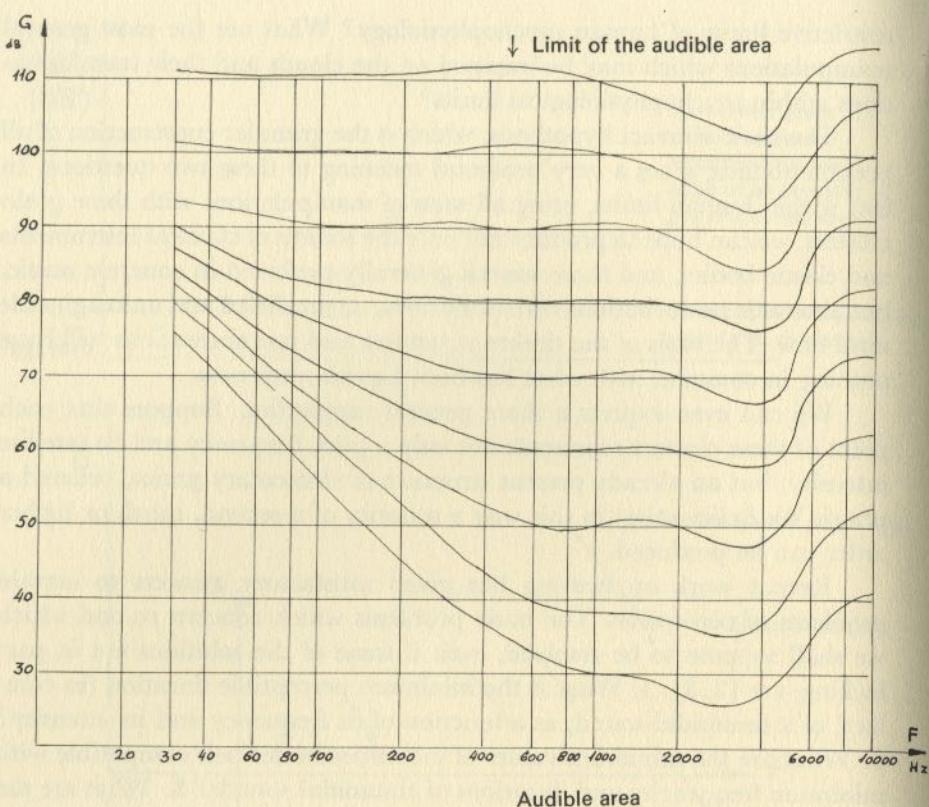


Fig. II-4. Fletcher-Munson Diagram

Equal Loudness Contours

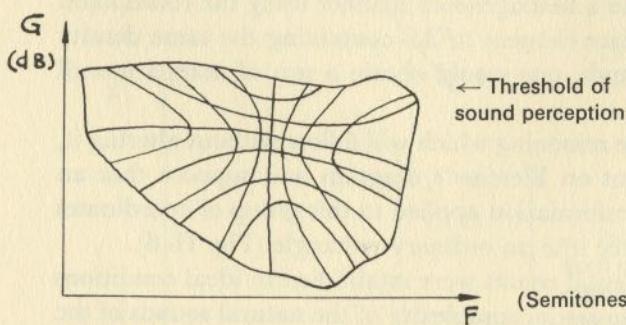
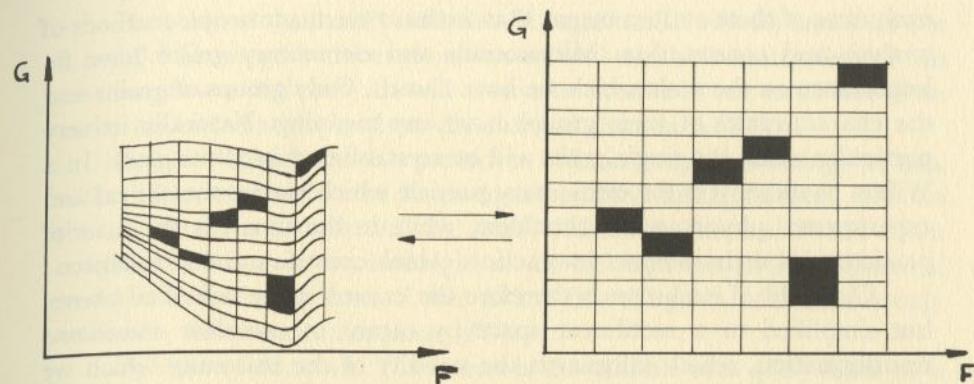


Fig. II-5



sounds of industry or of chaotic nature [4]. Theoretically [5] a complex sound can only be exhaustively represented on a three-dimensional diagram F, G, t , giving the instantaneous frequency and intensity as a function of time. But in practice this boils down to saying that in order to represent a momentary sound, such as a simple noise made by a car, months of calculations and graphs are necessary. This impasse is strikingly reminiscent of classical mechanics, which claimed that, given sufficient time, it could account for all physical and even biological phenomena using only a few formulae. But just to describe the state of a gaseous mass of greatly reduced volume at one instant t , even if simplifications are allowed at the beginning of the calculation, would require several centuries of human work!

This was a false problem because it is useless; and as far as gaseous masses are concerned, the Maxwell-Boltzmann kinetic theory of gases, with its statistical method, has been very fruitful [6]. This method re-established the value of scales of observation. For a macroscopic phenomenon it is the massed total result which counts, and each time a phenomenon is to be observed the scale relationship between observer and phenomenon must first be established. Thus if we observe galactic masses, we must decide whether it is the movement of the whole mass, the movement of a single star, or the molecular constitution of a minute region on a star that interests us.

The same thing holds true for complex as well as quite simple sounds. It would be a waste of effort to attempt to account analytically or graphically for the characteristics of complex sounds when they are to be used in an electromagnetic composition. For the manipulation of these sounds macroscopic methods are necessary.

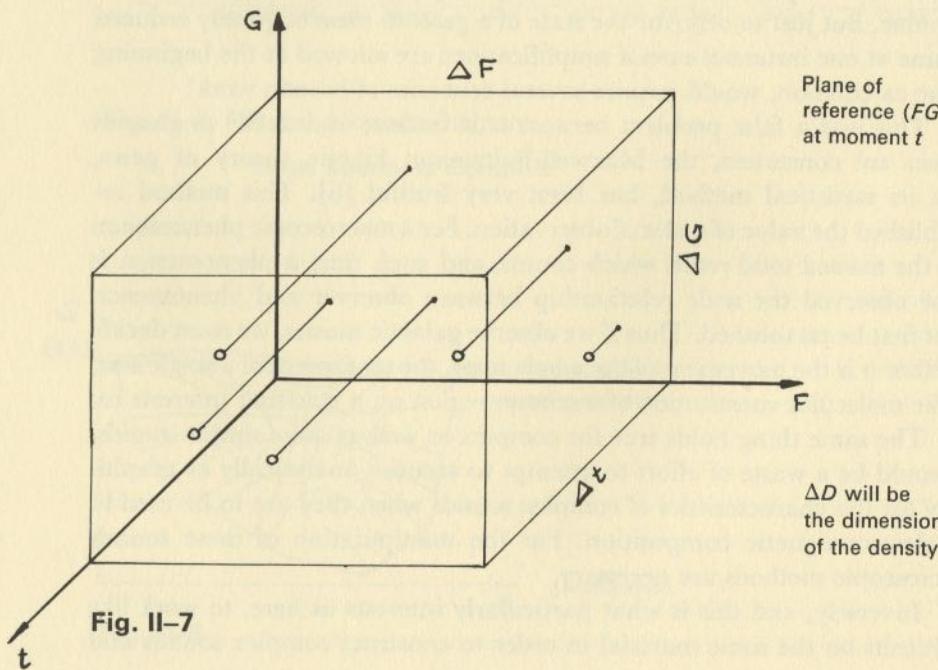
Inversely, and this is what particularly interests us here, to work like architects on the sonic material in order to construct complex sounds and

evolutions of these entities means that we must use macroscopic methods of analysis and construction. Microsounds and elementary grains have no importance on the scale which we have chosen. Only groups of grains and the characteristics of these groups have any meaning. Naturally in very particular cases, the single grain will be reestablished in all its glory. In a Wilson chamber it is the elementary particle which carries theoretical and experimental physics on its shoulders, while in the sun it is the mass of particles and their compact interactions which constitute the solar object.

Our field of evolution is therefore the curved space described above, but simplified to a rectilinear space by means of complete one-to-one transformation, which safeguards the validity of the reasoning which we shall pursue.

SCREENS

The graphical representation of a cloud of grains in a slice of time Δt examined earlier brings a new concept, that of the density of grains per unit of volume, $\Delta F \Delta G \Delta t$ (Fig. II-7). Every possible sound may therefore be cut up into a precise quantity of elements $\Delta F \Delta G \Delta t \Delta D$ in four dimensions, distributed in this space and following certain rules defining this sound, which are summarized by a function with four variables: $s(F, G, D, t)$.



The scale of the density will also be logarithmic with its base between 2 and 3.² To simplify the explanation we will make an abstraction of this new coordinate of density. It will always be present in our mind but as an entity associated with the three-dimensional element $\Delta F \Delta G \Delta t$.

If time is considered as a procedure of lexicographic ordering, we can, without loss, assume that the Δt are equal constants and quite small. We can thus reason on a two-dimensional space defined by the axes F and G , on condition that we do not lose sight of the fact that the cloud of grains of sound exists in the thickness of time Δt and that the grains of sound are only artificially flattened on the plane (FG) .

Definition of the screen. The screen is the audible area (FG) fixed by a sufficiently close and homogeneous grid as defined above, the cells of which may or may not be occupied by grains. In this way any sound and its history may be described by means of a sufficiently large number of sheets of paper carrying a given screen S . These sheets are placed in a fixed lexicographic order (see Fig. II-8).

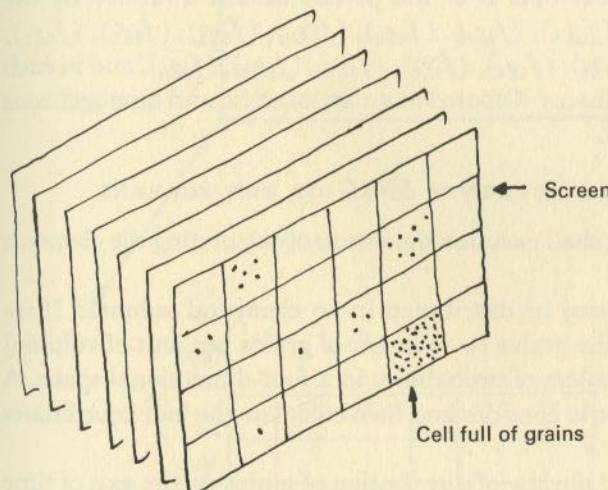


Fig. II-8

A book of screens equals the life of a complex sound

The clouds of grains drawn on the screens will differ from one screen to another by their geographical or topographical position and by their surface density (see Fig. II-9). Screen A contains a small elemental rectangle with a small cluster of density d of mean frequency f and mean amplitude g . It is almost a pure sound. Screen B represents a more complex sound with strong high and low areas but with a weak center. Screen C represents a

"white" sound of weak density which may therefore be perceived as a sonic sheen occupying the whole audible area.

What is important in all the statements made up to now is that nothing has been said about the topographic fixity of the grains on the screens. All natural or instrumental sounds are composed of small surface elements filled with grains which fluctuate around a mean frequency and intensity. The same holds for the density. This statement is fundamental, and it is very likely that the failure of electronic music to create new timbres, aside from the inadequacy of the serial method, is largely due to the fixity of the grains, which form structures like packets of spaghetti (Fig. II-10).

Topographic fixity of the grains is a very particular case, the most general case being mobility and the statistical distribution of grains around positions of equilibrium. Consequently in the majority of cases real sounds can be analyzed as quite small rectangles, $\Delta F \Delta G$, in which the topographic positions and the densities vary from one screen to another following more or less well-defined laws.

Thus the sound of example *D* at this precise instant is formed by the collection of rectangles $(f_2 g_4)$, $(f_2 g_5)$, $(f_4 g_2)$, $(f_4 g_3)$, $(f_5 g_1)$, $(f_5 g_2)$, $(f_6 g_1)$, $(f_6 g_2)$, $(f_6 g_5)$, $(f_7 g_2)$, $(f_7 g_3)$, $(f_7 g_4)$, $(f_7 g_5)$, $(f_8 g_3)$, $(f_8 g_4)$, $(f_8 g_5)$, and in each of the rectangles the grains are disposed in an asymmetric and homogeneous manner (see Fig. II-11).

CONSTRUCTION OF THE ELEMENTS $\Delta F \Delta G$ OF THE SCREENS

1. *By calculation.* We shall examine the means of calculating the elements $\Delta F \Delta G \Delta t \Delta D$.

How should the grains be distributed in an elemental volume? If we fix the mean density of the grains (= number of grains per unit of volume) we have to resolve a problem of probability in a four-dimensional space. A simpler method would be to consider and then calculate the four coordinates independently.

For the coordinate *t* the law of distribution of grains on the axis of time is:

$$P_x = c e^{-cx} dx \quad \text{or} \quad P_{x_i} = e^{-civ} c \Delta x_i. \quad (r) \quad (\text{See Appendix I.})$$

For the coordinates *G*, *F*, *D* the stochastic law will be:

$$f(j) dj = \frac{2}{a} \left(1 - \frac{j}{a}\right) dj \quad (r')$$

or $P_i = \frac{2}{2 \cdot 10^n} \left(1 - \frac{i}{2 \cdot 10^n - 1}\right). \quad (\text{See Appendix I.})$

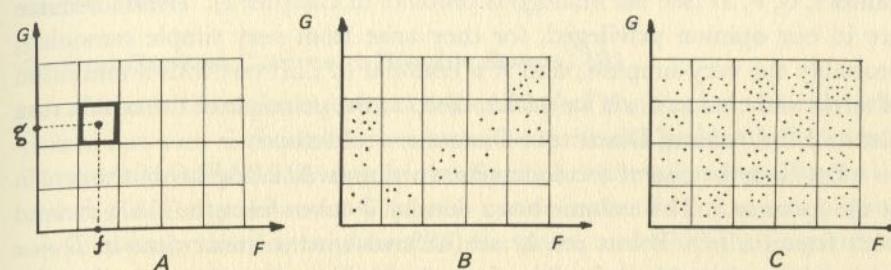


Fig. II-9

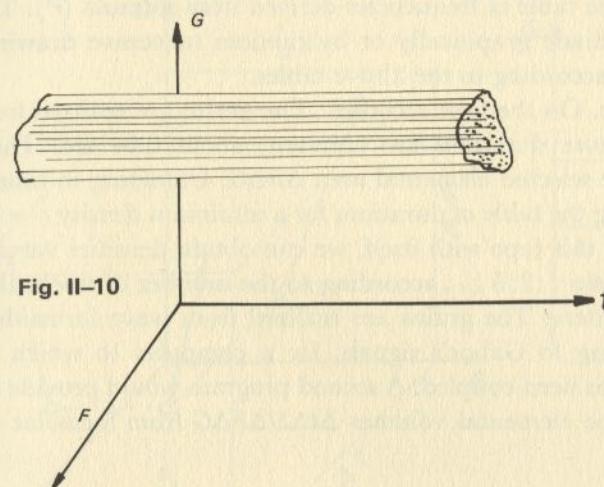


Fig. II-10

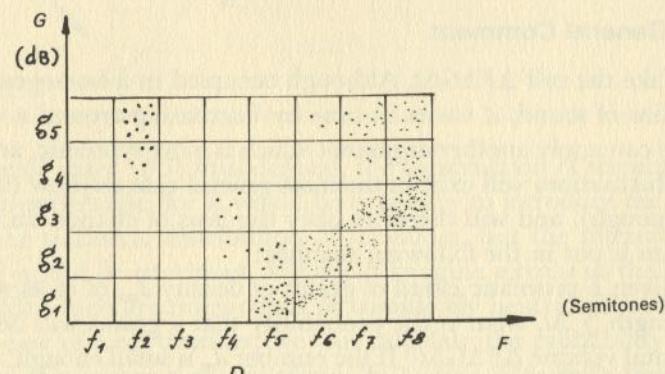


Fig. II-11

From these formulae we can draw up tables of the frequencies of the values t, G, F, D (see the analogous problem in Chapter I). These formulae are in our opinion privileged, for they arise from very simple reasoning, probably the very simplest; and it is essential to start out with a minimum of terms and constraints if we wish to keep to the principle of the *tabula rasa* (1st and 3rd rules of Descartes's Discourse on Method).

Let there be one of these elemental volumes $\Delta t \Delta D \Delta F \Delta G$ of the screen at the moment t . This volume has a density D taken from the table derived from formula (r') . Points on Δt are defined with a linear density $D = c$ according to the table defined by formula (r) . To each point is attributed a sonic grain of frequency f and intensity g , taken from within the rectangle $\Delta F \Delta G$ by means of the table of frequencies derived from formula (r') . The correspondences are made graphically or by random successive drawings from urns composed according to the above tables.

2. *Mechanically.* a. On the tape recorder: The grains are realized from sinusoidal sounds whose durations are constant, about 0.04 sec. These grains must cover the selected elemental area $\Delta F \Delta G$. Unfolding in time is accomplished by using the table of durations for a minimum density $c = D$. By mixing sections of this tape with itself, we can obtain densities varying geometrically with ratio 1:2:3... according to the number of tracks that we use. b. On computers: The grains are realized from wave forms duly programmed according to Gabor's signals, for a computer to which an analogue converter has been coupled. A second program would provide for the construction of the elemental volumes $\Delta t \Delta D \Delta F \Delta G$ from formulae (r) and (r') .

First General Comment

Take the cell $\Delta F \Delta G \Delta t$. Although occupied in a homogeneous manner by grains of sound, it varies in time by fluctuating around a mean density d_m . We can apply another argument which is more synthetic, and admit that these fluctuations will exist in the most general case anyhow (if the sound is long enough), and will therefore obey the laws of chance. In this case, the problem is put in the following manner:

Given a prismatic cloud of grains of density d_m , of cross section $\Delta F \Delta G$ and length $\sum \Delta t$, what is the probability that d grains will be found in an elemental volume $\Delta F \Delta G \Delta t$? If the number d_m is small enough, the probability is given by Poisson's formula:

$$P_k = \frac{(d_m)^k}{K!} e^{-d_m}.$$

For the definition of each grain we shall again use the methods described above.

Second General Comment (Vector Space) [8]

We can construct elemental cells $\Delta F \Delta G$ of the screens not only with points, but with elemental vectors associated with the grains (vector space). The mean density of 0.04 sec/grain really implies a small vector. The particular case of the grain occurs when the vector is parallel to the axis of time, when its projection on the plane (FG) is a point, and when the frequency of the grain is constant. In general, the frequencies and intensities of the grains can be variable and the grain a very short glissando (see Fig. II-12).

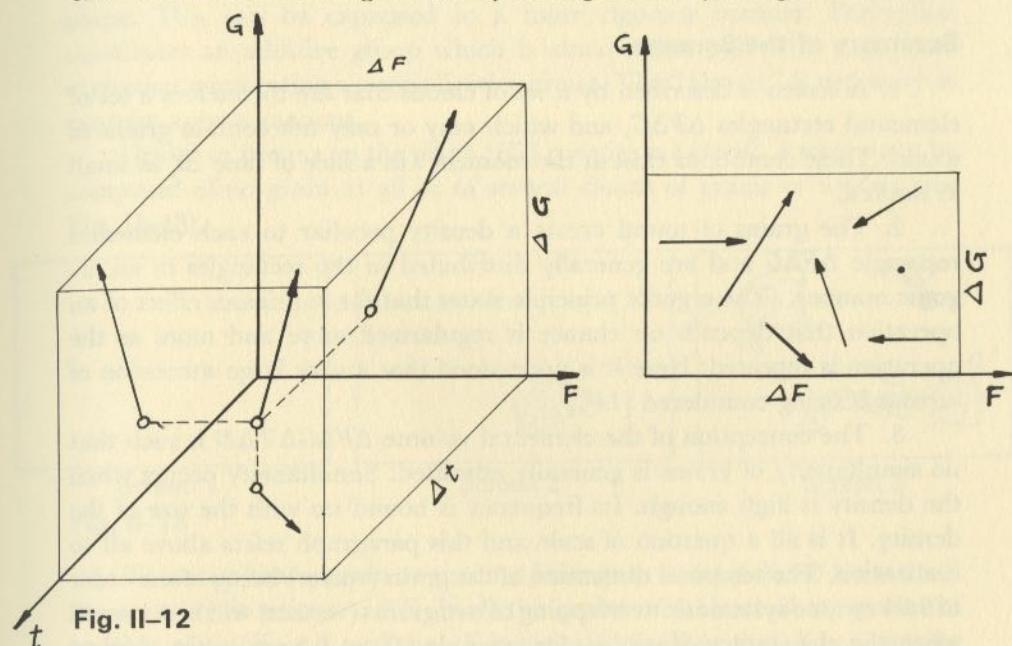


Fig. II-12

In a vector space (FG) thus defined, the construction of screens would perhaps be cumbersome, for it would be necessary to introduce the idea of speed and the statistical distribution of its values, but the interest in the undertaking would be enormous. We could imagine screens as the basis of granular fields which are magnetized or completely neutral (disordered).

In the case of total disorder, we can calculate the probability $f(v)$ of the existence of a vector v on the plane (FG) using Maxwell's formula as applied to two dimensions [11]:

$$f(v) = \frac{2v}{a^2} e^{-v^2/a^2}.$$

For the mean value $v_1 \leq v_m \leq v_2$,

$$P(v_m) = \frac{2\sqrt{\pi}}{a} \left\{ \theta(\lambda_1) - \theta(\lambda_2) \right\}$$

in which $\lambda_i = v_i/a$ and

$$\theta(\lambda_i) = \frac{1}{\sqrt{\pi}} \int_{-\lambda_i}^{+\lambda_i} e^{-\lambda^2} d\lambda,$$

for $\lambda_1 \leq \lambda \leq \lambda_2$ (normal Gaussian law) [12]. In any case, whether it is a matter of a vector space or a scalar space does not modify the arguments [13].

Summary of the Screens

1. A screen is described by a set of clouds that are themselves a set of elemental rectangles $\Delta F \Delta G$, and which may or may not contain grains of sound. These conditions exist at the moment t in a slice of time Δt , as small as desired.

2. The grains of sound create a density peculiar to each elemental rectangle $\Delta F \Delta G$ and are generally distributed in the rectangles in an ergodic manner. (The ergodic principle states that the capricious effect of an operation that depends on chance is regularized more and more as the operation is repeated. Here it is understood that a very large succession of screens is being considered [14].)

3. The conception of the elemental volume $\Delta F \Delta G \Delta T \Delta D$ is such that no simultaneity of grains is generally admitted. Simultaneity occurs when the density is high enough. Its frequency is bound up with the size of the density. It is all a question of scale and this paragraph refers above all to realization. The temporal dimension of the grain (vector) being of the order of 0.04 sec., no systematic overlapping of two grains (vectors) will be accepted when the elementary density is, for example, $D_0 = 1.5$ grains/sec. And as the surface distribution of the grains is homogeneous, only chance can create this overlapping.

4. The limit for a screen may be only one pure sound (sinusoidal), or even no sound at all (empty screen).

ELEMENTARY OPERATIONS ON SCREENS

Let there be a complex sound. At an instant t of its life during a thickness Δt it can be represented by one or several clouds of grains or vectors on the plane (FG). This is the definition which we gave for the

screen. The junction of several of these screens in a given order describes or prescribes the life of this sonic complex. It would be interesting to envisage in all its generality the manner of combining and juxtaposing screens to describe, and above all to construct, sonic evolutions, which may be continuous or discontinuous, with a view to playing with them in a composition. To this end we shall borrow the terminology and symbolism of modern algebra, but in an elementary manner and as a form of introduction to a further development which we shall not undertake at the moment.

Comment: It does not matter whether we place ourselves on the plane of physical phenomena or of perception. In general, on the plane of perception we consider arithmetically that which is geometrical on the physical plane. This can be expressed in a more rigorous manner. Perception constitutes an additive group which is *almost* isomorphic with a physical excitation constituting a multiplicative group. The “*almost*” is necessary to exorcise approximations.

Grains or vectors on the plane (FG) constitute a cloud. A screen can be composed of no grain at all or of several clouds of grains or vectors (see Fig. II-13).

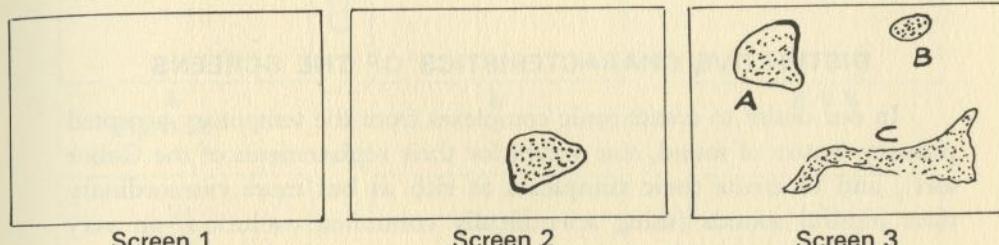


Fig. II-13

To note that a grain or vector a belongs to a cloud E , we write $a \in E$ the contrary is written $a \notin E$. If all the grains of a cloud X are grains of another cloud Y , it is said that X is included in Y or that X is a part or sub-cloud of Y . This relation is notated $X \subset Y$ (inclusion).

Consequently we have the following properties:

$$X \subset X \text{ for any } X.$$

$$X \subset Y \text{ and } Y \subset Z \text{ imply } X \subset Z.$$

When $X \subset Y$ and $Y \subset X$, the clouds X and Y consist of the same grains; they are indistinguishable and the relation is written: $X = Y$ (equation).

A cloud may contain as little as a single grain. A cloud X is said to be empty when it contains no grain a , such that $a \in X$. The empty cloud is notated \emptyset .

ELEMENTARY OPERATIONS

These operations apply equally well to clouds and to screens. We can therefore use the terms "screen" and "cloud" indiscriminately, with cloud and grain as "constitutive elements."

The *intersection* of two screens A and B is the screen of clouds which belong to both A and B . This is notated as $A \cap B$ and read as " A inter B " (Fig. II-14). When $A \cap B = \emptyset$, A and B are said to be *disjoint* (Fig. II-15). The *union* of two screens A and B is the set of clouds which belong to both A and/or B (Fig. II-16). The *complement* of a screen A in relation to a screen E containing A is the set of clouds in E which do *not* belong to A . This is notated $C_E A$ when there is no possible uncertainty about E (Fig. II-17). The *difference* $(A - B)$ of A and B is the set of clouds of A which do not belong to B . The immediate consequence is $A - B = A - (A \cap B) = C_A(A \cap B)$ (Fig. II-18).

We shall stop this borrowing here; however, it will afford a stronger, more precise conception on the whole, better adapted for the manipulations and arguments which follow.

DISTINCTIVE CHARACTERISTICS OF THE SCREENS

In our desire to create sonic complexes from the temporary accepted primary matter of sound, sine waves (or their replacements of the Gabor sort), and to create sonic complexes as rich as but more extraordinary than natural sounds (using scientifically controlled evolutions on very general abstract planes), we have implicitly recognized the importance of three basic factors which seem to be able to dominate both the theoretical construction of a sonic process and its sensory effectiveness: 1. the density of the elementary elements, 2. the topographic situation of events on the screens, and 3. the order or disorder of events.

At first sight then the density of grains or vectors, their topography, and their degree of order are the indirect entities and aspects perceived by our macroscopic ears. It is wonderful that the ear and the mind follow objective reality and react directly in spite of gross inherent or cultural imperfections. Measurement has been the foundation of the experimental sciences. Man voluntarily treats himself as a sensory invalid, and it is for this reason that he has armed himself, justifiably, with machines that measure other machines. His ears and eyes do measure entities or physical phenomena, but they are transformed as if a distorting filter came between immediate perception and consciousness. About a century ago the logarithmic

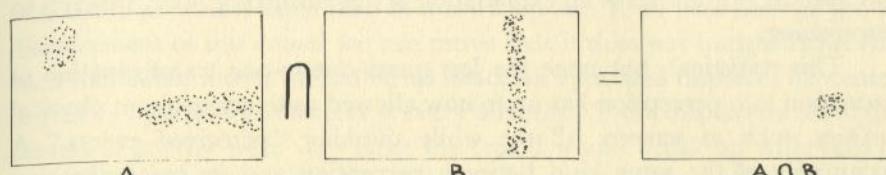


Fig. II-14

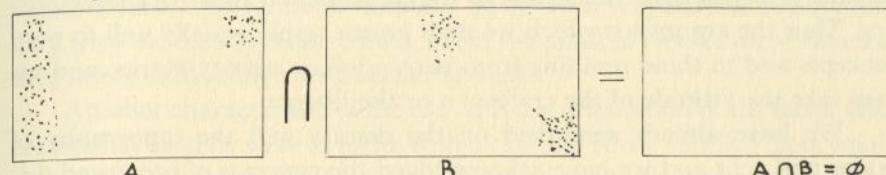


Fig. II-15

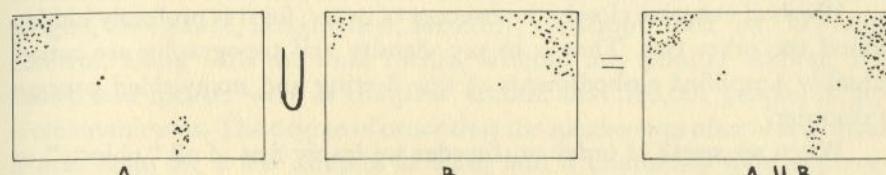


Fig. II-16

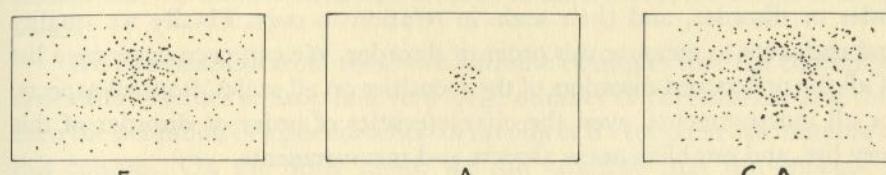


Fig. II-17

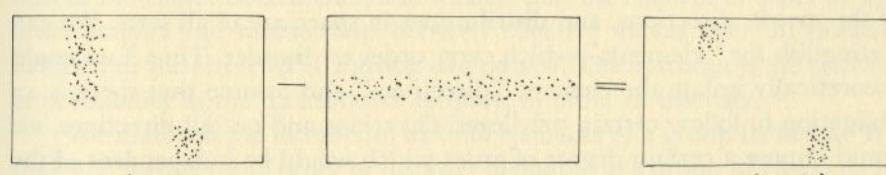


Fig. II-18

law of sensation was discovered; until now it has not been contradicted. But as knowledge never stops in its advance, tomorrow's science will without doubt find not only a greater flexibility and exactitude for this law, but also the beginnings of an explanation of this distorting filter, which is so astonishing.

This statistical, but none the less quasi-one-to-one transformation of excitation into perception has up to now allowed us to argue about physical entities, such as screens, all the while thinking "perceived events." A reciprocity of the same kind between perception and its comprehension permits us to pass from the screens to the consequent distinctive characteristics. Thus the arguments which we shall pursue apply equally well to pure concepts and to those resulting from perception or sensory events, and we may take the attitude of the craftsman or the listener.

We have already remarked on the density and the topography of grains and cells and we have acknowledged the concepts of order and disorder in the homogeneous superficial distribution or grains.

We shall examine closely the concept of order, for it is probably hidden behind the other two. That is to say, density and topography are rather palpably simplified embodiments of this fleeting and many-sided concept of disorder.

When we speak of order or disorder we imply first of all "objects" or "elements." Then, and this is already more complex, we define the very "elements" which we wish to study and from which we wish to construct order or disorder, and their scale in relation to ours. Finally we qualify and endeavor to measure this order or disorder. We can even draw up a list of all the orders and disorders of these entities on all scales, from all aspects, for all measurements, even the characteristics of order or disorder of this very list, and establish anew aspects and measurements.

Take the example of the gases mentioned above. On the molecular scale (and we could have descended to the atomic level), the absolute values of the speeds, directions, and distributions in space are of all sorts. We can distinguish the "elements" which carry order or disorder. Thus if we could theoretically isolate the element "directions" and assume that there is an obligation to follow certain privileged directions and not all directions, we could impose a certain degree of order which would be independent of the other elements constituting the concept "gas." In the same way, given enough time, the values of the speeds of a single molecule will be distributed around a mean value and the size of the deviations will follow Gauss's law. There we will have a certain order since these values are vastly more

numerous in the neighborhood of their mean than anywhere else, from infinitely small to infinitely large.

Let us take another example, more obvious and equally true. A crowd of 500,000 persons is assembled in a town square. If we examine the group displacement of this crowd we can prove that it does not budge. However, each individual moves his limbs, his head, his eyes, and displaces his center of gravity by a few centimeters in every direction. If the displacements of the centers of gravity were very large the crowd would break up with yells of terror because of the multiple collisions between individuals. The statistical values of these displacements normally lie between very narrow limits which vary with the density of the crowd. From the point of view of these values as they affect immobility, the disorder is weak.

Another characteristic of the crowd is the orientation of the faces. If an orator on a balcony were to speak with a calming effect, 499,000 faces would look at the balcony and 998,000 ears would listen to the honeyed words. A thousand or so faces and 2000 ears would be distracted for various reasons: fatigue, annoyance, imagination, sexuality, contempt, theft, etc. We could confirm, along with the mass media, without any possible dispute, that crowd and speaker were in complete accord, that 500,001 people, in fact, were unanimous. The degree of order that the speaker was after would attain a maximum for a few minutes at least, and if unanimity were expressed equally strongly at the conclusion of the meeting, the orator could be persuaded that the ideas were as well ordered in the heads of the crowd as in his own.

We can establish from these two extreme examples that the concept of order and disorder is basic to a very large number of phenomena, and that even the definition of a phenomenon or an object is very often attributable to this concept. On the other hand, we can establish that this concept is founded on precise and distinct groups of elements; that the scale is important in the choice of elements; and finally, that the concept of order or disorder implies the relationship between effective values over all possible values that the elements of a group can possess. This introduces the concept of probability in the quantitative estimate of order or disorder.

We shall call the number of distinct elements in a group its *variety*. We shall call the degree of order or disorder definable in a group of elements its *entropy*. Entropy is linked with the concept of variety, and for that very reason, it is linked to the probability of an element in the group. These concepts are those of the theory of communications, which itself borrows from the second law of thermodynamics (Boltzmann's theorem *H*) [15].

Variety is expressed as a pure number or as its logarithm to the base 2. Thus human sex has two elements, male and female, and its variety is 2, or 1 bit: 1 bit = $\log_2 2$.

Let there be a group of probabilities (a group of real numbers p , positive or zero, whose sum is 1). The entropy H of this group is defined as

$$H = -K \sum p_i \log p_i.$$

If the logarithmic base is 2, the entropy is expressed in bits. Thus if we have a sequence of heads and tails, the probability of each is $\frac{1}{2}$, and the entropy of this sequence, i.e., its uncertainty at each throw, will be 1 bit. If both sides of the coin were heads, the uncertainty would be removed and the entropy H would be zero.

Let us suppose that the advent of a head or a tail is not controlled by tossing the coin, but by a fixed, univocal law, e.g., heads at each even toss and tails at each odd toss. Uncertainty or disorder is always absent and the entropy is zero. If the law becomes very complex the appearance of heads or tails will seem to a human observer to be ruled by the law of chance, and disorder and uncertainty will be reestablished. What the observer could do would be to count the appearances of heads and tails, add up their respective frequencies, deduce their probabilities, and then calculate the entropy in bits. If the frequency of heads is equal to that of tails the uncertainty will be maximum and equal to 1 bit.

This typical example shows roughly the passage from order to disorder and the means of calibrating this disorder so that it may be compared with other states of disorder. It also shows the importance of scale. The intelligence of the observer would assimilate a deterministic complexity up to a certain limit. Beyond that, in his eyes, the complexity would swing over into unforeseeability and would become chance or disorder; and the visible (or macroscopic) would slide into the invisible (or microscopic). Other methods and points of view would be necessary to observe and control the phenomena.

At the beginning of this chapter we admitted that the mind and especially the ear were very sensitive to the order or disorder of phenomena. The laws of perception and judgment are probably in a geometrical or logarithmic relation to the laws of excitation. We do not know much about this, and we shall again confine ourselves to examining general entities and to tracing an overall orientation of the poetic processes of a very general kind of music, without giving figures, moduli, or determinisms. We are still optimistic enough to think that the interdependent experiment and action of abstract

hypotheses can cut biologically into the living conflict between ignorance and reality (if there is any reality).

Study of Ataxy (order or disorder) on the Plane of a Cloud of Grains or Vectors

Axis of time: The degree of ataxy, or the entropy, is a function of the simultaneity of the grains and of the distinct intervals of time between the emission of each grain. If the variety of the durations of the emissions is weak, the entropy is also weak. If, for example, in a given Δt each grain is emitted at regular intervals of time, the temporal variety will be 1 and the entropy zero. The cloud will have zero ataxy and will be completely ordered. Conversely, if in a fairly long succession of Δt the grains are emitted according to the law $P_x = \delta e^{-\delta x} dx$, the degree of ataxy will be much larger. The limit of entropy is infinity, for we can imagine all possible values of time intervals with an equal probability. Thus, if the variety is $n \rightarrow \infty$, the probability for each time interval is $p_i = 1/n$, and the entropy is

$$H = -K \sum_{i=0}^n p_i \log p_i$$

$$H = -K \sum_{i=0}^n \frac{1}{n} \log \frac{1}{n} = -K n \frac{1}{n} \log \frac{1}{n} = -K \log \frac{1}{n} = K \log n$$

for $n \rightarrow \infty$, $H \rightarrow \infty$.

This is less true in practice, for a Δt will never offer a very great variety of durations and its entropy will be weak. Furthermore a sonic composition will rarely have more than 100,000 Δt 's, so that $H \leq \log 100,000$ and $H \leq 16.6$ bits.

Axis of frequencies (melodic): The same arguments are valid here but with greater restriction on the variety of melodic intervals and on the absolute frequencies because of the limits of the audible area.

Entropy is zero when the variety of frequencies of grains is 1, i.e., when the cloud contains only one pure sound.

Axis of intensity and density: The above observations are valid. Therefore, if at the limit, the entropies following the three axes of an element $\Delta F \Delta G \Delta t \Delta D$ are zero, this element will only contain one pure sound of constant intensity emitted at regular intervals.

In conclusion, a cloud may contain just one single pure sound emitted at regular intervals of time (see Fig. II-19), in which case its mean entropy (arithmetic mean of the three entropies) would be zero. It may contain chaotically distributed grains, with maximum ataxy and maximum mean

entropy (theoretically ∞). Between these two limits the grains may be distributed in an infinite number of ways with mean entropies between 0 and the maximum and able to produce both the Marseillaise and a raw, dodecaphonic series.

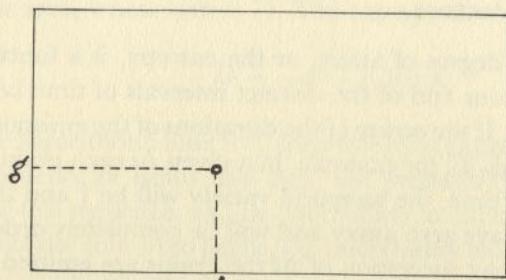


Fig. II-19

A single grain emitted at regular intervals of time

Parentheses

GENERAL OBSERVATIONS ON ATAXY

Taking this last possibility as a basis, we shall examine the very general formal processes in all realms of thought, in all physical and psychic realities.

To this end we shall imagine a "Primary Thing," malleable at will; capable of deforming instantaneously, progressively, or step-by-step; extendible or retractable; unique or plural; as simple as an electron (!) or as complex as the universe (as compared to man, that is).

It will have a given mean entropy. At a defined time we will cause it to undergo a transformation. From the point of view of ataxy this transformation can have one of three effects:

1. The degree of complexity (variety) does not change; the transformation is neutral; and the overall entropy does not change.
2. The degree of complexity increases and so does the entropy.
3. The transformation is a simplifying one, and the entropy diminishes.

Thus the neutral transformation may act on and transform: perfect disorder into perfect disorder (fluctuations), partial disorder into partial disorder, and perfect order into perfect order.

Multiplicative transformation transforms: perfect disorder into perfect disorder, partial disorder into greater disorder, and perfect order into partial disorder.

And simplifying transformation transforms: perfect disorder into partial disorder, partial order into greater order, and partial order into perfect order. Fig. II-20 shows these transformations in the form of a kinematic diagram.

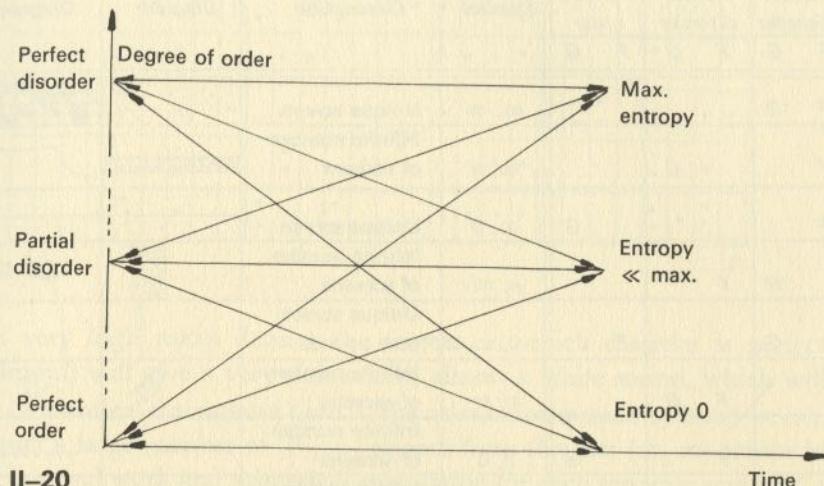


Fig. II-20

STUDY OF ATAXY AT THE LEVEL OF SCREENS (SET OF CLOUDS)

From the above discussion, a screen which is composed of a set of cells $\Delta F \Delta G$ associated with densities during a slice of time Δt , may be dissociated according to the two characters of the grains, frequency and amplitude, and affected by a mean entropy. Thus we can classify screens according to the criterion of ataxy by means of two parameters of disorder: the variety of the frequencies and the variety of the intensities. We shall make an abstraction of the temporal distribution of the grains in Δt and of the density, which is implicitly bound up with the varieties of the two fundamental sizes of the grain. In symbolic form:

$$\begin{aligned} \text{Perfect disorder} &= \infty \\ \text{Partial disorder} &= n \text{ or } m \\ \text{Partial order} &= m \text{ or } n \\ \text{Perfect order} &= 0. \end{aligned}$$

From the point of view of ataxy a screen is formulated by a pair of entropy values ascribed to a pair of frequencies and intensities of its grains. Thus the pair (n, ∞) means a screen whose frequencies have quite a small entropy (partial order or disorder) and whose intensities have maximum entropy (more or less perfect disorder).

CONSTRUCTION OF THE SCREENS

We shall quickly survey some of the screens in the entropy table in Fig. II-21.

Perfect disorder	Partial disorder	Perfect order	Symbol	Description	Diagram	Diagram
F	G	F	G			
F	G		∞, ∞	Unique screen		
F		G	∞, n	Infinite number of screens		
F			$\infty, 0$	Unique screen		
G	F		n, ∞	Infinite number of screens		
G		F	$0, \infty$	Unique screen, pure sounds		
	F	G	n, m	Infinite number of screens		
F		G	$n, 0$	Infinite number of screens		
	G	F	$0, n$	Infinite number of screens		
		F	$0, 0$	Unique screen, pure sound		

Fig. II-21. Screen Entropy Table

SCREEN (∞, ∞)

Let there be a very large number of grains distributed at random over the whole range of the audible area and lasting an interval of time equal to Δt . Let there also be a grid fine enough so that the average density will not be more than 30 grains per cell. The distribution law is then given by Poisson's formula

$$P_k = \frac{(d_m)^k}{K!} e^{-d_m},$$

where d_m is the mean density and P_k the probability that there will be k grains in a cell. If d_m becomes greater than about 30, the distribution law will become *normal*.

Fig. II-22 is an example of a Poisson distribution for a mean density $d_m = 0.6$ grains/cell in a grid of 196 cells for a screen (∞, ∞) .

Thus we may construct the (∞, ∞) screens by hand, according to the distributions for the rows and columns, or with suitable computer programs.

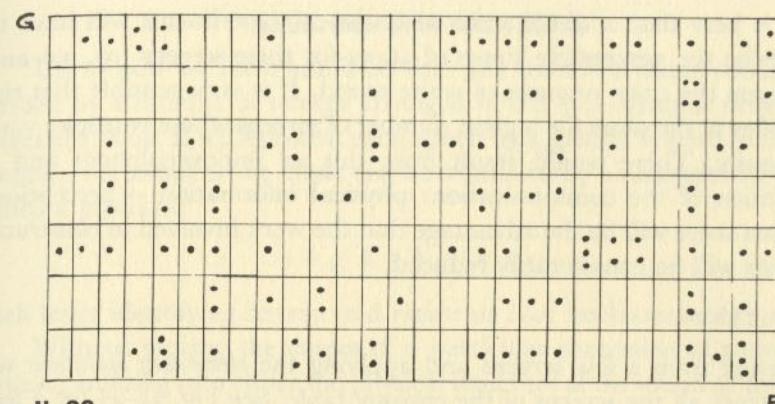


Fig. II-22

For a very high mean density the screens in which disorder is perfect (maximum) will give a very rich sound, almost a white sound, which will never be identical throughout time. If the calculation is done by hand we can construct a large number of (∞, ∞) screens from the first (∞, ∞) screen in order to avoid work and numerical calculation for each separate screen. To this end we permute the cells by column and row (see Fig. II-23).

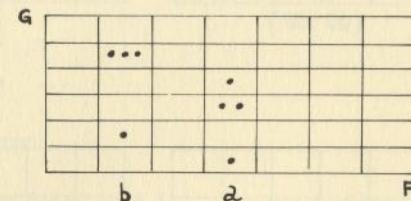
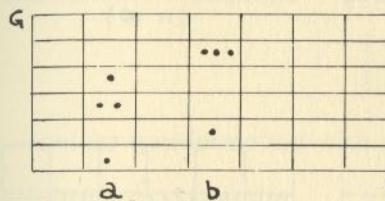


Fig. II-23. Example of Permutation by Columns

Discussion. It is obvious that for a high mean density, the greater the number of cells, the more the distribution of grains in one region of the screen tends to regularize (ergodicism) and the weaker are the fluctuations from one cell or cloud to another. But the absolute limits of the density in the cells in the audible area will be a function of the technical means available: slide rules, tables, calculating machines, computers, ruled paper, orchestral instruments, tape recorders, scissors, programmed impulses of pure sounds, automatic splicing devices, programmed recordings, analogue converters, etc.

If each cell is considered as a symbol defined by the number of grains k , the entropy of the screen (for a given fineness of grid) will naturally be affected by the mean density of the grains per cell and will grow at the same

time. It is here that a whole series of statistical experiments will have to circumscribe the perceptible limits of ataxy for these screens (∞, ∞) and even express the color nuances of white sound. It is very possible that the ear classifies in the same file a great number of screens whose entropies vary tremendously. There would result from this an impoverishment and a simplification of the communication: physical information \rightarrow perception, but at least there will be the advantage that the work involved in constructing screens will be considerably reduced.

ALL SCREENS

Starting from a few screens and applying the *elementary operations* we can construct all the screens of the entropy table. See Fig. II-24 for a few examples. In practice, frequency and intensity filters imitate these elementary operations perfectly.

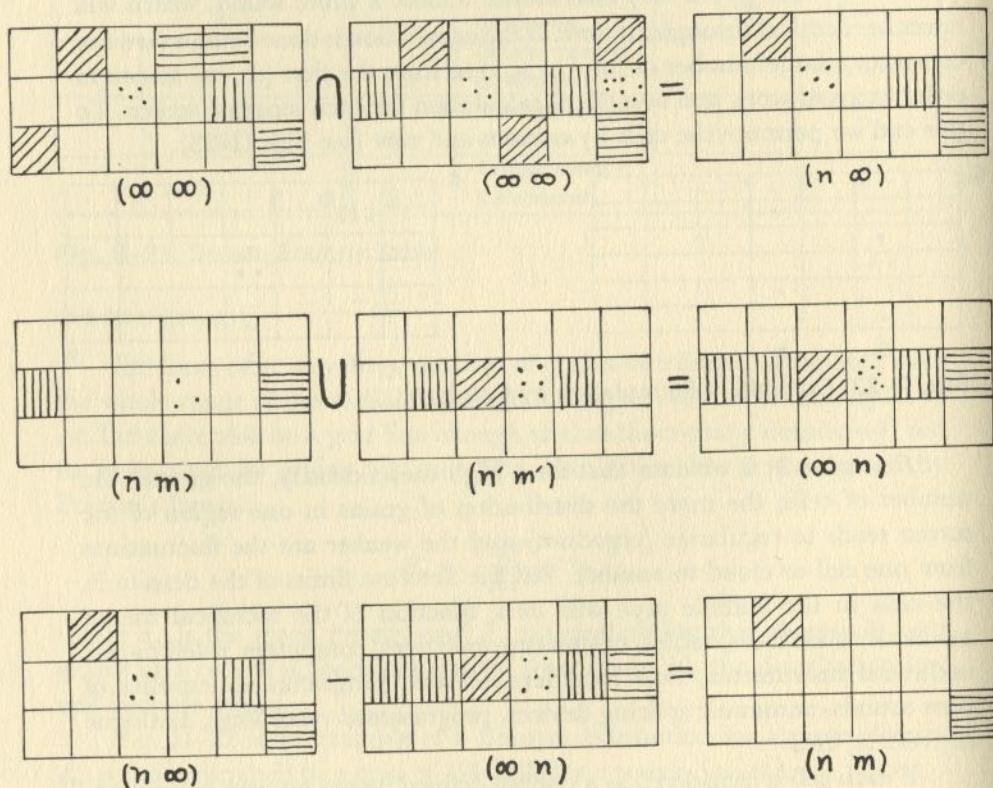


Fig. II-24

LINKING THE SCREENS

Up to now we have admitted that any sound or music could be described by a number of screens arranged in the lexicographic order of the pages of a book. If we represent each screen by a specific symbol (one-to-one coding), the sound or the music can be translated by a succession of symbols called a protocol:

$a \ b \ g \ k \ a \ b \dots b \ g \dots$

each letter identifying screens and moments t for isochronous Δt 's.

Without seeking the causes of a particular succession of screens, i.e., without entering into either the physical structure of the sound or the logical structure of the composition, we can disengage certain modes of succession and species of protocols [16]. We shall quickly review the elementary definitions.

Any matter or its unique symbol is called a *term*. Two successive terms cause a *transition* to materialize. The second term is called the *transform* and the change effected is represented by term $A \rightarrow$ term B , or $A \rightarrow B$.

A *transformation* is a collection of transitions. The following example is drawn from the above protocol:

$\begin{array}{c} a \ b \ g \ k \dots \\ \downarrow \\ b \ g \ k \ a \dots \end{array}$

another transformation with musical notes:

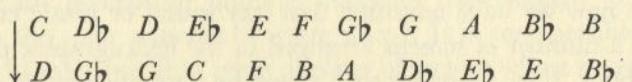
$\begin{array}{c} C \ D \ E \dots \\ \downarrow \\ B \ G \ A \dots \end{array}$

$\begin{array}{c} \downarrow \\ \text{♩} \quad \text{♩} \text{♩} \text{♩} \quad \text{♩} \quad \dots \end{array}$

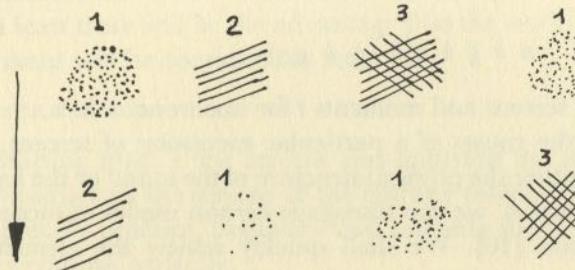
A transformation is said to be *closed* when the collection of transforms contains only elements belonging to the collection of terms, for example: the alphabet,

$\begin{array}{c} a \ b \ c \dots z \\ \downarrow \\ b \ c \ d \dots a \end{array}$

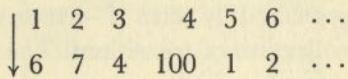
musical notes,



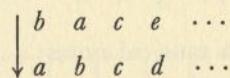
musical sounds,



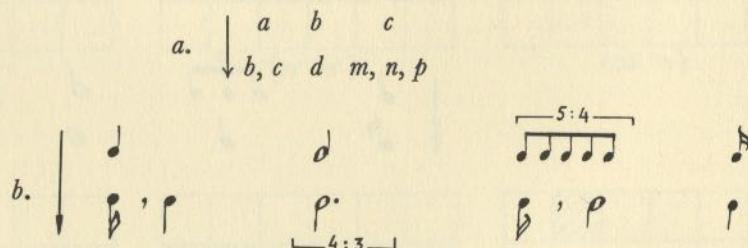
an infinity of terms,



A transformation is *univocal* or single-valued (mapping) when each term has a single transform, for example:



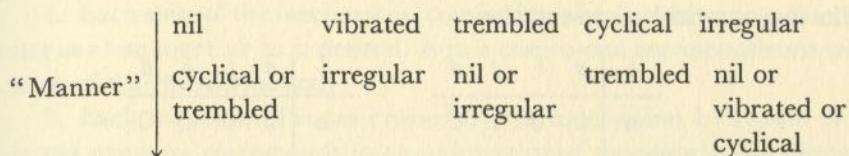
The following are examples of transformations that are not univocal:



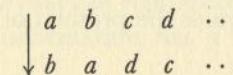
c. timbre change of a group of values

Timbres	clarinets	oboes	strings	timpani	brass
	timpani,	timpani,	brass	oboes	strings,
	strings	bassoon			oboes

and d. concrete music characteriology [4, 5]

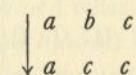


A transformation is a one-to-one mapping when each term has a single transform and when each transform is derived from a single term, for example:



MATRICAL REPRESENTATION

A transformation:



can be represented by a table as follows:

\downarrow	a	b	c	\downarrow	a	b	c
a	+	0	0	a	1	0	0
b	0	0	0	or	b	0	0
c	0	+	+	c	0	1	1

This table is a matrix of the transitions of the collection of terms to a collection of transforms.

PRODUCT

Let there be two transformations *T* and *U*:

$$T: \begin{array}{cccc} a & b & c & d \\ \downarrow & & & \\ b & d & a & b \end{array} \quad \text{and} \quad U: \begin{array}{cccc} a & b & c & d \\ \downarrow & & & \\ d & c & d & b \end{array}$$

In certain cases we can apply to a term *n* of *T* a transformation *T*, then a transformation *U*. This is written: *U*[*T*(*n*)], and is the product of the two transformations *T* and *U*, on condition that the transforms of *T* are terms of *U*. Thus, first *T*: *a* \rightarrow *b*, then *U*: *b* \rightarrow *c*, which is summarized as *V* = *UT*: *a* \rightarrow *c*.

To calculate the product applied to all the terms of T we shall use the following matrical representation:

$$T: \begin{array}{c|cccc} \downarrow & a & b & c & d \\ \hline a & 0 & 0 & 1 & 0 \\ b & 1 & 0 & 0 & 1 \\ c & 0 & 0 & 0 & 0 \\ d & 0 & 1 & 0 & 0 \end{array} \quad U: \begin{array}{c|cccc} \downarrow & a & b & c & d \\ \hline a & 0 & 0 & 0 & 0 \\ b & 0 & 0 & 0 & 1 \\ c & 0 & 1 & 0 & 0 \\ d & 1 & 0 & 1 & 0 \end{array}$$

the total transformation V equals the product of the two matrices T and U in the order U, T .

$$U \quad T \quad V$$

$$\begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{vmatrix} \times \begin{vmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

KINEMATIC DIAGRAM

The kinematic or transition diagram is a graphical expression of transformation. To draw it each term is connected to its transform by an arrow pointed at the transform. The *representative point* of a kinematic diagram is an imaginary point which moves in jumps from term to term following the arrows of the diagram; for an example see Fig. II-25.

$$T: \begin{array}{ccccccccc} & A & C & D & I & L & N & P & A \\ \downarrow & D & D & I & A & N & A & N & N \end{array}$$

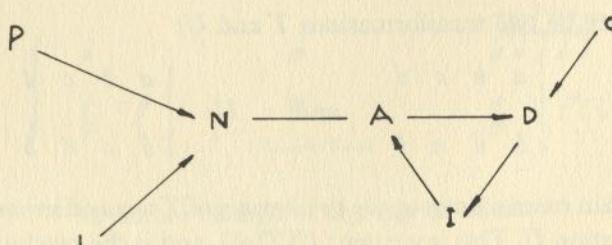


Fig. II-25

A transformation is really a mechanism and theoretically all the mechanisms of the physical or biological universes can be represented by

transformations under five conditions of correspondence:

1. Each state of the mechanism (continuity is broken down into discrete states as close together as is desired) is in a one-to-one correspondence with a term of the transformation.
2. Each sequence of states crossed by the mechanism by reason of its internal structure corresponds to an uninterrupted sequence of the terms of the transformation.
3. If the mechanism reaches a state and remains there (absorbing or stationary state), the term which corresponds to this state has no transform.
4. If the states of a mechanism reproduce themselves in the same manner without end, the transformation has a kinematic diagram in closed circuit.
5. A halt of the mechanism and its start from another state is represented in the diagram by a displacement of the representative point, which is not due to an arrow but to an arbitrary action on the paper.

The mechanism is determined when the corresponding transformation is univocal and closed. The mechanism is not determined when the corresponding transformation is many-valued. In this case the transformation is said to be *stochastic*. In a stochastic mechanism the numbers 0 and 1 in the transformation matrix must be replaced by relative frequencies. These are the alternative probabilities of various transformations. The determined mechanism is a particular case of the stochastic mechanism, in which the probabilities of transition are 0 and 1.

Example: All the harmonic or polyphonic rules of classical music could be represented by mechanisms. The fugue is one of the most accomplished and determined mechanisms. One could even generalize and say that the avant-garde composer is not content with following the mechanisms of his age but proposes new ones, for both detail and general form.

If these probabilities are constant over a long period of time, and if they are independent of the states of origin, the stochastic sequence is called, more particularly, a Markov chain.

Let there be two screens A and B and a protocol of 50 transitions:

ABABBBBABAABABAABABBBABAABABBAABABBABAABABBA
ABBABBA.

The real frequencies of the transitions are:

$$\begin{array}{ll} A \rightarrow B & 17 \text{ times} \\ A \rightarrow A & 6 \text{ times} \\ \hline & 23 \text{ times} \end{array} \quad \begin{array}{ll} B \rightarrow A & 17 \text{ times} \\ B \rightarrow B & 10 \text{ times} \\ \hline & 27 \text{ times} \end{array}$$

The matrix of the frequencies of real transitions is:

↓	A	B
A	6	17
B	17	10

The matrix of transition probabilities (relative frequencies) (MTP) is:

↓	A	B
A	0.26	0.63
B	0.74	0.37
	1.00	1.00

If the preceding matrix were:

↓	A	B
A	0.5	0.5
B	0.5	0.5

the unpredictability of the succession of *A* and *B* would be maximum and so would the entropy. Inversely, the constraint would be zero.

If the preceding matrix were:

↓	A	B
A	0	1
B	1	0

the transformation would be absolutely determined and the entropy of the sequence zero. The constraint would be maximum.

It may be that the symbols of a protocol depend in a certain way on the preceding terms, e.g., digram, trigram, etc., protocols. In this case, the matrix of transition probabilities (MTP) may be made independent by using an appropriate coding.

We can have (MTP) with *a*, *b*, and *c* as parameters, e.g.:

↓	A	B	↓	A	B	↓	A	B
	A	0.26	0.63	A	0.5	0.5	A	1
B	0.74	0.37	B	0.5	0.5	B	0	1

We can couple two or several (MTP) of different symbols on condition that we introduce a determined or stochastic transformation between the various parameters. Thus a protocol of timbres may be coupled with a protocol of intensity and a protocol of frequencies, etc., and each of the protocols may be linked in pairs with all the others. Isolated or coupled mechanisms can have one or several stationary or absorbing states towards which they tend in a manner which may or may not be unique. The stochastic mechanism may be a completely closed one on the same grounds as a determined mechanism.

If a Matrix of Transition Probabilities (MTP) is regular, that is, if all the entries of some power of this matrix are positive, the (MTP) has a unique *fixed* probability vector *t*; and in the long run, the sequence of increasing powers of (MTP) approaches the matrix *T*, whose rows are each the *fixed point* *t*. This *T* matrix is called the *stationary distribution* of the Markov chain represented by the stochastic (MTP) matrix. In the next chapter we shall see two methods of calculating this stationary distribution, or state of stability, from one (MTP) and the definition of a mean entropy. It is with the aid of this mean entropy that we shall be able to define and then compare the degrees of ataxy of a particular mechanism that we have applied to a set of screens.

In this way all that has been said about the ataxy of grains and clouds may be generalized and transposed to the sets (books) of screens. A fundamental criterion of the evolution of a piece of music can be shaped by the transformations of ataxy over time.

For example, it is very common in musical composition not to unleash at one go all the riches of which one is capable but to reserve them and introduce them little by little. It is also possible to imagine a piece of music which would give, at one stroke in the beginning, all the variety available and then break it into its separate elements in time.

The elementary evolutions of ataxy are diagrammed in Fig. II-26. Diagram F can be given in the form of a protocol. Ataxy can therefore be put in a matrical form with parameters, etc., and all the logical rules of transformation which we have allowed so far are applicable to the (MTP) of entropy.

How is this ataxy perceived? In many ways. If the grains of a given surface of a screen are distributed in a homogeneous manner, increasing the density will increase the richness, unpredictability, and entropy. In the same ergodic distribution of grains, if certain symmetries in the disposition of the grains appear and are discernible, a constraint is felt, and consequently

a diminution of the entropy. If melodic or harmonic liaisons are effected and perceived in the same distribution, unpredictability and entropy are both diminished.

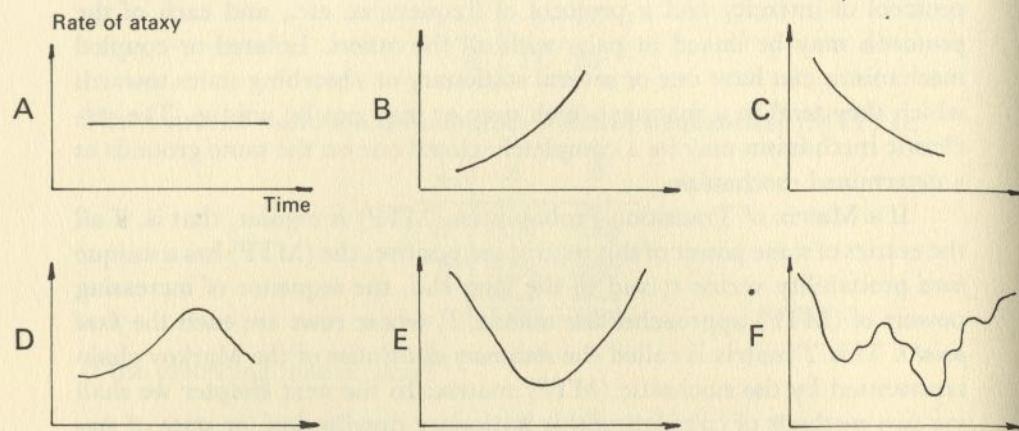


Fig. II-26

A. The evolution is nil. B. The rate of disorder and the richness increase. C. Ataxy decreases. D. Ataxy increases and then decreases. E. Ataxy decreases and then increases. F. The evolution of the ataxy is very complex, but it may be analyzed from the first three diagrams.

Thus after the first unfolding of a series of twelve sounds of the tempered scale, the unpredictability has fallen to zero, the constraint is maximum, the choice is nil, and the entropy is zero. Richness and hence interest are displaced to other fields, such as harmonies, timbres, and durations, and many other compositional wiles are aimed at reviving entropy. In fact sonic discourse is nothing but a perpetual fluctuation of entropy in all its forms [17].

However, human sensitivity does not necessarily follow the variation in entropy even if it is logarithmic to an appropriate base. It is rather a succession or a protocol of strains and relaxations of every degree that often excites the listener in a direction contrary to that of entropy. Thus Ravel's *Bolero*, in which the only variation is in the dynamics, has a virtually zero entropy after the third or fourth repetition of the fundamental idea. However, the interest, or rather the psychological agitation, grows with time through the very fact of this immobility and banality.

All incantatory manifestations aim at an effect of maximum tension with minimum entropy. The inverse is equally true, and seen from a certain

angle, white noise with its maximum entropy is soon tiresome. It would seem that there is no correspondence aesthetics \leftrightarrow entropy. These two entities are linked in quite an independent manner at each occasion. This statement still leaves some respite for the free will of the composer even if this free will is buried under the rubbish of culture and civilization and is only a shadow, at the least a tendency, a simple stochasm.

The great obstacle to a too hasty generalization is chiefly one of logical order; for an object is only an object as a function of its definition, and there is, especially in art, a near-infinity of definitions and hence a near-infinity of entropies, for the notion of entropy is an epiphenomenon of the definition. Which of these is valid? The ear, the eye, and the brain unravel sometimes inextricable situations with what is called intuition, taste, and intelligence. Two definitions with two different entropies can be perceived as identical, but it is also true that the set of definitions of an object has its own degree of disorder. We are not concerned here with investigating such a difficult, complex, and unexplored situation, but simply with looking over the possibilities that connected realms of contemporary thought promise, with a view to action.

To conclude briefly, since the applications which follow are more eloquent than explanatory texts, we shall accept that a collection or book of screens can be expressed by matrices of transition probabilities having parameters. They are affected by a degree of ataxy or entropy which is calculable under certain conditions. However, in order to render the analysis and then the synthesis of a sonic work within reach of understanding and the slide rule, we shall establish three criteria for a screen:

1. TOPOGRAPHIC CRITERION

The position of the cells $\Delta F \Delta G$ on the audible area is qualitatively important, and an enumeration of their possible combinations is capable of creating a group of well defined terms to which we can apply the concept of entropy and its calculation.

2. DENSITY CRITERION

The superficial density of the grains of a cell $\Delta F \Delta G$ also constitutes a quality which is immediately perceptible, and we could equally well define terms to which the concept and calculation of entropy would be applicable.

3. CRITERION OF PURE ATAXY (defined in relation to the grains of a screen)

A cell has three variables: mean frequency, mean amplitude, and mean

density of the grains. For a screen we can therefore establish three independent or connected protocols, then three matrices of transition probabilities which may or may not be coupled. Each of the matrices will have its entropy and the three coupled matrices will have a mean entropy. In the procession of sound we can establish several series of three matrices and hence several series of mean entropies, their variations constituting the criterion of ataxy.

The first two criteria, which are general and on the scale of screens or cells, will not concern us in what follows. But the third, more conventional criterion will be taken up in detail in the next chapter.

Chapter III

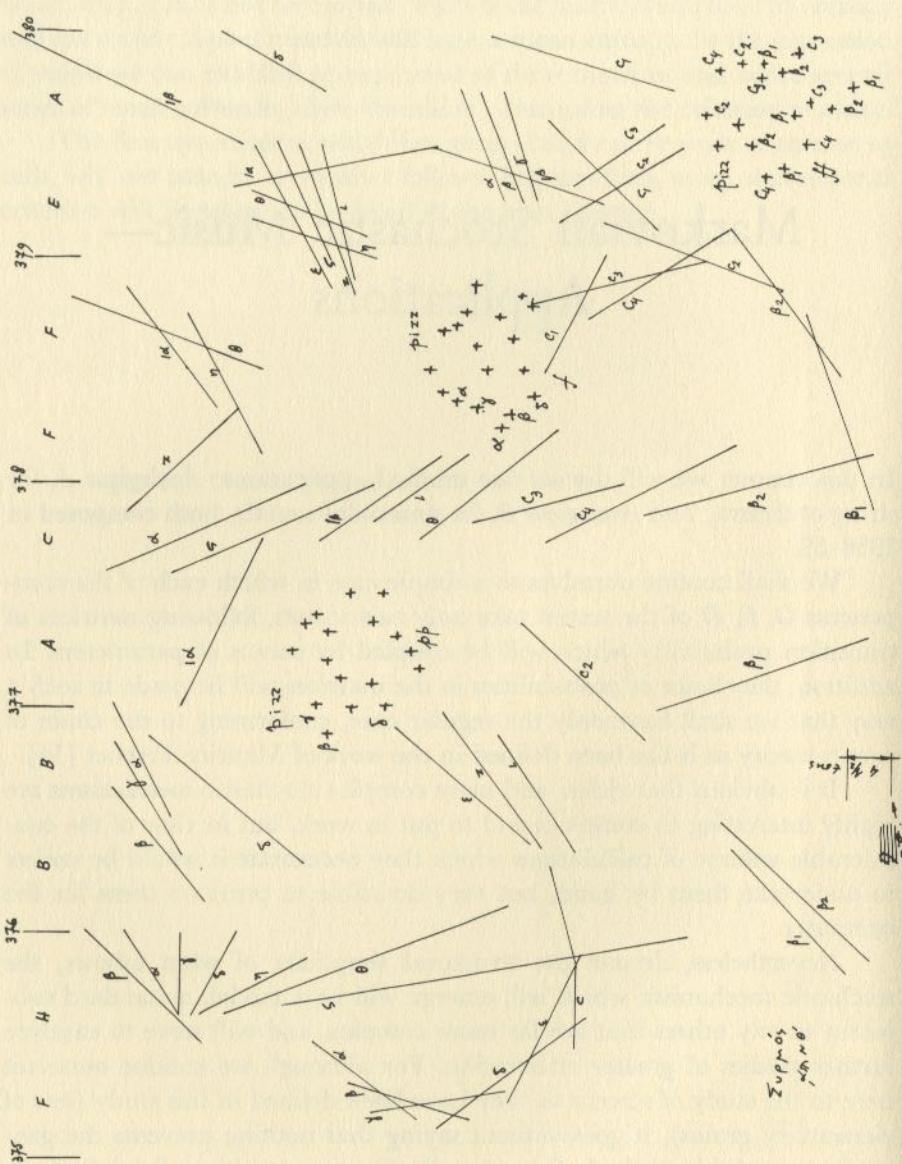
Markovian Stochastic Music— Applications

In this chapter we will discuss two musical applications: *Analogue A*, for string orchestra, and *Analogue B*, for sinusoidal sounds, both composed in 1958–59.

We shall confine ourselves to a simple case in which each of the components *G*, *F*, *D* of the screen take only two values, following matrices of transition probability which will be coupled by means of parameters. In addition, the choice of probabilities in the matrices will be made in such a way that we shall have only the regular case, conforming to the chain of events theory as it has been defined in the work of Maurice Fréchet [14].

It is obvious that richer and more complex stochastic mechanisms are highly interesting to construct and to put in work, but in view of the considerable volume of calculations which they necessitate it would be useless to undertake them by hand, but very desirable to program them for the computer.

Nevertheless, despite the structural simplicity of what follows, the stochastic mechanism which will emerge will be a model, a standard subjacent to any others that are far more complex, and will serve to catalyze further studies of greater elaboration. For although we confine ourselves here to the study of screens as they have been defined in this study (sets of elementary grains), it goes without saying that nothing prevents the generalization of this method of structuralization (composition) for definitions of sonic entities of more than three dimensions. Thus, let us no longer suppose screens, but *criteria* of definitions of a sonic entity, such that for the timbre, degree of order, density, variation, and even the *criteria* of more or less

Fig. III-1. *Syrmos* for 18 strings

complex elementary structures (e.g., melodic and temporal structures of groups of sounds, and instrumental, spatial, and kinematic structures) the same stochastic scheme is adaptable. It is enough to define the variations well and to be able to classify them even in a rough manner.

The sonic result thus obtained is not guaranteed a priori by calculation. Intuition and experience must always play their part in guiding, deciding, and testing.

ANALYSIS (definition of the scheme of a mechanism)

We shall define the scheme of a mechanism as the "analogue" of a stochastic process. It will serve for the production of sonic entities and for their transformations over time. These sonic entities will have screens which will show the following characteristics freely chosen:

1. They will permit two distinct combinations of frequency regions f_0 and f_1 (see Fig. III-2).

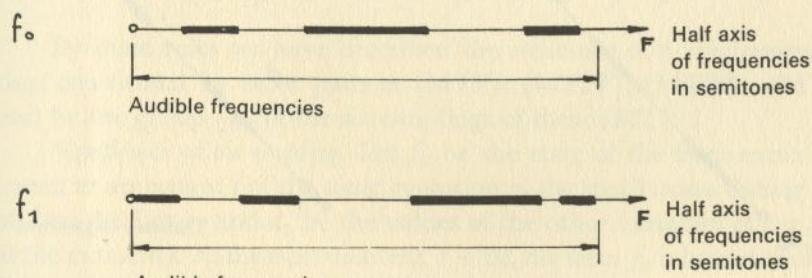


Fig. III-2

Syrmos, written in 1959, is built on stochastic transformations of eight basic textures: parallel horizontal bowed notes, parallel ascending bowed glissandi, parallel descending bowed glissandi, crossed (ascending and descending) parallel bowed notes, pizzicato clouds, atmospheres made up of col legno struck notes with short col legno glissandi, geometric configurations of convergent or divergent glissandi, and glissando configurations treated as undevelopable ruled surfaces. The mathematical structure of this work is the same as that of *Analogique A* and *Analogique B*.

2. They will permit two distinct combinations of intensity regions (see Fig. III-3).

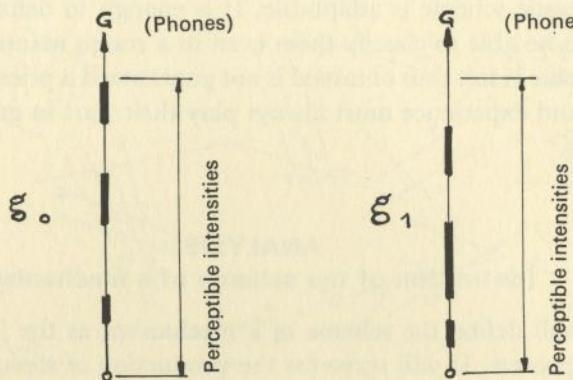


Fig. III-3

3. They will permit two distinct combinations of density regions (see Fig. III-4).

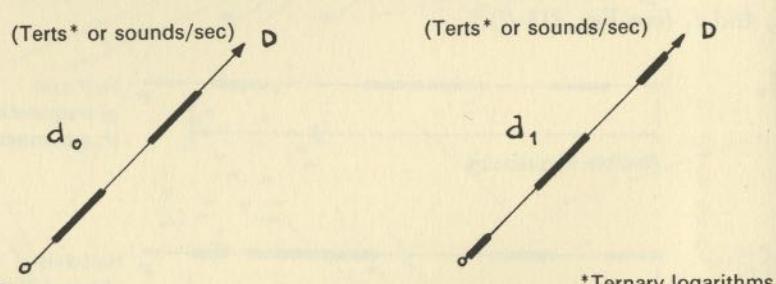


Fig. III-4

4. Each of these three variables will present a protocol which may be summarized by two matrices of transition probabilities (MTP).

	\downarrow	X	Y		\downarrow	X	Y	
(ρ)	X	0.2	0.8	(σ)	X	0.85	0.4	(μ)
	Y	0.8	0.2		Y	0.15	0.6	

The letters (ρ) and (σ) constitute the parameters of the (MTP).

MTPF (of frequencies)								
	\downarrow	f_0	f_1		\downarrow	f_0	f_1	
(α)	f_0	0.2	0.8	(β)	f_0	0.85	0.4	(μ)
	f_1	0.8	0.2		f_1	0.15	0.6	

MTPG (of intensities)

	\downarrow	g_0	g_1		\downarrow	g_0	g_1
(γ)	g_0	0.2	0.8	(ε)	g_0	0.85	0.4
	g_1	0.8	0.2		g_1	0.15	0.6

MTPD (of densities)

	\downarrow	d_0	d_1		\downarrow	d_0	d_1
(λ)	d_0	0.2	0.8	(μ)	d_0	0.85	0.4
	d_1	0.8	0.2		d_1	0.15	0.6

5. The transformations of the variables are indeterminate at the interior of each (MTP) (diagram processes), but on the other hand their (MTP) will be connected by means of a determined coupling of parameters. The coupling is given by the following transformations:

$$(e_0) \quad \begin{array}{cccccccccc} f_0 & f_1 & d_0 & d_1 & g_0 & g_1 & g_0 & g_1 & f_0 & f_1 & d_0 & d_1 \\ \downarrow & & \downarrow & \\ \lambda & \mu & \alpha & \beta & \lambda & \mu & \beta & \alpha & \gamma & \varepsilon & \gamma & \varepsilon \end{array}$$

By these rules we have described the structure of a mechanism. It is thus constituted by three pairs of (MTP): (MTPF), (MTPG), (MTPD), and by the group (e_0) of the six couplings of these (MTP).

Significance of the coupling. Let f_0 be the state of the frequencies of the screen at an instant t of the sonic evolution of the mechanism during a slice of time Δt . Let g_1 and d_1 be the values of the other variables of the screen at the moment t . At the next moment, $t + \Delta t$, the term f_0 is bound to change, for it obeys one of the two (MTPF), (α) or (β) . The choice of (α) or (β) is conditioned by the values g_1 and d_1 of the moment t , conforming to the transformation of the coupling. Thus g_1 proposes the parameter (α) and d_1 the parameter (β) simultaneously. In other words the term f_0 must either remain f_0 or yield its place to f_1 according to mechanism (α) or mechanism (β) . Imagine the term f_0 standing before two urns (α) and (β) , each containing two colors of balls, red for f_0 and blue for f_1 , in the following proportions:

Urn (α)	Urn (β)
red balls (f_0) , 0.2	red balls (f_0) , 0.85
blue balls (f_1) , 0.8	blue balls (f_1) , 0.15

The choice is free and the term f_0 can take its successor from either urn (α)

or urn (β) with a probability equal to $\frac{1}{2}$ (total probabilities).

Once the urn has been chosen, the choice of a blue or a red ball will have a probability equal to the proportion of colors in the chosen urn. Applying the law of compound probabilities, the probability that f_0 from moment t will remain f_0 at the moment $t + \Delta t$ is $(0.20 + 0.85)/2 = 0.525$, and the probability that it will change to f_1 is $(0.80 + 0.15)/2 = 0.475$.

The five characteristics of the composition of the screens have established a stochastic mechanism. Thus in each of the slices Δt of the sonic evolution of the created mechanism, the three variables f_i , g_i , d_i follow a round of unforeseeable combinations, always changing according to the three (MTP) and the coupling which connects terms and parameters.

We have established this mechanism without taking into consideration any of the screen criteria. That is to say, we have implied a topographic distribution of grain regions at the time of the choice of f_0 , f_1 and g_0 , g_1 , but without specifying it. The same is true for the density distribution. We shall give two examples of very different realizations in which these two criteria will be effective. But before setting them out we shall pursue further the study of the criterion of ataxy.

We shall neglect the entropies of the three variables at the grain level, for what matters is the macroscopic mechanism at the screen level. The fundamental questions posed by these mechanisms are, "Where does the transformation summarized by an (MTP) go? What is its destiny?"

Let us consider the (MTP):

	X	Y
X	0.2	0.8
Y	0.8	0.2

and suppose one hundred mechanisms identified by the law of this single (MTP). We shall allow them all to set out from X and evolve freely. The preceding question then becomes, "Is there a general tendency for the states of the hundred mechanisms, and if so, what is it?" (See Appendix II.)

After the first stage the $100X$ will be transformed into $0.2(100X) \rightarrow 20X$, and $0.8(100X) \rightarrow 80Y$. At the third stage 0.2 of the X 's and 0.8 of the Y 's will become X 's. Conversely 0.8 of the X 's will become Y 's and 0.2 of the Y 's will remain Y 's. This general argument is true for all stages and can be written:

$$\begin{aligned} X' &= 0.2X + 0.8Y \\ Y' &= 0.8X + 0.2Y. \end{aligned}$$

If this is to be applied to the 100 mechanisms X as above, we shall have:

Stage	Mechanisms	
	X	Y
0	100	0
1	20	80
2	68	32
3	39	61
4	57	43
5	46	54
6	52	48
7	49	51
8	50	50
9	50	50
:	:	:

We notice oscillations that show a general tendency towards a stationary state at the 8th stage. We may conclude, then, that of the 100 mechanisms that leave from X , the 8th stage will in all probability send 50 to X and 50 to Y . The same stationary probability distribution of the Markov chain, or the fixed probability vector, is calculated in the following manner:

At equilibrium the two probability values X and Y remain unchanged and the preceding system becomes

$$X = 0.2X + 0.8Y$$

$$Y = 0.8Y + 0.2X$$

or

$$0 = -0.8X + 0.8Y$$

$$0 = +0.8X - 0.8Y.$$

Since the number of mechanisms is constant, in this case 100 (or 1), one of the two equations may be replaced at the stationary distribution by $1 = X + Y$. The system then becomes

$$0 = 0.8X - 0.8Y$$

$$1 = X + Y$$

and the stationary probability values X , Y are $X = 0.50$ and $Y = 0.50$.

The same method can be applied to the (MTP)(σ), which will give us stationary probabilities $X = 0.73$ and $Y = 0.27$.

Another method, particularly interesting in the case of an (MTP) with many terms, which forces us to resolve a large system of linear equations in order to find the stationary probabilities, is that which makes use of matrix calculus.

Thus the first stage may be considered as the matrix product of the (MTP) with the unicolumn matrix $\begin{vmatrix} 100 \\ 0 \end{vmatrix}$

$$X: \begin{vmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{vmatrix} \times \begin{vmatrix} 100 \\ 0 \end{vmatrix} = \begin{vmatrix} 20 \\ 80 \end{vmatrix}.$$

The second stage will be

$$\begin{vmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{vmatrix} \times \begin{vmatrix} 20 \\ 80 \end{vmatrix} = \begin{vmatrix} 4 + 64 \\ 16 + 16 \end{vmatrix} = \begin{vmatrix} 68 \\ 32 \end{vmatrix},$$

and the n th stage

$$\begin{vmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{vmatrix}^n \times \begin{vmatrix} 100 \\ 0 \end{vmatrix}.$$

Now that we know how to calculate the stationary probabilities of a Markov chain we can easily calculate its mean entropy. The definition of the entropy of a system is

$$H = - \sum p_i \log p_i.$$

The calculation of the entropy of an (MTP) is made first by columns ($\sum p_i = 1$), the p_i being the probability of the transition for the (MTP); then this result is weighted with the corresponding stationary probabilities. Thus for the (MTP)(σ):

↓	X	Y
X	0.85	0.4
Y	0.15	0.6

The entropy of the states of X will be $-0.85 \log 0.85 - 0.15 \log 0.15 = 0.611$ bits; the entropy of the states of Y , $-0.4 \log 0.4 - 0.6 \log 0.6 = 0.970$ bits; the stationary probability of $X = 0.73$; the stationary probability of $Y = 0.27$; the mean entropy at the stationary stage is

$$H\sigma = 0.611(0.73) + 0.970(0.27) = 0.707 \text{ bits};$$

and the mean entropy of the (MTP)(ρ) at the stationary stage is

$$H\rho = 0.722 \text{ bits.}$$

The two entropies do not differ by much, and this is to be expected, for if we look at the respective (MTP) we observe that the great contrasts of probabilities inside the matrix (ρ) are compensated by an external equality of stationary probabilities, and conversely in the (MTP)(σ) the interior quasi-equality, 0.4 and 0.6, succeeds in counteracting the interior contrast, 0.85 and 0.15, and the exterior contrast, 0.73 and 0.27.

At this level we may modify the (MTP) of the three variables f_i, g_i, d_i in such a way as to obtain a new pair of entropies. As this operation is repeatable we can form a protocol of pairs of entropies and therefore an (MTP) of pairs of entropies. These speculations and investigations are no doubt interesting, but we shall confine ourselves to the first calculation made above and we shall pursue the investigation on an even more general plane.

MARKOV CHAIN EXTENDED SIMULTANEOUSLY FOR f_i, g_i, d_i

On p. 83 we analyzed the mechanism of transformation of f_0 to f_0 or f_1 when the probabilities of the two variables g_i and d_i are given. We can apply the same arguments for each of the three variables f_i, g_i, d_i when the two others are given.

Example for g_i . Let there be a screen at the moment t whose variables have the values (f_0, g_1, d_1) . At the moment $t + \Delta t$ the value of g_1 will be transformed into g_1 or g_0 . From f_0 comes the parameter (γ) , and from d_1 comes the parameter (ε) .

With (MTP)(γ) the probability that g_1 will remain g_1 is 0.2. With (MTP)(ε) the probability that g_1 will remain g_1 is 0.6. Applying the rules of compound probabilities and/or probabilities of mutually exclusive events as on p. 83, we find that the probability that g_1 will remain g_1 at the moment $t + \Delta t$ under the simultaneous effects of f_0 and d_1 is equal to $(0.2 + 0.6)/2 = 0.4$. The same holds for the calculation of the transformation from g_1 into g_0 and for the transformations of d_1 .

We shall now attempt to emerge from this jungle of probability combinations, which is impossible to manage, and look for a more general viewpoint, if it exists.

In general, each screen is constituted by a triad of specific values of the variables F, G, D so that we can enumerate the different screens emerging from the mechanism that we are given (see Fig. III-5). The possible

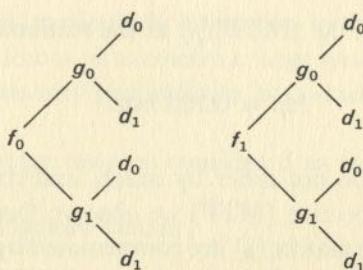


Fig. III-5

combinations are: $(f_0g_0d_0)$, $(f_0g_0d_1)$, $(f_0g_1d_0)$, $(f_0g_1d_1)$, $(f_1g_0d_0)$, $(f_1g_0d_1)$, $(f_1g_1d_0)$, $(f_1g_1d_1)$; i.e., eight different screens, which, with their protocols, will make up the sonic evolution. At each moment t of the composition we shall encounter one of these eight screens and no others.

What are the rules for the passage from one combination to another? Can one construct a matrix of transition probabilities for these eight screens?

Let there be a screen $(f_0g_1d_1)$ at the moment t . Can one calculate the probability that at the moment $t + \Delta t$ this screen will be transformed into $(f_1g_1d_0)$? The above operations have enabled us to calculate the probability that f_0 will be transformed into f_1 under the influence of g_1 and d_1 and that g_1 will remain g_1 under the influence of f_0 and d_1 . These operations are schematized in Fig. III-6, and the probability that screen $(f_0g_1d_1)$ will be transformed into $(f_1g_1d_0)$ is 0.114.

Screen at the moment t :

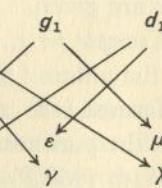
Parameters derived from the coupling transformations :

Screen at the moment $t + \Delta t$:

	f_1	g_1	d_0
Values of probabilities taken from the (MTP) corresponding to the coupling parameters :	0.80	0.6	0.4
	0.15	0.2	0.8
Compound probabilities :	0.475	0.4	0.6
Compound probabilities for independent events :	0.475	0.4	0.6 = 0.114

Fig. III-6

We can therefore extend the calculation to the eight screens and construct the matrix of transition probabilities. It will be square and will have eight rows and eight columns.



	MTPZ							
	$A(f_0g_0d_0)$	$B(f_0g_0d_1)$	$C(f_0g_1d_0)$	$D(f_0g_1d_1)$	$E(f_1g_0d_0)$	$F(f_1g_0d_1)$	$G(f_1g_1d_0)$	$H(f_1g_1d_1)$
$A(f_0g_0d_0)$	0.021	0.357	0.084	0.189	0.165	0.204	0.408	0.096
$B(f_0g_0d_1)$	0.084	0.089	0.076	0.126	0.150	0.136	0.072	0.144
$C(f_0g_1d_0)$	0.084	0.323	0.021	0.126	0.150	0.036	0.272	0.144
$D(f_0g_1d_1)$	0.336	0.081	0.019	0.084	0.135	0.024	0.048	0.216
$E(f_1g_0d_0)$	0.019	0.063	0.336	0.171	0.110	0.306	0.102	0.064
$F(f_1g_0d_1)$	0.076	0.016	0.304	0.114	0.100	0.204	0.018	0.096
$G(f_1g_1d_0)$	0.076	0.057	0.084	0.114	0.100	0.054	0.068	0.096
$H(f_1g_1d_1)$	0.304	0.014	0.076	0.076	0.090	0.036	0.012	0.144

Does the matrix have a region of stability? Let there be 100 mechanisms Z whose scheme is summarized by (MTPZ). At the moment t , d_A mechanisms will have a screen A , d_B a screen B , ..., d_H a screen H . At the moment $t + \Delta t$ all 100 mechanisms will produce screens according to the probabilities written in (MTPZ). Thus,

0.021 d_A will stay in A ,
 0.357 d_B will be transformed to A ,
 0.084 d_C will be transformed to A ,
 ...
 0.096 d_H will be transformed to A .

The d_A screens at the moment t will become d'_A screens at the moment $t + \Delta t$, and this number will be equal to the sum of all the screens that will be produced by the remaining mechanisms, in accordance with the corresponding probabilities.

Therefore:

$$(e_1) \quad \begin{cases} d'_A = 0.021d_A + 0.357d_B + 0.084d_C + \dots + 0.096d_H \\ d'_B = 0.084d_A + 0.089d_B + 0.076d_C + \dots + 0.144d_H \\ d'_C = 0.084d_A + 0.323d_B + 0.021d_C + \dots + 0.144d_H \\ \dots \\ d'_H = 0.304d_A + 0.014d_B + 0.076d_C + \dots + 0.144d_H. \end{cases}$$

At the stationary state the frequency of the screens A, B, C, \dots, H will remain constant and the eight preceding equations will become:

$$(d'_A = d_A, d'_B = d_B, d'_C = d_C, \dots, d'_H = d_H)$$

$$(e_2) \quad \begin{cases} 0 = -0.979d_A + 0.357d_B + 0.084d_C + \dots + 0.096d_H \\ 0 = 0.084d_A - 0.911d_B + 0.076d_C + \dots + 0.144d_H \\ 0 = 0.084d_A + 0.323d_B - 0.979d_C + \dots + 0.144d_H \\ \dots \\ 0 = 0.304d_A + 0.014d_B + 0.076d_C + \dots - 0.856d_H \end{cases}$$

On the other hand

$$d_A + d_B + d_C + \cdots + d_H = 1.$$

If we replace one of the eight equations by the last, we obtain a system of eight linear equations with eight unknowns. Solution by the classic method of determinants gives the values:

$$(e_3) \quad \begin{cases} d_A = 0.17, d_B = 0.13, d_C = 0.13, d_D = 0.11, d_E = 0.14, d_F = 0.12, \\ d_G = 0.10, d_H = 0.10. \end{cases}$$

which are the probabilities of the screens at the stationary stage. This method is very laborious, for the chance of error is very high (unless a calculating machine is available).

The second method (see p. 85), which is more approximate but adequate, consists in making all 100 mechanisms Z set out from a single screen and letting them evolve by themselves. After several more or less long oscillations, the stationary state, if it exists, will be attained and the proportions of the screens will remain invariable.

We notice that the system of equations (e₁) may be broken down into:

1. Two vectors V' and V which may be represented by two unicolumn matrices:

$$V' = \begin{vmatrix} d'_A \\ d'_B \\ \vdots \\ d'_G \\ d'_H \end{vmatrix}$$

$$\text{and } V = \begin{vmatrix} d_A \\ d_B \\ \vdots \\ d_G \\ d_H \end{vmatrix}$$

2. A linear operator, the matrix of transition probabilities Z . Consequently system (e_1) can be summarized in a matrix equation:

$$(e_4) \quad V' = ZV.$$

To cause all 100 mechanisms Z to leave screen X and evolve "freely" means allowing a linear operator:

Z =	0.021	0.357	0.084	0.189	0.165	0.204	0.408	0.096
	0.084	0.089	0.076	0.126	0.150	0.136	0.072	0.144
	0.084	0.323	0.021	0.126	0.150	0.036	0.272	0.144
	0.336	0.081	0.019	0.084	0.135	0.024	0.048	0.216
	0.019	0.063	0.336	0.171	0.110	0.306	0.102	0.064
	0.076	0.016	0.304	0.114	0.100	0.204	0.018	0.096
	0.076	0.057	0.084	0.114	0.100	0.054	0.068	0.096
	0.304	0.014	0.076	0.076	0.090	0.036	0.012	0.144

to perform on the column vector

$$V = \begin{vmatrix} 0 \\ 0 \\ \vdots \\ 100 \\ \vdots \\ 0 \\ 0 \end{vmatrix}$$

in a continuous manner at each moment t . Since we have broken down continuity into a discontinuous succession of thickness in time Δt , the equation (e₄) will be applied to each stage Δt .

Thus at the beginning (moment $t = 0$) the population vector of the mechanisms will be V^0 . After the first stage (moment $0 + \Delta t$) it will be $V' = ZV^0$; after the second stage (moment $0 + 2\Delta t$), $V'' = ZV' = Z^2V^0$; and at the n th stage (moment $n\Delta t$), $V^{(n)} = Z^nV^0$. In applying these data to the vector

$$V_H^0 = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 100 \end{vmatrix}$$

after the first stage at the moment Δt :

$$V'_H = ZV_H^0 = \begin{vmatrix} 9.6 \\ 14.4 \\ 14.4 \\ 21.6 \\ 6.4 \\ 9.6 \\ 9.6 \\ 14.4 \end{vmatrix}$$

after the third stage at the moment $3\Delta t$:

$$V'''_H = ZV_H'' = \begin{vmatrix} 16.860 \\ 10.867 \\ 13.118 \\ 13.143 \\ 14.575 \\ 12.257 \\ 8.145 \\ 11.046 \end{vmatrix}$$

after the second stage at the moment $2\Delta t$:

$$V''_H = ZV'_H = \begin{vmatrix} 18.941 \\ 10.934 \\ 14.472 \\ 11.146 \\ 15.164 \\ 11.954 \\ 8.416 \\ 8.966 \end{vmatrix}$$

and after the fourth stage at the moment $4\Delta t$:

$$V'''_H = ZV'''_H = \begin{vmatrix} 17.111 \\ 11.069 \\ 13.792 \\ 12.942 \\ 14.558 \\ 12.111 \\ 8.238 \\ 10.716 \end{vmatrix}$$

Thus after the fourth stage, an average of 17 out of the 100 mechanisms will have screen A , 11 screen B , 14 screen C , ..., 11 screen H .

If we compare the components of the vector V'''_H with the values (e_3) we notice that by the fourth stage we have almost attained the stationary state. Consequently the mechanism we have built shows a very rapid abatement of the oscillations, and a very great convergence towards final stability, the goal (stochos). The perturbation P_H , which was imposed on the mechanism (MPTZ) when we considered that all the mechanisms (here 100) left from a single screen, was one of the strongest we could create.

Let us now calculate the state of the 100 mechanisms Z after the first stage with the maximal perturbations P applied.

$$V_A^0 = \begin{vmatrix} 100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} \quad V'_A = \begin{vmatrix} 2.1 \\ 8.4 \\ 8.4 \\ 33.6 \\ 1.9 \\ 7.6 \\ 7.6 \\ 30.4 \end{vmatrix} \quad V_B^0 = \begin{vmatrix} 0 \\ 100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} \quad V'_B = \begin{vmatrix} 35.7 \\ 8.9 \\ 32.3 \\ 8.1 \\ 6.3 \\ 1.6 \\ 5.7 \\ 1.4 \end{vmatrix}$$

$$V_C^0 = \begin{vmatrix} 0 \\ 0 \\ 100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} \quad V'_C = \begin{vmatrix} 8.4 \\ 7.6 \\ 2.1 \\ 1.9 \\ 33.6 \\ 30.4 \\ 8.4 \\ 7.6 \end{vmatrix} \quad V_D^0 = \begin{vmatrix} 0 \\ 100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} \quad V'_D = \begin{vmatrix} 18.9 \\ 12.6 \\ 12.6 \\ 8.4 \\ 17.1 \\ 11.4 \\ 11.4 \\ 7.6 \end{vmatrix}$$

$$V_E^0 = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 100 \\ 0 \\ 0 \\ 0 \end{vmatrix} \quad V'_E = \begin{vmatrix} 16.5 \\ 15.0 \\ 15.0 \\ 13.5 \\ 11.0 \\ 10.0 \\ 10.0 \\ 9.0 \end{vmatrix} \quad V_F^0 = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 100 \\ 0 \\ 0 \end{vmatrix} \quad V'_F = \begin{vmatrix} 20.4 \\ 13.6 \\ 3.6 \\ 2.4 \\ 30.6 \\ 20.4 \\ 5.4 \\ 3.6 \end{vmatrix}$$

$$V_G^0 = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 100 \\ 0 \end{vmatrix} \quad V'_G = \begin{vmatrix} 40.8 \\ 7.2 \\ 27.2 \\ 4.8 \\ 10.2 \\ 1.8 \\ 6.8 \\ 1.2 \end{vmatrix}$$

Recapitulation of the Analysis

Having arrived at this stage of the analysis we must take our bearings. On the level of the screen cells we now have: 1. partial mechanisms of transformation for frequency, intensity, and density ranges, which are expressed by the (MTPF), (MTPG), (MTPD); and 2. an interaction between the three fundamental variables F , G , D of the screen (transformations of the coupling (e_0)).

On the level of the screens we now have: 1. eight different screens, A, B, C, D, E, F, G, H ; 2. a general mechanism, the (MTPZ), which summarizes all the partial mechanisms and their interactions; 3. a final state of equilibrium (the goal, stochos) of the system Z towards which it tends quite quickly, the stationary distribution; and 4. a procedure of disequilibrium in system Z with the help of the perturbations P which are imposed on it.

SYNTHESIS

Mechanism Z which we have just constructed does not imply a real evolution of the screens. It only establishes a dynamic situation and a potential evolution. The natural process is that provoked by a perturbation P imposed on the system Z and the advancement of this system towards its goal, its stationary state, once the perturbation has ceased its action. We can therefore act on this mechanism through the intermediary of a perturbation such as P , which is stronger or weaker as the case may be. From this it is only a brief step to imagining a whole series of successive perturbations which would force the apparatus Z to be displaced towards exceptional regions at odds with its behavior at equilibrium.

In effect the intrinsic value of the organism thus created lies in the fact that it must manifest itself, be. The perturbations which apparently change its structure represent so many negations of this existence. And if we create a succession of perturbations or negations, on the one hand, and stationary states or existences on the other, we are only *affirming* mechanism Z . In other words, at first we argue positively by proposing and offering as evidence the existence itself; and then we confirm it negatively by opposing it with perturbatory states.

The bi-pole of being a thing and not being this thing creates the whole—the object which we intended to construct at the beginning of Chapter III. A dual dialectics is thus at the basis of this compositional attitude, a dialectics that sets the pace to be followed. The “experimental” sciences are an expression of this argument on an analogous plane. An experiment establishes a body of data, a web which it disentangles from the magma of

objective reality with the help of negations and transformations imposed on this body. The repetition of these dual operations is a fundamental condition on which the whole universe of knowledge rests. To state something once is not to define it; the causality is confounded with the repetition of phenomena considered to be identical.

In conclusion, this dual dialectics with which we are armed in order to compose within the framework of our mechanism is homothetic with that of the experimental sciences; and we can extend the comparison to the dialectics of biological beings or to nothing more than the dialectics of being. This brings us back to the point of departure.

Thus an entity must be proposed and then a modification imposed on it. It goes without saying that to propose the entity or its modification in our particular case of musical composition is to give a human observer the means to perceive the two propositions and to compare them. Then the antitheses, entity and modification, are repeated enough times for the entity to be identified.

What does identification mean in the case of our mechanism Z ?

Parenthesis. We have supposed in the course of the analysis that 100 mechanisms Z were present simultaneously, and that we were following the rules of the game of these mechanisms at each moment of an evolution created by a displacement beyond the stationary zone. We were therefore comparing the states of 100 mechanisms in a Δt with the states of these 100 mechanisms in the next t , so that in comparing two successive stages of the group of 100 simultaneous states, we *enumerate* 100 states twice. Enumeration, that is, insofar as abstract action implies ordered operations, means to observe the 100 mechanisms one by one, classify them, and test them; then start again with 100 at the following stage, and finally compare the classes number by number. And if the observation of each mechanism necessitates a fraction of time x , it would take $200x$ of time to enumerate 200 mechanisms.

This argument therefore allows us to transpose abstractly a simultaneity into a lexicographic (temporal) succession without subtracting anything, however little, from the definition of transformations engendered by scheme Z . Thus to compare two successive stages of the 100 mechanisms Z comes down to comparing 100 states produced in an interval of time $100x$ with 100 others produced in an equal interval of time $100x$ (see Fig. III-7).

MATERIAL IDENTIFICATION OF MECHANISM Z

Identification of mechanism Z means essentially a comparison between

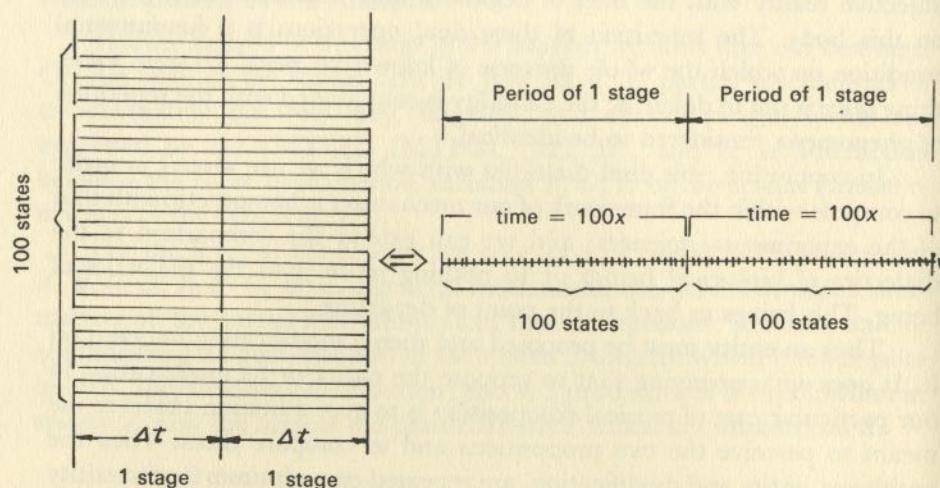


Fig. III-7

all its possibilities of being: perturbed states compared to stationary states, independent of order.

Identification will be established over equal periods of time $100x$ following the diagram:

Phenomenon: $P_N \rightarrow E \rightarrow P_M \rightarrow E \dots$
 Time: $100x \quad 100x \quad 100x \quad 100x \dots$

in which P_N and P_M represent any perturbations and E is the state of Z at equilibrium (stationary state).

An alternation of P and E is a protocol in which $100x$ is the unit of time ($100x$ = period of the stage), for example:

$P_A \quad P_A \quad E \quad E \quad E \quad P_H \quad P_G \quad P_G \quad E \quad P_C \dots$

A new mechanism W may be constructed with an (MTP), etc., which would control the identification and evolution of the composition over more general time-sets. We shall not pursue the investigation along these lines for it would lead us too far afield.

A realization which will follow will use a very simple kinematic diagram of perturbations P and equilibrium E , conditioned on one hand by the degrees of perturbation P , and on the other by a freely agreed selection.

(e₅) $E \rightarrow P_A^0 \rightarrow P_A' \rightarrow E \rightarrow P_C' \rightarrow P_C^0 \rightarrow P_B^0 \rightarrow P_B' \rightarrow E \rightarrow P_A'$

Definition of State E and of the Perturbations P

From the above, the stationary state E will be expressed by a sequence of screens such as:

Protocol $E(Z)$

*ADFFECBDBC FEFADGCHCCHBEDFEFFECFEHBFFFBC
 HDBABADDBADADAHHBGADGAHDADGF BEBGABEBB ...*

To carry out this protocol we shall utilize eight urns $[A]$, $[B]$, $[C]$, $[D]$, $[E]$, $[F]$, $[G]$, $[H]$, each containing balls of eight different colors, whose proportions are given by the probabilities of (MTPZ). For example, urn $[G]$ will contain 40.8% red balls A , 7.2% orange balls B , 27.2% yellow balls C , 4.8% maroon balls D , 10.2% green balls E , 1.8% blue balls F , 6.8% white balls G , and 1.2% black balls H . The composition of the other seven urns can be read from (MTPZ) in similar fashion.

We take a yellow ball C at random from urn $[G]$. We note the result and return the ball to urn $[G]$. We take a green ball E at random from urn $[C]$. We note the result and return the ball to urn $[C]$. We take a black ball H at random from urn $[E]$, note the result, and return the ball to urn $[E]$. From urn $[H]$ we take ... The protocol so far is: *GCEH ...*

Protocol $P_A^0 (V_A^0)$ is obviously

AAAA ...

Protocol $P_A' (V_A')$. Consider an urn $[Y]$ in which the eight colors of balls are in the following proportions: 2.1% color A , 8.4% color B , 8.4% color C , 33.6% color D , 1.9% color E , 7.6% color F , 7.6% color G , and 30.4% color H . After each draw return the ball to urn Y . A likely protocol might be the following:

*GFFGHDDCBHGGHDDHBBHCDDDCGDDDFDDHHHBF
 FHDBHDHHCHHECHDBHHDHHFHDGDCAFHHHDFDG ...*

Protocol $P_C' (V_C')$. The same method furnishes us with a protocol of P' :

*EEGFGEFEEFADFE BECGEEAEFBFBEADEFAAEEFH
 ABFECHFEBEFEEFHFAEBFFFEEAFHFBEFEEB ...*

Protocol $P_C^0 (V_C^0)$:

CCCC ...

Protocol $P_B^0 (V_B^0)$:

BBBB ...

Protocol $P'_C (V'_C)$:

AAADCCECDAACEBAFGBCAAADGCDDCGCADGAAGEC
CAACAAHAACGCDAACDAABDCCCCACACAACACB...

REALIZATION OF ANALOGIQUE A FOR ORCHESTRA

The instrumental composition follows the preceding exposition point by point, within the limits of orchestral instruments and conventional execution and notation. The mechanism which will be used is system Z , which has already been treated numerically. The choice of variables for the screens are shown in Figs. III-8, 9, 10.

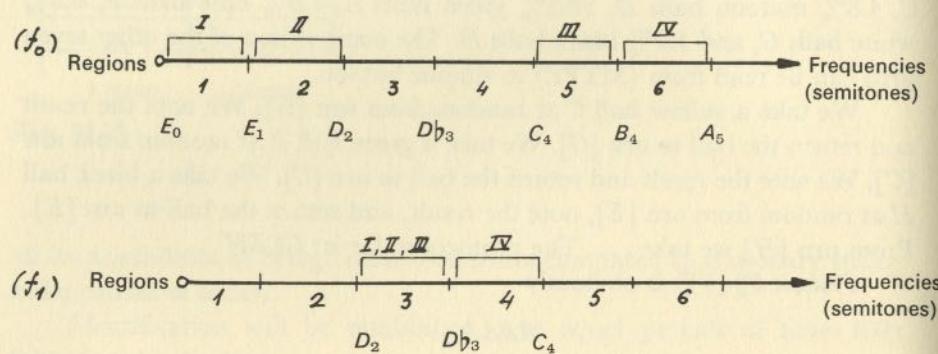


Fig. III-8. Frequencies

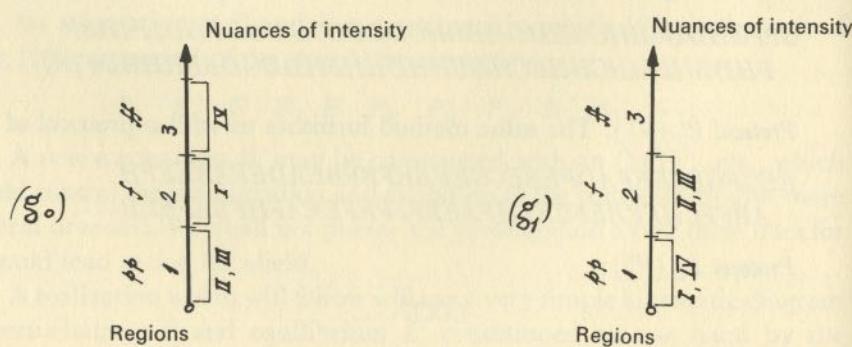


Fig. III-9. Intensities

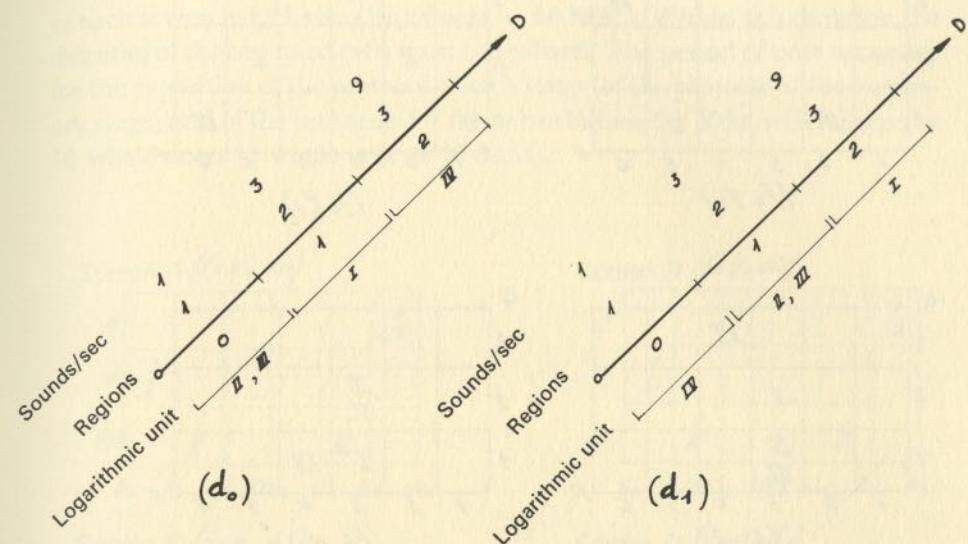


Fig. III-10. Densities

This choice gives us the partial screens FG (Fig. III-11) and FD (Fig. III-12), the partial screens GD being a consequence of FG and FD . The Roman numerals are the liaison agents between all the cells of the three planes of reference, FG , FD , and GD , so that the different combinations (f_i , g_j , d_k) which are perceived theoretically are made possible.

For example, let there be a screen (f_1, g_1, d_0) and the sonic entity C_3 corresponding to frequency region no. 3. From the partial screens above, this entity will be the arithmetic sum in three dimensions of the grains of cells I, II, and III, lying on frequency region no. 3. $C_3 = I + II + III$.

The dimensions of the cell corresponding to I are: $\Delta F = \text{region 3}$, $\Delta G = \text{region 1}$, $\Delta D = \text{region 2}$. The dimensions of the cell corresponding to II are: $\Delta F = \text{region 3}$, $\Delta G = \text{region 2}$, $\Delta D = \text{region 1}$. The dimensions of the cell corresponding to III are: $\Delta F = \text{region 3}$, $\Delta G = \text{region 2}$, $\Delta D = \text{region 1}$. Consequently in this sonic entity the grains will have frequencies included in region 3, intensities included in regions 1 and 2, and they will form densities included in regions 1 and 2, with the correspondences set forth above.

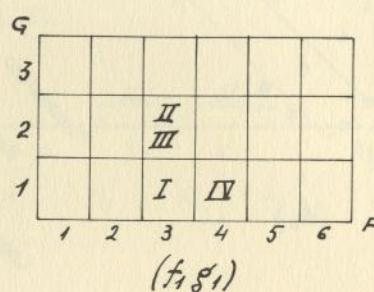
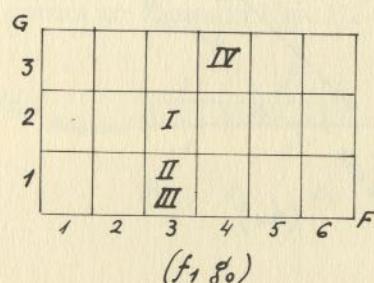
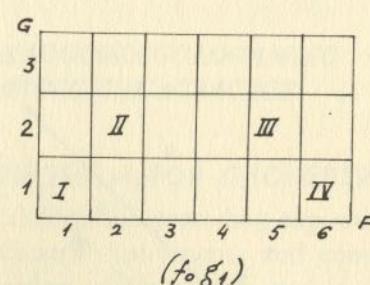
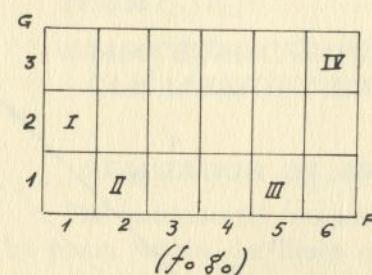


Fig. III-11. Partial Screens for FG

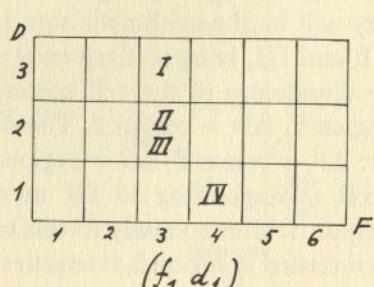
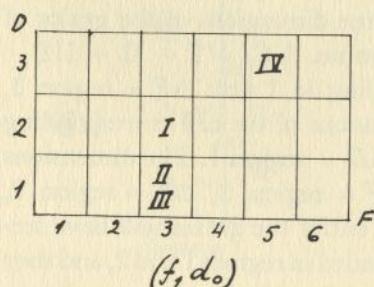
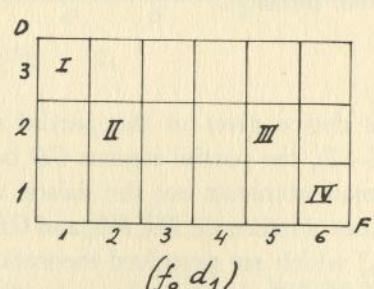
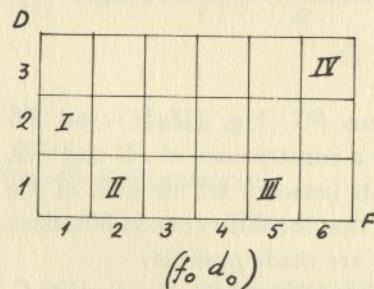


Fig. III-12. Partial Screens for FD

The eight principal screens *A*, *B*, *C*, *D*, *E*, *F*, *G*, *H* which derive from the combinations in Fig. III-5 are shown in Fig. III-13. The duration Δt of each screen is 1.11 sec. (1 half note = 54 MM). Within this duration the densities of the occupied cells must be realized. The period of time necessary for the exposition of the protocol of each stage (of the protocol at the stationary stage, and of the protocols for the perturbations) is $30\Delta t$, which becomes 15 whole notes (1 whole note = 27 MM).

Screen A ($f_0 g_0 d_0$)

#						
f	3					IV
pp	I					
		II			III	

Screen C ($f_1 g_1 d_0$)

#						
f		4			4	
pp	3	I				IV
			II		III	

Screen E ($f_1 g_0 d_0$)

#						
f		4			4	
pp	3	I				IV
			II+III			

Screen G ($f_1 g_1 d_0$)

#						
f		4			4	
pp	3	I			4	
			II+III			

Screen B ($f_0 g_0 d_1$)

#						
f	9					IV
pp	I					
		II			III	

Screen D ($f_1 g_1 d_1$)

#						
f	3					IV
pp	I					
		II			III	

Screen F ($f_1 g_0 d_1$)

#						
f		9			4	
pp	I					
		II+III				

Screen H ($f_1 g_1 d_1$)

#						
f		3+3				
pp	I					
		II+III				

Fig. III-13

NOTE: The numbers written in the cells are the mean densities in grains/sec.

The linkage of the perturbations and the stationary state of (MTPZ) is given by the following kinematic diagram, which was chosen for this purpose:



Fig. III-14. Bars 105–15 of *Analogique A*

Fig. III-14, bars 105-15 of the score of *Analogique A*, comprises a section of perturbations P_B^0 and P_B' . The change of period occurs at bar 109. The disposition of the screens is given in Fig. III-15. For technical reasons screens E , F , G , and H have been simplified slightly.

Fig. III-15

Analogique A replaces elementary sinusoidal sounds by very ordered clouds of elementary grains, restoring the string timbres. In any case a realization with classical instruments could not produce screens having a timbre other than that of strings because of the limits of human playing. The hypothesis of a sonority of a second order cannot, therefore, be confirmed or invalidated under these conditions.

On the other hand, a realization using electromagnetic devices as mighty as computers and adequate converters would enable one to prove the existence of a second order sonority with elementary sinusoidal grains or grains of the Gabor type as a base.

While anticipating some such technique, which has yet to be developed, we shall demonstrate how more complex screens are realizable with the resources of an ordinary electroacoustic studio equipped with several magnetic tapes or synchronous recorders, filters, and sine-wave generators.

ELECTROMAGNETIC MUSIC (sinusoidal sounds)—EXAMPLE TAKEN FROM ANALOGIQUE B

We choose: 1. Two groups of frequency regions f_0, f_1 , as in Fig. III-16. The protocols of these two groups will be such that they will obey the preceding (MTP)'s:

		f_0	f_1			f_0	f_1
		0.2	0.8			0.85	0.4
(α)	f_0	0.2	0.8	(β)	f_0	0.85	0.4
	f_1	0.8	0.2		f_1	0.15	0.6

in which (α) and (β) are the parameters.

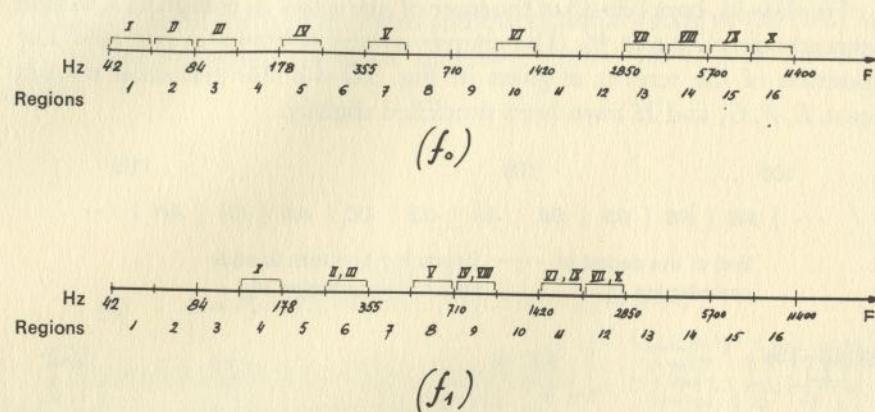


Fig. III-16

2. Two groups of intensity regions g_0, g_1 , as in Fig. III-17. The protocols of this group will again obey the same (MTP)'s with their parameters (γ) and (ε) :

	g_0	g_1
(γ)	0.2	0.8
	g_0	g_1
(ε)	0.85	0.4
	g_0	g_1
(ε)	0.15	0.6

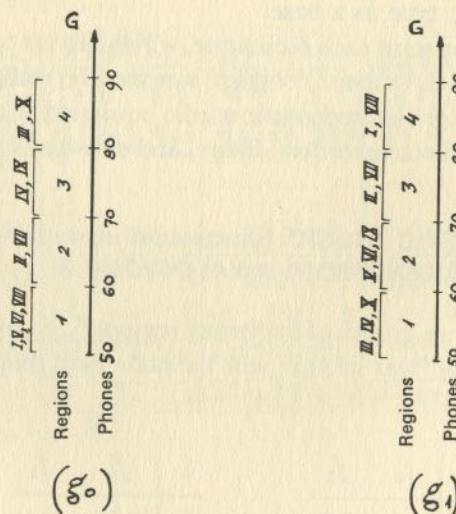


Fig. III-17

3. Two groups of density regions d_0, d_1 , as in Fig. III-18. The protocols of this group will have the same (MTP)'s with parameters (λ) and (μ) :

	d_0	d_1
(λ)	0.2	0.8
	d_0	d_1
(μ)	0.85	0.4
	d_0	d_1
(μ)	0.15	0.6

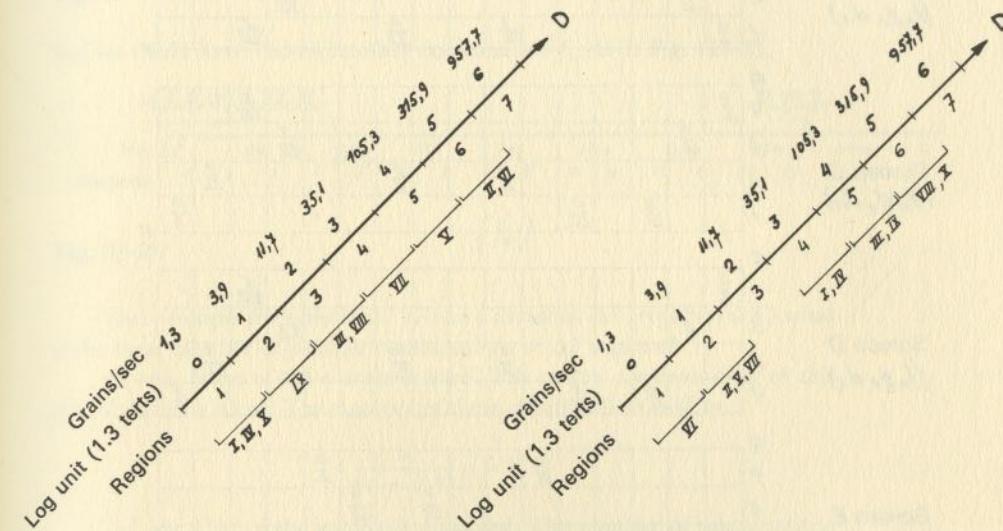


Fig. III-18

This choice gives us the principal screens A, B, C, D, E, F, G, H , as shown in Fig. III-19. The duration Δt of each screen is about 0.5 sec. The period of exposition of a perturbation or of a stationary state is about 15 sec.

We shall choose the same protocol of exchanges between perturbations and stationary states of (MTPZ), that of *Analogue A*.

$$(e_5) \quad E \rightarrow P_A^0 \rightarrow P'_A \rightarrow E \rightarrow P_C \rightarrow P_C^0 \rightarrow P_B^0 \rightarrow P'_B \rightarrow E \rightarrow P'_A$$

The screens of *Analogue B* calculated up to now constitute a special choice. Later in the course of this composition other screens will be used more particularly, but they will always obey the same rules of coupling and the same (MTPZ). In fact, if we consider the combinations of regions of the variable f_i of a screen, we notice that without tampering with the name of the variable f_i its structure may be changed.

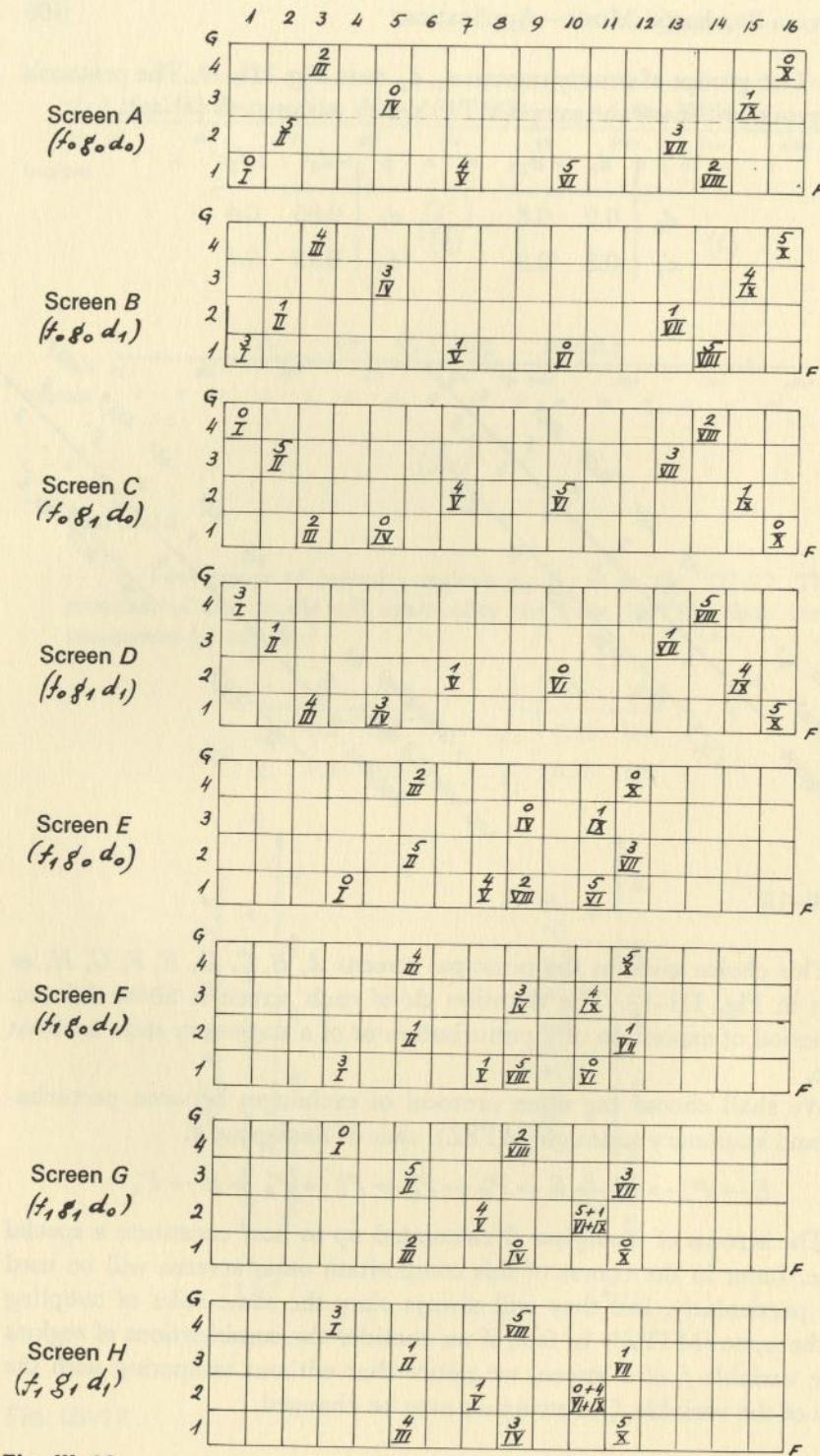


Fig. III-19

Thus for f_0 we may have the regions shown in Fig. III-20. The Roman numerals establish the liaison with the regions of the other two variables.

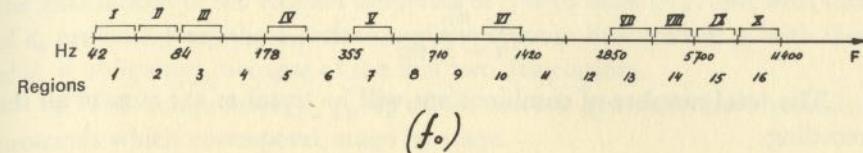


Fig. III-20

But we could have chosen another combination f_0 , as in Fig. III-21.

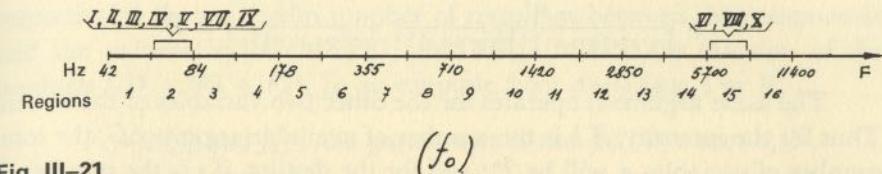


Fig. III-21

This prompts the question: "Given n divisions ΔF (regions on F) what is the total number of possible combinations of ΔF regions?

1st case. None of the n areas is used. The screen corresponding to this combination is silent. The number of these combinations will be

$$\frac{n!}{(n-0)!0!} (=1).$$

2nd case. One of the n areas is occupied. The number of combinations will be

$$\frac{n!}{(n-1)!1!}.$$

3rd case. Two of the n areas are occupied. The number of combinations will be

$$\frac{n!}{(n-2)!2!}.$$

m th case. m of the n areas are occupied. The number of combinations will be

$$\frac{n!}{(n-m)!m!}.$$

FIG. III-19: The Arabic numbers above the Roman numerals in the cells indicate the density in logarithmic units. Thus cell (10,1) will have a density of $[(\log 1.3/\log 3) + 5]$ terts, which is 315.9 grains/sec on the average.

nth case. n of the areas are occupied. The number of the combinations will be

$$\frac{n!}{(n-n)!n!}.$$

The total number of combinations will be equal to the sum of all the preceding:

$$\begin{aligned} \frac{n!}{(n-0)!0!} + \frac{n!}{(n-1)!1!} + \frac{n!}{(n-2)!2!} + \dots \\ + \frac{n!}{[n-(n-1)]!(n-1)!} + \frac{n!}{(n-n)!n!} = 2^n \end{aligned}$$

The same argument operates for the other two variables of the screen. Thus for the intensity, if k is the number of available regions ΔG , the total number of variables g_i will be 2^k ; and for the density, if r is the number of available regions ΔD , the total number of variables d_i will be 2^r .

Consequently the total number of possible screens will be

$$T = 2^{(n+k+r)}.$$

In the case of *Analogique B* we could obtain $2^{(16+4+7)} = 2^{27} = 134,217,728$ different screens.

Important comment. At the start of this chapter we would have accepted the richness of a musical evolution, an evolution based on the method of stochastic protocols of the coupled screen variables, as a function of the transformations of the entropies of these variables. From the preceding calculation, we now see that without modifying the entropies of the (MTPF), (MTPG), and (MTPD) we may obtain a supplementary subsidiary evolution by utilizing the different combinations of regions (topographic criterion).

Thus in *Analogique B* the (MTPF), (MTPG), and (MTPD) will not vary. On the contrary, in time the f_i, g_i, d_i will have new structures, corollaries of the changing combinations of their regions.

Complementary Conclusions about Screens and Their Transformations

1. *Rule.* To form a screen one may choose any combination of regions on F, G , and D , the f_i, g_i, d_i .

2. *Fundamental Criterion.* Each region of one of the variables F, G, D must be associative with a region corresponding to the other two variables in all the chosen couplings. (This is accomplished by the Roman numerals.)

3. The preceding association is arbitrary (free choice) for two pairs, but obligatory for the third pair, a consequence of the first two. For example, the associations of the Roman numerals of f_i with those of g_j and with those of d_k are both free; the association of the Roman numerals of g_j with those of d_k is obligatory, because of the first two associations.

4. The components f_i, g_j, d_k of the screens generally have stochastic protocols which correspond, stage by stage.

5. The (MTP) of these protocols will, in general, be coupled with the help of parameters.

6. If F, G, D are the “variations” (number of components f_i, g_i, d_i , respectively) the maximum number of couplings between the components and the parameters of (MTPF), (MTPG), (MTPD) is the sum of the products $GD + FG + FD$. In an example from *Analogique A* or *B*:

$$\begin{aligned} F = 2 \text{ } (f_0 \text{ and } f_1) & \text{ the parameters of the (MTP)'s are: } \alpha, \beta \\ G = 2 \text{ } (g_0 \text{ and } g_1) & \gamma, \varepsilon \\ D = 2 \text{ } (d_0 \text{ and } d_1) & \lambda, \mu \end{aligned}$$

and there are 12 couplings:

$$\begin{array}{cccccccccccc} f_0 & f_1 & f_0 & f_1 & g_0 & g_1 & g_0 & g_1 & d_0 & d_1 & d_0 & d_1 \\ \downarrow & & \downarrow & \\ \gamma & \varepsilon & \lambda & \mu & \beta & \alpha & \lambda & \mu & \alpha & \beta & \gamma & \varepsilon \end{array}$$

$$\text{Indeed, } FG + FD + GD = 4 + 4 + 4 = 12.$$

7. If F, G, D are the “variations” (number of components f_i, g_j, d_k , respectively), the number of possible screens T is the product FGD . For example, if $F = 2 \text{ } (f_0 \text{ and } f_1)$, $G = 2 \text{ } (g_0 \text{ and } g_1)$, $D = 2 \text{ } (d_0 \text{ and } d_1)$, $T = 2 \times 2 \times 2 = 8$.

8. The protocol of the screens is stochastic (in the broad sense) and can be summarized when the chain is ergodic (tending to regularity), by an (MTPZ). This matrix will have FGD rows and FGD columns.

SPATIAL PROJECTION

No mention at all has been made in this chapter of the spatialization of sound. The subject was confined to the fundamental concept of a sonic complex and of its evolution in itself. However nothing would prevent broadening of the technique set out in this chapter and “leaping” into space. We can, for example, imagine protocols of screens attached to a particular point in space, with transition probabilities, space-sound couplings, etc. The method is ready and the general application is possible, along with the reciprocal enrichments it can create.

Musical Strategy—Strategy, Linear Programming, and Musical Composition

Before passing to the problem of the mechanization of stochastic music by the use of computers, we shall take a stroll in a more enjoyable realm, that of games, their theory, and application in musical composition.

AUTONOMOUS MUSIC

The musical composer establishes a scheme or pattern which the conductor and the instrumentalists are called upon to follow more or less rigorously. From the final details—attacks, notes, intensities, timbres, and styles of performance—to the form of the whole work, virtually everything is written into the score. And even in the case where the composer leaves a margin of improvisation to the conductor, the instrumentalist, the machine, or to all three together, the unfolding of the sonic discourse follows an open line without *loops*. The score-model which is presented to them once and for all does not give rise to any *conflict* other than that between a "good" performance in the technical sense, and its "musical expression" as desired or suggested by the writer of the score. This opposition between the sonic realization and the symbolic schema which plots its course might be called *internal conflict*; and the role of the conductors, instrumentalists, and their machines is to control the output by feedback and comparison with the input signals, a role analogous to that of servo-mechanisms that reproduce profiles by such means as grinding machines. In general we can state that

the nature of the technical oppositions (instrumental and conductorial) or even those relating to the aesthetic logic of the musical discourse, is *internal* to the works written until now. The tensions are shut up in the score even when more or less defined stochastic processes are utilized. This traditional class of *internal conflict* might be qualified as *autonomous music*.

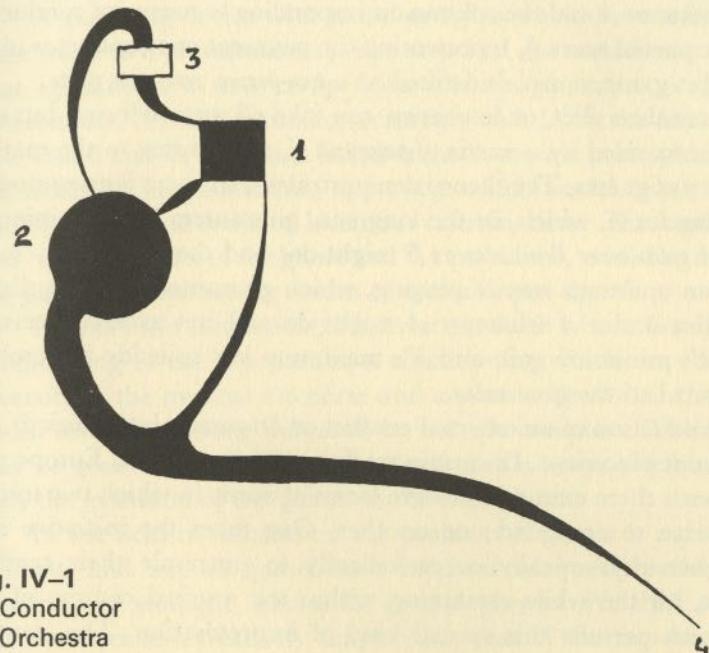


Fig. IV-1
 1. Conductor
 2. Orchestra
 3. Score
 4. Audience

HETERONOMOUS MUSIC

It would be interesting and probably very fruitful to imagine another class of musical discourse, which would introduce a concept of *external conflict* between, for instance, two opposing orchestras or instrumentalists. One party's move would influence and condition that of the other. The sonic discourse would then be identified as a very strict, although often stochastic, succession of sets of acts of sonic opposition. These acts would derive from both the will of the two (or more) conductors as well as from the will of the composer, all in a higher dialectical harmony.

Let us imagine a competitive situation between two orchestras, each having one conductor. Each of the conductors directs sonic operations *against* the operations of the other. Each operation represents a *move* or a *tactic* and the encounter between two moves has a numerical and/or a qualitative value which benefits one and harms the other. This value is written in a grid or matrix at the intersection of the row corresponding to move *i* of conductor *A* and the column corresponding to move *j* of conductor *B*. This is the partial score *ij*, representing the payment one conductor gives the other. This game, a *duel*, is defined as a *two-person zero-sum game*.

The external conflict, or *heteronomy*, can take all sorts of forms, but can always be summarized by a *matrix of payments* *ij*, conforming to the mathematical theory of games. The theory demonstrates that there is an optimum way of playing for *A*, which, in the long run, guarantees him a minimum advantage or gain over *B* whatever *B* might do; and that conversely there exists for *B* an optimum way of playing, which guarantees that his disadvantage or loss under *A* whatever *A* might do will not exceed a certain maximum. *A*'s minimum gain and *B*'s maximum loss coincide in absolute value; this is called the *game value*.

The introduction of an external conflict or *heteronomy* into music is not entirely without precedent. In certain traditional folk music in Europe and other continents there exist competitive forms of music in which two instrumentalists strive to confound one another. One takes the initiative and attempts either rhythmically or melodically to uncouple their tandem arrangement, all the while remaining within the musical context of the tradition which permits this special kind of improvisation. This contradictory virtuosity is particularly prevalent among the Indians, especially among tabla and sarod (or sitar) players.

A musical *heteronomy* based on modern science is thus legitimate even to the most conformist eye. But the problem is not the historical justification of a new adventure; quite the contrary, it is the enrichment and the leap forward that count. Just as stochastic processes brought a beautiful generalization to the complexity of linear polyphony and the deterministic logic of musical discourse, and at the same time disclosed an unsuspected opening on a totally asymmetric aesthetic form hitherto qualified as nonsense; in the same way *heteronomy* introduces into stochastic music a complement of dialectical structure.

We could equally well imagine setting up conflicts between two or more instrumentalists, between one player and what we agree to call natural environment, or between an orchestra or several orchestras and the public. But the fundamental characteristic of this situation is that there exists a gain

and a loss, a victory and a defeat, which may be expressed by a moral or material reward such as a prize, medal, or cup for one side, and by a penalty for the other.

A *degenerate* game is one in which the parties play arbitrarily following a more or less improvised route, without any conditioning for conflict, and therefore without any new compositional argument. This is a false game.

A gambling device with sound or lights would have a *trivial* sense if it were made in a gratuitous way, like the usual slot machines and juke boxes, that is, without a new competitive inner organization inspired by any heteronomy. A sharp manufacturer might cash in on this idea and produce new sound and light devices based on heteronomic principles. A less trivial use would be an educational apparatus which would require children (or adults) to react to sonic or luminous combinations. The aesthetic interest, and hence the rules of the game and the payments, would be determined by the players themselves by means of special input signals.

In short the fundamental interest set forth above lies in the mutual conditioning of the two parties, a conditioning which respects the greater diversity of the musical discourse and a certain liberty for the players, but which involves a strong influence by a single composer. This point of view may be generalized with the introduction of a spatial factor in music and with the extension of the games to the art of light.

In the field of calculation the problem of games is rapidly becoming difficult, and not all games have received adequate mathematical clarification, for example, games for several players. We shall therefore confine ourselves to a relatively simple case, that of the two-person zero-sum game.

ANALYSIS OF DUEL

This work for two conductors and two orchestras was composed in 1958-59. It appeals to relatively simple concepts: sonic constructions put into mutual correspondence by the will of the conductors, who are themselves conditioned by the composer. The following events can occur:

Event I: A cluster of sonic grains such as pizzicati, blows with the wooden part of the bow, and very brief arco sounds distributed stochastically.

Event II: Parallel sustained strings with fluctuations.

Event III: Networks of intertwined string glissandi.

Event IV: Stochastic percussion sounds.

Event V: Stochastic wind instrument sounds.

Event VI: Silence.

Each of these events is written in the score in a very precise manner and with sufficient length, so that at any moment, following his instantaneous choice, the conductor is able to cut out a slice without destroying the identity of the event. We therefore imply an overall homogeneity in the writing of each event, at the same time maintaining local fluctuations.

We can make up a list of couples of simultaneous events x, y issuing from the two orchestras X and Y , with our subjective evaluations. We can also write this list in the form of a qualitative matrix (M_1) .

Table of Evaluations

Couple $(x, y) = (y, x)$	Evaluation	
(I, I)	passable	(p)
(I, II) = (II, I)	good	(g)
(I, III) = (III, I)	good ⁺	(g^+)
(I, IV) = (IV, I)	passable ⁺	(p^+)
(I, V) = (V, I)	very good	(g^{++})
(II, II)	passable	(p)
(II, III) = (III, II)	passable	(p)
(II, IV) = (IV, II)	good	(g)
(II, V) = (V, II)	passable ⁺	(p^+)
(III, III)	passable	(p)
(III, IV) = (IV, III)	good ⁺	(g^+)
(III, V) = (V, III)	good	(g)
(IV, IV)	passable	(p)
(IV, V) = (V, IV)	good	(g)
(V, V)	passable	(p)

Conductor Y

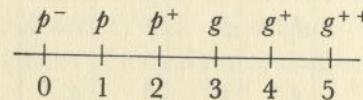
	I	II	III	IV	V	Minimum per row
Conductor X	p	g	g^+	p^+	g^{++}	p
	g	p	p	g	p^+	p
	g^+	p	p	g^+	g	p
	p^+	g	g^+	p	g	p
	g^{++}	p^+	g	g	p	p
		Maximum per column	g^{++}	g	g^+	g^+
						g^{++}

In (M_1) the largest minimum per row and the smallest maximum per column do not coincide ($g \neq p$), and consequently the game has no saddle point and no pure strategy. The introduction of the move of silence (VI) modifies (M_1) , and matrix (M_2) results.

Conductor Y

	I	II	III	IV	V	VI	
Conductor X	p	g	g^{++}	g^+	g^+	p	p
	g	p	p	g	p^+	p	p
	g^{++}	p	p	g^+	g	p	p
	g^+	g	g^+	p	g	p	p
	g^+	p^+	g	g	p	p	p
	p	p	p	p	p	p^-	p^-
		g^{++}	g	g^{++}	g^+	g^+	p

This time the game has several saddle points. All tactics are possible, but a closer study shows that the conflict is still too slack: Conductor Y is interested in playing tactic VI only, whereas conductor X can choose freely among I, II, III, IV, and V. It must not be forgotten that the rules of this matrix were established for the benefit of conductor X and that the game in this form is not fair. Moreover the rules are too vague. In order to pursue our study we shall attempt to specify the qualitative values by ordering them on an axis and making them correspond to a rough numerical scale:



If, in addition, we modify the value of the couple (VI, VI) the matrix becomes (M_3) .

Conductor Y								
	I	II	III	IV	V	VI		
Conductor X	I	1	3	5	4	4	1	1
	II	3	1	1	3	2	1	1
	III	5	1	1	4	3	1	1
	IV	4	3	4	1	3	1	1
	V	4	2	3	3	1	1	1
	VI	1	1	1	1	1	3	1
	5	3	5	4	4	3		

(M_3) has no saddle point and no recessive rows or columns. To find the solution we apply an approximation method, which lends itself easily to computer treatment but modifies the relative equilibrium of the entries as little as possible. The purpose of this method is to find a mixed strategy; that is to say, a weighted multiplicity of tactics of which none may be zero. It is not possible to give all the calculations here [21], but the matrix that results from this method is (M_4) , with the two unique strategies for X and for Y written in the margin of the matrix. Conductor X must therefore play

Conductor Y								
	I	II	III	IV	V	VI		
Conductor X	I	2	3	4	2	3	2	18
	II	3	2	2	3	3	2	4
	III	4	2	1	4	3	1	5
	IV	2	4	4	2	2	2	5
	V	3	2	3	3	2	2	11
	VI	2	2	1	2	2	4	15
	9	6	8	12	9	14	58	Total

tactics I, II, III, IV, V, VI in proportions 18/58, 4/58, 5/58, 5/58, 11/58, 15/58, respectively; while conductor Y plays these six tactics in the proportions 9/58, 6/58, 8/58, 12/58, 9/58, 14/58, respectively. The game value from this method is about 2.5 in favor of conductor X (game with zero-sum but still not fair).

We notice immediately that the matrix is no longer symmetrical about its diagonal, which means that the tactic couples are not commutative, e.g., $(IV, II = 4) \neq (II, IV = 3)$. There is an orientation derived from the adjustment of the calculation which is, in fact, an enrichment of the game.

The following stage is the experimental control of the matrix.

Two methods are possible:

1. Simulate the game, i.e., mentally substitute oneself for the two conductors, X and Y , by following the matrix entries stage by stage, without memory and without bluff, in order to test the least interesting case.

Stages:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Cond. X	I	III	I	VI	I	III	VI	IV	III	III	IV	IV	III	III	IV	III	III	IV	IV	IV	
Cond. Y	IV	III	VI	III	I	VI	III	V	V	II	III	IV	IV	IV	IV	IV	IV	IV	IV	IV	
Scores:	2	4	1	4	2	4	1	4	2	4	1	4	1	4	2	3	2	2	1	4	2

Game value: 52/20 = 2.6 points in X 's favor.

2. Choose tactics at random, but with frequencies proportional to the marginal numbers in (M_4) .

Stages:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Cond. X	I	VI	VI	II	I	II	V	IV		I	V	V									
Cond. Y	VI	VI	V	III	I	IV	VI	V	IV	VI	VI										
Scores:	2	4	4	4	2	3	2	4	2	3	3	2	2	2	3	2	3	2	2	3	

Game value: $57/21 = 2.7$ points in X 's favor.

We now establish that the experimental game values are very close to the value calculated by approximation. The sonic processes derived from the two experiments are, moreover, satisfactory.

We may now apply a rigorous method for the definition of the optimum strategies for X and Y and the value of the game by using methods of linear programming, in particular the simplex method [22]. This method is based on two theses:

1. The fundamental theorem of game theory (the "minimax theorem") is that the minimum score (maximin) corresponding to X 's optimum strategy is always equal to the maximum score (minimax) corresponding to Y 's optimum strategy.

2. The calculation of the maximin or minimax value, just as the probabilities of the optimum strategies of a two-person zero-sum game, comes down to the resolution of a pair of dual problems of linear programming (dual simplex method).

Here we shall simply state the system of linear equations for the player of the minimum, Y . Let $y_1, y_2, y_3, y_4, y_5, y_6$ be the probabilities corresponding to tactics I, II, III, IV, V, VI of Y ; $y_7, y_8, y_9, y_{10}, y_{11}, y_{12}$ be the "slack" variables; and v be the game value which must be minimized. We then have the following liaisons:

$$\begin{aligned}
 y_1 + y_2 + y_3 + y_4 + y_5 + y_6 &= 1 \\
 2y_1 + 3y_2 + 4y_3 + 2y_4 + 3y_5 + 2y_6 + y_7 &= v \\
 3y_1 + 2y_2 + 2y_3 + 2y_4 + 3y_5 + 2y_6 + y_8 &= v \\
 2y_1 + 4y_2 + 4y_3 + 2y_4 + 2y_5 + 2y_6 + y_9 &= v \\
 3y_1 + 2y_2 + 3y_3 + 3y_4 + 2y_5 + 2y_6 + y_{10} &= v \\
 2y_1 + 2y_2 + y_3 + 2y_4 + 2y_5 + 4y_6 + y_{11} &= v \\
 4y_1 + 2y_2 + y_3 + 4y_4 + 3y_5 + y_6 + y_{12} &= v.
 \end{aligned}$$

To arrive at a unique strategy, the calculation leads to the modification

of the score (III, IV = 4) into (III, IV = 5). The solution gives the following optimum strategies:

For X		For Y	
Tactics	Probabilities	Tactics	Probabilities
I	2/17	I	5/17
II	6/17	II	2/17
III	0	III	2/17
IV	3/17	IV	1/17
V	2/17	V	2/17
VI	4/17	VI	5/17

and for the game value, $v = 42/17 \approx 2.47$. We have established that X must completely abandon tactic III (probability of III = 0), and this we must avoid.

Modifying score (II, IV = 3) to (II, IV = 2), we obtain the following optimum strategies:

For X		For Y	
Tactics	Probabilities	Tactics	Probabilities
I	14/56	I	19/56
II	6/56	II	7/56
III	6/56	III	6/56
IV	6/56	IV	1/56
V	8/56	V	7/56
VI	16/56	VI	16/56

and for the game value, $v = 138/56 \approx 2.47$ points.

Although the scores have been modified a little, the game value has, in fact, not moved. But on the other hand the optimum strategies have varied widely. A rigorous calculation is therefore necessary, and the final matrix accompanied by its calculated strategies is (M_5) .

		Conductor Y							
		I	II	III	IV	V	VI		
Conductor X	I	2	3	4	2	3	2	14	
	II	3	2	2	2	3	2	6	
	III	4	2	1	5	3	1	6	
	IV	2	4	4	2	2	2	6	
	V	3	2	3	3	2	2	8	
	VI	2	2	1	2	2	4	16	
		19	7	6	1	7	16	56	Total

By applying the elementary matrix operations to the rows and columns in such a way as to make the game fair (game value = 0), we obtain the equivalent matrix (M_6) with a zero game value.

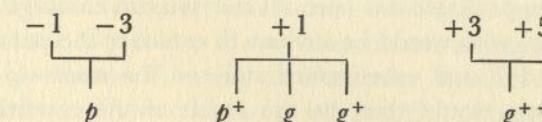
		Conductor <i>Y</i>						(M_6)
		I	II	III	IV	V	VI	
Conductor <i>X</i>	I	-13	15	43	-13	15	-13	$\frac{14}{56}$
	II	15	-13	-13	-13	15	-13	$\frac{6}{56}$
	III	43	-13	-41	71	15	-41	$\frac{6}{56}$
	IV	-13	43	43	-13	-13	-13	$\frac{6}{56}$
	V	15	-13	15	15	-13	-13	$\frac{8}{56}$
	VI	-13	-13	-41	-13	-13	43	$\frac{16}{56}$
		$\frac{19}{56}$	$\frac{7}{56}$	$\frac{6}{56}$	$\frac{1}{56}$	$\frac{7}{56}$	$\frac{16}{56}$	

As this matrix is difficult to read, it is simplified by dividing all the scores by +13. It then becomes (M_7) with a game value $v = -0.07$, which

		Conductor <i>Y</i>						(M_7)
		I	II	III	IV	V	VI	
Conductor <i>X</i>	I	-1	+1	+3	-1	+1	-1	$\frac{14}{56}$
	II	+1	-1	-1	-1	+1	-1	$\frac{6}{56}$
	III	+3	-1	-3	+5	+1	-3	$\frac{6}{56}$
	IV	-1	+3	+3	-1	-1	-1	$\frac{6}{56}$
	V	+1	-1	+1	+1	-1	-1	$\frac{8}{56}$
	VI	-1	-1	-3	-1	-1	+3	$\frac{16}{56}$
		$\frac{19}{56}$	$\frac{7}{56}$	$\frac{6}{56}$	$\frac{1}{56}$	$\frac{7}{56}$	$\frac{16}{56}$	

means that at the end of the game, at the final score, conductor Y should give $0.07m$ points to conductor X , where m is the total number of moves.

If we convert the numerical matrix (M_7) into a qualitative matrix according to the correspondence:



we obtain (M_8) , which is not very different from (M_2) , except for the silence couple, VI, VI, which is the opposite of the first value. The calculation is now finished.

(M₈)

Mathematical manipulation has brought about a refinement of the duel and the emergence of a paradox: the couple VI, VI, characterizing total silence. Silence is to be avoided, but to do this it is necessary to augment its potentiality.

It is impossible to describe in these pages the fundamental role of the mathematical treatment of this problem, or the subtle arguments we are forced to make on the way. We must be vigilant at every moment and over every part of the matrix area. It is an instance of the kind of work where detail is dominated by the whole, and the whole is dominated by detail. It was to show the value of this intellectual labor that we judged it useful to set out the processes of calculation.

The conductors direct with their backs to each other, using finger or light signals that are invisible to the opposing orchestra. If the conductors use illuminated signals operated by buttons, the successive partial scores can be announced automatically on lighted panels in the hall, the way the score is displayed at football games. If the conductors just use their fingers, then a referee can count the points and put up the partial scores manually so they are visible in the hall. At the end of a certain number of exchanges or minutes, as agreed upon by the conductors, one of the two is declared the winner and is awarded a prize.

Now that the principle has been set out, we can envisage the intervention of the public, who would be invited to evaluate the pairs of tactics of conductors *X* and *Y* and vote immediately on the make-up of the game matrix. The music would then be the result of the conditioning of the composer who established the musical score, conductors *X* and *Y*, and the public who construct the matrix of points.

RULES OF THE WORK STRATÉGIE

The two-headed flow chart of *Duel* is shown in Fig. IV-2. It is equally valid for *Stratégie*, composed in 1962. The two orchestras are placed on either side of the stage, the conductors back-to-back (Fig. IV-3), or on platforms on opposite sides of the auditorium. They may choose and play one of six sonic constructions, numbered in the score from I to VI. We call them tactics and they are of stochastic structure. They were calculated on the IBM-7090 in Paris. In addition, each conductor can make his orchestra play simultaneous combinations of two or three of these fundamental tactics. The six fundamental tactics are:

- I. Winds
- II. Percussion
- III. String sound-box struck with the hand
- IV. String pointillistic effects
- V. String glissandi
- VI. Sustained string harmonics.

The following are 13 compatible and simultaneous combinations of these tactics:

I & II = VII	II & III = XII	I & II & III = XVI
I & III = VIII	II & IV = XIII	I & II & IV = XVII
I & IV = IX	II & V = XIV	I & II & V = XVIII
I & V = X	II & VI = XV	I & II & VI = XIX
I & VI = XI		

Thus there exist in all 19 tactics which each conductor can make his orchestra play, $361 (19 \times 19)$ possible pairs that may be played simultaneously.

The Game

1. *Choosing tactics.* How will the conductors choose which tactics to play?

a. A first solution consists of arbitrary choice. For example, conductor *X* chooses tactic I. Conductor *Y* may then choose any one of the 19 tactics including I. Conductor *X*, acting on *Y*'s choice, then chooses a new tactic (see Rule 7 below). *X*'s second choice is a function of both his taste and *Y*'s choice. In his turn, conductor *Y*, acting on *X*'s choice and his own taste, either chooses a new tactic or keeps on with the old one, and plays it for a certain optional length of time. And so on. We thus obtain a continuous succession of couplings of the 19 structures.

b. The conductors draw lots, choosing a new tactic by taking one card from a pack of 19; or they might make a drawing from an urn containing balls numbered from I to XIX in different proportions. These operations can be carried out before the performance and the results of the successive draws set down in the form of a sequential plan which each of the conductors will have before him during the performance.

c. The conductors get together in advance and choose a fixed succession which they will direct.

d. Both orchestras are directed by a single conductor who establishes the succession of tactics according to one of the above methods and sets them down on a master plan, which he will follow during the performance.

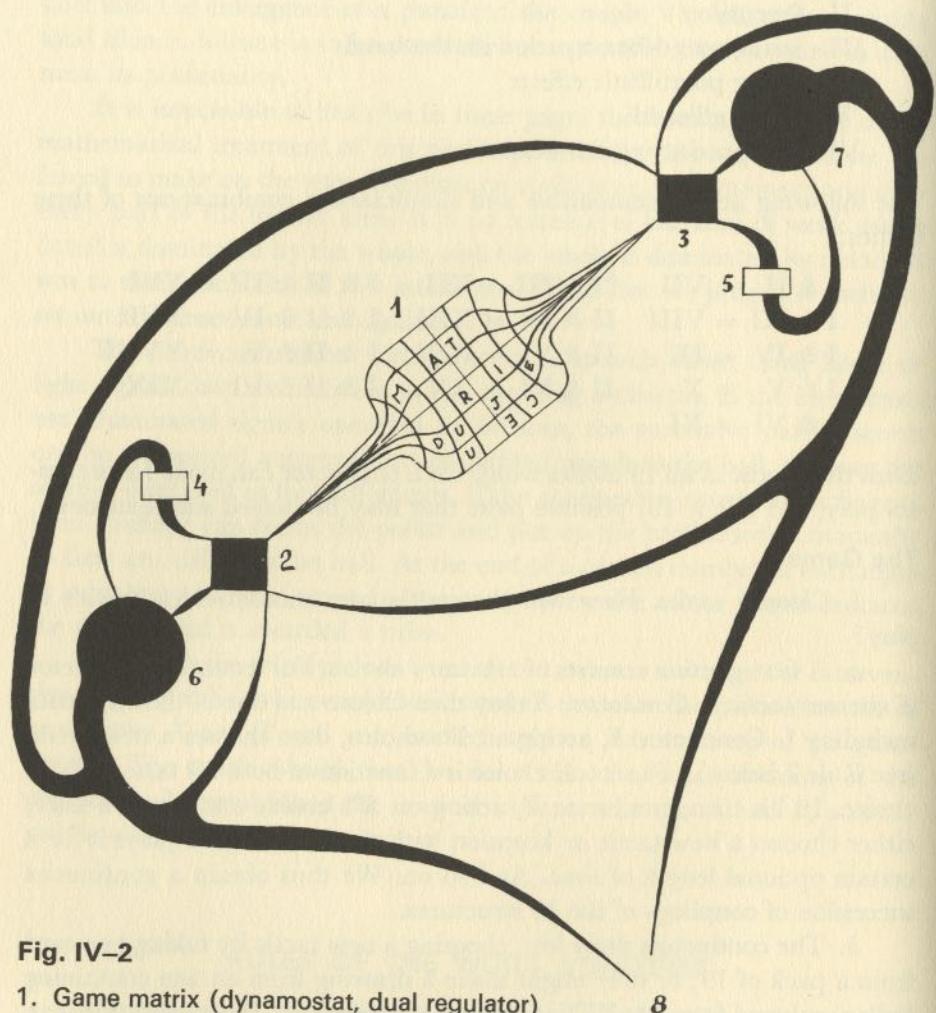


Fig. IV-2

1. Game matrix (dynamostat, dual regulator)
2. Conductor *A* (device for comparison and decision)
3. Conductor *B* (device for comparison and decision)
4. Score *A* (symbolic excitation)
5. Score *B* (symbolic excitation)
6. Orchestra *A* (human or electronic transforming device)
7. Orchestra *B* (human or electronic transforming device)
8. Audience

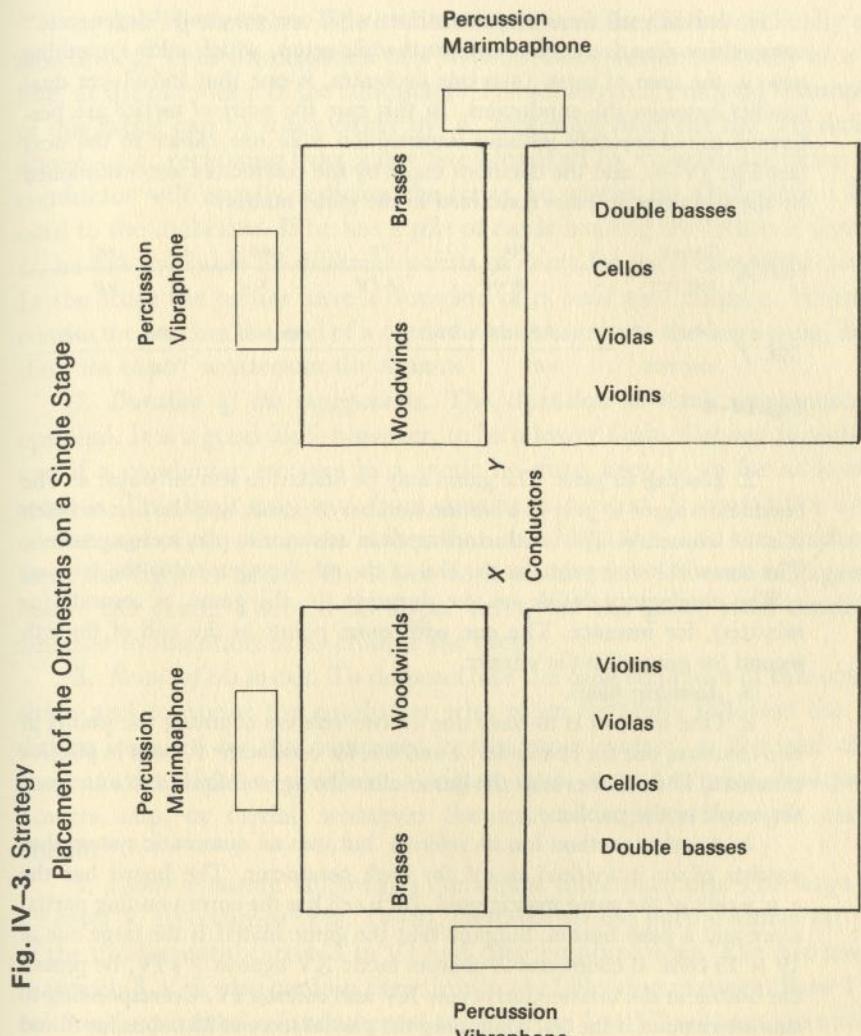


Fig. IV-3. Strategy
Placement of the Orchestras on a Single Stage

*Composition of the
orchestra:*

88 *players* in both
orchestras

← Public →

NOTE: If two stages are used, each orchestra is arranged in the classic manner.

e. Actually all these ways constitute what one may call "degenerate" competitive situations. The only worthwhile setup, which adds something new in the case of more than one orchestra, is one that introduces dual conflict between the conductors. In this case the pairs of tactics are performed simultaneously without interruption from one choice to the next (see Fig. IV-4), and the decisions made by the conductors are conditioned by the winnings or losses contained in the game matrix.

CON	OR X	GAINS				36	
		TACTICS	IX	XVIII	XIV		
CON	OR Y	GAINS	52	40	48	28	48
		TACTICS	VII	XIX	XV	V	

Fig. IV-4

2. *Limiting the game.* The game may be limited in several ways: a. The conductors agree to play to a certain number of points, and the first to reach it is the winner. b. The conductors agree in advance to play n engagements. The one with more points at the end of the n th engagement is the winner. c. The conductors decide on the duration for the game, m seconds (or minutes), for instance. The one with more points at the end of the m th second (or minute) is the winner.

3. *Awarding points.*

a. One method is to have one or two referees counting the points in two columns, one for conductor X and one for conductor Y , both in positive numbers. The referees stop the game after the agreed limit and announce the result to the public.

b. Another method has no referees, but uses an automatic system that consists of an individual board for each conductor. The board has the $n \times n$ cells of the game matrix used. Each cell has the corresponding partial score and a push button. Suppose that the game matrix is the large one of 19×19 cells. If conductor X chooses tactic XV against Y 's IV, he presses the button at the intersection of row XV and column IV. Corresponding to this intersection is the cell containing the partial score of 28 points for X and the button that X must push. Each button is connected to a small adding machine which totals up the results on an electric panel so that they can be seen by the public as the game proceeds, just like the panels in the football stadium, but on a smaller scale.

4. *Assigning of rows or columns* is made by the conductors tossing a coin.

5. *Deciding who starts the game* is determined by a second toss.

6. *Reading the tactics.* The orchestras perform the tactics cyclically on a closed loop. Thus the cessation of a tactic is made instantaneously at a bar line, at the discretion of the conductor. The subsequent eventual resumption of this tactic can be made either by: a. reckoning from the bar line defined above, or b. reckoning from a bar line identified by a particular letter. The conductor will usually indicate the letter he wishes by displaying a large card to the orchestra. If he has a pile of cards bearing the letters A through U , he has available 22 different points of entry for each one of the tactics. In the score the tactics have a duration of at least two minutes. When the conductor reaches the end of a tactic he starts again at the beginning, hence the "da capo" written on the score.

7. *Duration of the engagements.* The duration of each engagement is optional. It is a good idea, however, to fix a lower limit of about 10 seconds; i.e., if a conductor engages in a tactic he must keep it up for at least 10 seconds. This limit may vary from concert to concert. It constitutes a wish on the part of the composer rather than an obligation, and the conductors have the right to decide the lower limit of duration for each engagement before the game. There is no upper limit, for the game itself conditions whether to maintain or to change the tactic.

8. *Result of the contest.* To demonstrate the dual structure of this composition and to honor the conductor who more faithfully followed the conditions imposed by the composer in the game matrix, at the end of the combat one might a. proclaim a victor, or b. award a prize, bouquet of flowers, cup, or medal, whatever the concert impresario might care to donate.

9. *Choice of matrix.* In *Stratégie* there exist three matrices. The large one, 19 rows \times 19 columns (Fig. IV-5), contains all the partial scores for pairs of the fundamental tactics I to VI and their combinations. The two smaller matrices, 3×3 , also contain these but in the following manner: Row 1 and column 1 contain the fundamental tactics from I to VI without discrimination; row 2 and column 2 contain the two-by-two compatible combinations of the fundamental tactics; and row 3 and column 3 contain the three-by-three compatible combinations of these tactics. The choice between the large 19×19 matrix and one of the 3×3 matrices depends on the ease with which the conductors can read a matrix. The cells with positive scores mean a gain for conductor X and automatically a symmetrical loss for conductor Y . Conversely, the cells with negative scores mean a loss for conductor X and automatically a symmetrical gain for conductor Y . The two simpler, 3×3 matrices with different strategies are shown in Fig. IV-6.

MATRIX OF THE GAME

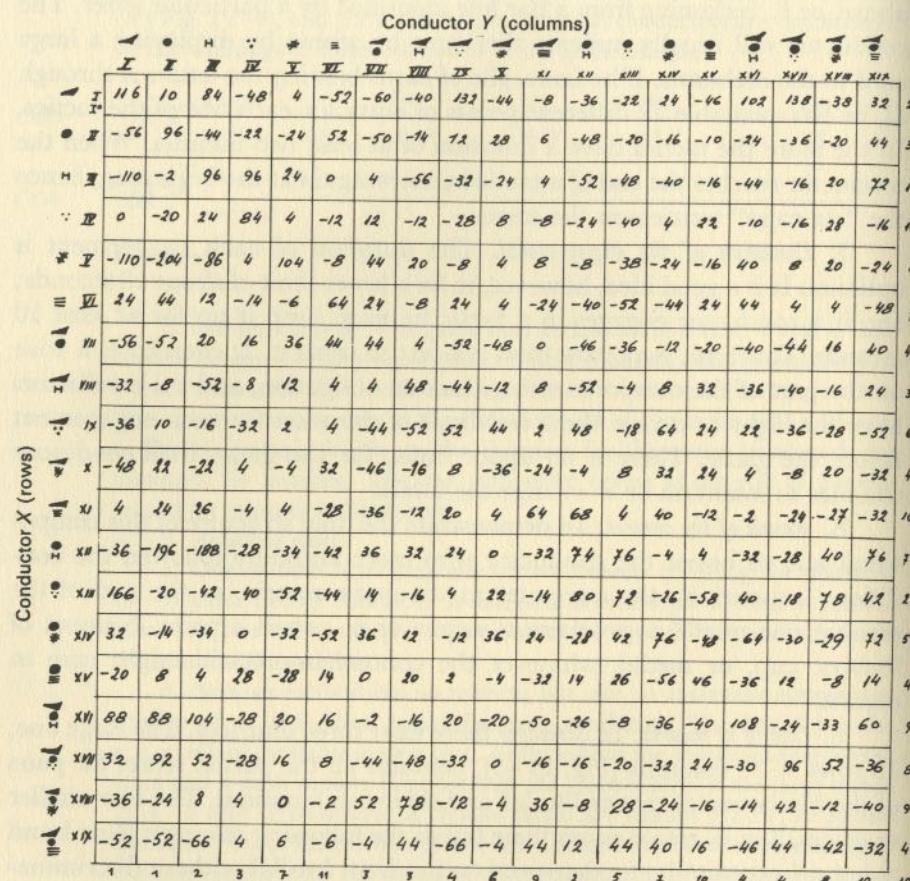


Fig. IV-5. Strategy

Two-person Game. Value of the Game = 0.

- ▲ Woodwinds
- Normal percussion
- Strings striking sound-boxes
- ◆ Strings pizzicato
- # Strings glissando
- ≡ Strings sustained

- Combinations of two and three different tactics
- Combinations of two distinct tactics
- ◆ Combinations of three distinct tactics
- Combinations of three distinct tactics

Conductor Y (columns) I : * : * for X			Strategy
Conductor X (rows)			Strategy for Y
3	0	-1	$\frac{2}{11}$
3	-3	3	$\frac{5}{11}$
-5	4	-3	$\frac{4}{11}$

Fig. IV-6

Two-person Zero-sum Game.
Value of the Game = 1/11.
This game is not fair for Y.

▼ Woodwinds	● Combinations of two distinct tactics
● Normal percussion	■ Combinations of three distinct tactics
■ Strings striking sound-boxes	◆ Combinations of three distinct tactics
◆ Strings pizzicato	■ Combinations of two distinct tactics
# Strings glissando	● Combinations of two and three different tactics
≡ Strings sustained	■ Combinations of two and three different tactics

Conductor Y (columns) I : * : * for X			Strategy
Conductor X (rows)			Strategy for Y
3	-1	0	$\frac{1}{6}$
-5	-1	4	$\frac{3}{6}$
3	1	-3	$\frac{4}{6}$

Simplification of the 19×19 Matrix

To make first performances easier, the conductors might use an equivalent 3×3 matrix derived from the 19×19 matrix in the following manner:

Let there be a fragment of the matrix containing row tactics $r+1, \dots, r+m$ and column tactics $s+1, \dots, s+n$ with the respective probabilities q_{r+1}, \dots, q_{r+m} and k_{s+1}, \dots, k_{s+n} .

$$\begin{array}{cccc}
 & k_{s+1} & k_{s+j} & k_{s+n} \\
 q_{r+1} & a_{r+1,s+1} & \cdots & a_{r+1,s+j} & \cdots & a_{r+1,s+n} \\
 & \vdots & & \vdots & & \vdots \\
 q_{r+i} & a_{r+i,s+1} & \cdots & a_{r+i,s+j} & \cdots & a_{r+i,s+n} \\
 & \vdots & & \vdots & & \vdots \\
 q_{r+m} & a_{r+m,s+1} & \cdots & a_{r+m,s+j} & \cdots & a_{r+m,s+n}
 \end{array}$$

This fragment can be replaced by the single score

$$A_{r+m, s+n} = \frac{\sum_{i,j=1}^{i=m, j=n} (a_{r+i, s+j})(q_{r+i})(k_{s+j})}{\sum_i^m q_{r+i} \sum_j^n k_{s+j}}$$

and by the probabilities

$$Q = \sum_{i=1}^m q_{r+i}$$

and

$$K = \sum_{j=i}^n k_{s+j}.$$

Operating in this way with the 19×19 matrix we obtain the following matrix (the tactics will be the same as in the matrices in Fig. IV-6):

$\frac{7704}{25 \times 25}$	$-\frac{8296}{49 \times 25}$	$\frac{592}{25 \times 26}$	25
$-\frac{14522}{25 \times 45}$	$\frac{17610}{49 \times 45}$	$-\frac{3088}{45 \times 26}$	45
$\frac{6818}{25 \times 30}$	$-\frac{9314}{49 \times 30}$	$\frac{2496}{30 \times 26}$	30

25 49 26

	2465	-1354	182	25
or	-2581	1597	-528	45
	1818	-1267	640	30

25 49 26

Chapter V

Free Stochastic Music by Computer

After this interlude, we return to the treatment of composition by machines.

The theory put forward by *Achorripsis* had to wait four years before being realized mechanically. This realization occurred thanks to M. François Génouys of IBM-France and to M. Jacques Barraud of the Régie Autonome des Transports Parisiens.

THE PARADOX: MUSIC AND COMPUTERS

A STOCHASTIC WORK EXECUTED BY THE IBM-7090

The general public has a number of different reactions when faced by the alliance of the machine with artistic creation. They fall into three categories:

"It is impossible to obtain a *work of art*, since by definition it is a handicraft and requires moment-by-moment "creation" for each detail and for the entire structure, while a machine is an inert thing and cannot invent."

"Yes, one may play games with a machine or use it for speculative purposes, but the result will not be "finished": it will represent only an experiment—interesting, perhaps, but no more."

The enthusiasts who at the outset accept without flinching the whole frantic brouhaha of science fiction. "The moon? Well, yes, it's within our reach. Prolonged life will also be with us tomorrow—why not a creative machine?" These people are among the credulous, who, in their idiosyncratic optimism, have replaced the myths of Icarus and the fairies, which have decayed, by the scientific civilization of the twentieth century, and science partly agrees with them. In reality, science is neither all paradox nor all animism, for it progresses in limited stages that are not foreseeable at too great a distance.

There exists in all the arts what we may call rationalism in the etymological sense: the search for proportion. The *artist* has always called upon it out of *necessity*. The rules of construction have varied widely over the centuries, but there have always been rules in every epoch because of the necessity of making oneself understood. Those who believe the first statement above are the first to refuse to apply the qualification *artistic* to a product which they do not *understand* at all.

Thus the musical scale is a convention which circumscribes the area of potentiality and permits construction within those limits in its own particular symmetry. The rules of Christian hymnography, of harmony, and of counterpoint in the various ages have allowed artists to construct and to make themselves understood by those who adopted the same constraints—through traditions, through collective taste or imitation, or through sympathetic resonance. The rules of serialism, for instance, those that banned the traditional octave doublings of tonality, imposed constraints which were partly new but none the less real.

Now everything that is rule or repeated constraint is part of the mental machine. A little “imaginary machine,” Philippot would have said—a choice, a set of decisions. A musical work can be analyzed as a multitude of mental machines. A melodic theme in a symphony is a mold, a mental machine, in the same way as its structure is. These mental machines are something very restrictive and deterministic, and sometimes very vague and indecisive. In the last few years we have seen that this idea of mechanism is really a very general one. It flows through every area of human knowledge and action, from strict logic to artistic manifestations.

Just as the wheel was once one of the greatest products of human intelligence, a mechanism which allowed one to travel farther and faster with more luggage, so is the computer, which today allows the transformation of man’s ideas. Computers resolve logical problems by heuristic methods. But computers are not really responsible for the introduction of mathematics into music; rather it is mathematics that makes use of the computer in composition. Yet if people’s minds are in general ready to recognize the usefulness of geometry in the plastic arts (architecture, painting, etc.), they have only one more stream to cross to be able to conceive of using more abstract, non-visual mathematics and machines as aids to musical composition, which is more abstract than the plastic arts.

To summarize:

1. The creative thought of man gives birth to mental mechanisms,

which, in the last analysis, are merely sets of constraints and choices. This process takes place in all realms of thought, including the arts.

2. Some of these mechanisms can be expressed in mathematical terms.
3. Some of them are physically realizable: the wheel, motors, bombs, digital computers, analogue computers, etc.
4. Certain mental mechanisms may correspond to certain mechanisms of nature.
5. Certain mechanizable aspects of artistic creation may be simulated by certain physical mechanisms or machines which exist or may be created.
6. It happens that computers can be useful in certain ways.

Here then is the theoretical point of departure for a utilization of electronic computers in musical composition.

We may further establish that the role of the living composer seems to have evolved, on the one hand, to one of inventing schemes (previously forms) and exploring the limits of these schemes, and on the other, to effecting the scientific synthesis of the new methods of construction and of sound emission. In a short while these methods must comprise all the ancient and modern means of musical instrument making, whether acoustic or electronic, with the help, for example, of digital-to-analogue converters; these have already been used in communication studies by N. Guttman, J. R. Pierce, and M. V. Mathews of Bell Telephone Laboratories in New Jersey. Now these explorations necessitate impressive mathematical, logical, physical, and psychological impedimenta, especially computers that accelerate the mental processes necessary for clearing the way for new fields by providing immediate experimental verifications at all stages of musical construction.

Music, by its very abstract nature, is the first of the arts to have attempted the conciliation of artistic creation with scientific thought. Its industrialization is inevitable and irreversible. Have we not already seen attempts to industrialize serial and popular music by the Parisian team of P. Barbaud, P. Blanchard, and Jeanine Charbonnier, as well as by the musicological research of Hiller and Isaacson at the University of Illinois?

In the preceding chapters we demonstrated some new areas of musical creation: Poisson, Markov processes, musical games, the thesis of the minimum of constraints, etc. They are all based on mathematics and especially on the theory of probability. They therefore lend themselves to being treated and explored by computers. The simplest and most meaningful scheme is one of minimum constraints in composition, as exemplified by *Achorripsis*.

Thanks to my friend Georges Boudouris of the C.N.R.S. I made the

acquaintance of Jacques Barraud, Engineer of the Ecole des Mines, then director of the Ensemble Electroniques de Gestion de la Société des Petroles Shell-Berre, and François Génuy, agrégé in mathematics, and head of the Etudes Scientifiques Nouvelles at IBM-France. All three are scientists, yet they consented to attempt an experiment which seemed at first far-fetched—that of a marriage of music with one of the most powerful machines in the world.

In most human relations it is rarely pure logical persuasion which is important; usually the paramount consideration is material interest. Now in this case it was not logic, much less self-interest, that arranged the betrothal, but purely experiment for experiment's sake, or game for game's sake, that induced collaboration. Stochastically speaking, my venture should have encountered failure. Yet the doors were opened, and at the end of a year and a half of contacts and hard work "the most unusual event witnessed by the firm or by this musical season [in Paris]" took place on 24 May 1962 at the headquarters of IBM-France. It was a live concert presenting a work of stochastic instrumental music entitled *ST/10-1, 080262*, which had been calculated on the IBM-7090. It was brilliantly performed by the conductor C. Simonovic and his Ensemble de Musique Contemporaine de Paris. By its passage through the machine, this work made tangible a stochastic method of composition, that of the minimum of constraints and rules.

Position of the Problem

The first working phase was the drawing up of the flow chart, i.e., writing down clearly and in order the stages of the operations of the scheme of *Achorripsis*,¹ and adapting it to the machine structure. In the first chapter we set out the entire synthetic method of this minimal structure. Since the machine is an iterative apparatus and performs these iterations with extraordinary speed, the thesis had to be broken down into a sequential series of operations reiterated in loops. An excerpt from the first flow chart is shown in Fig. V-1.

The statement of the thesis of *Achorripsis* receives its first machine-oriented interpretation in the following manner:

1. The work consists of a succession of sequences or movements each a_i seconds long. Their durations are totally independent (asymmetric) but have a fixed mean duration, which is introduced in the form of a parameter. These durations and their stochastic succession are given by the formula

$$P_{a_i} = ce^{-ca_i} da_i.$$

(See Appendix I.)

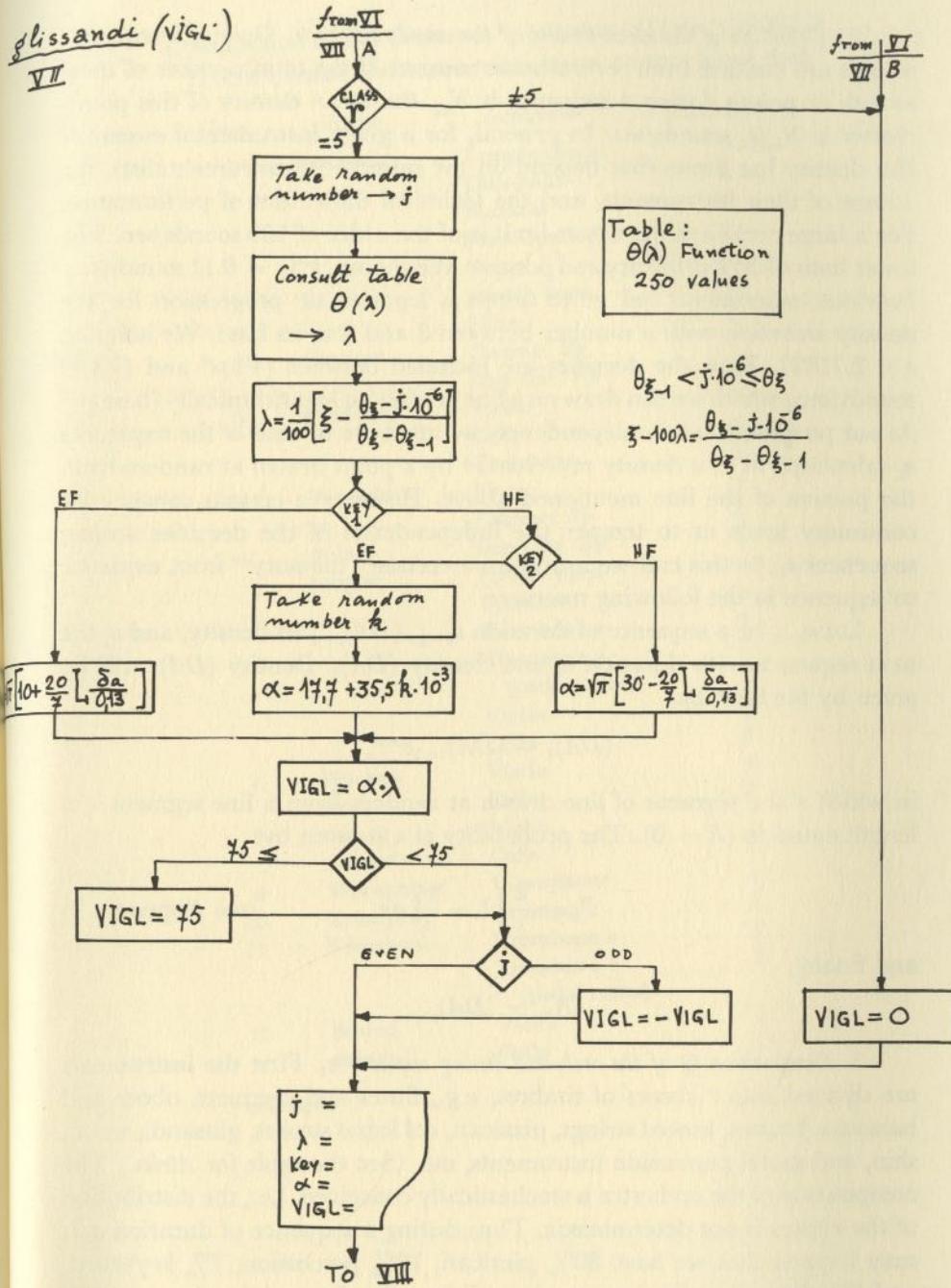


Fig. V-1. Excerpt from the First Flow Chart of *Achorripsis*

2. *Definition of the mean density of the sounds during a_i .* During a sequence sounds are emitted from several sonic sources. If the total number of these sounds or points during a sequence is N_{a_i} , the mean density of this point-cluster is N_{a_i}/a_i sounds/sec. In general, for a given instrumental ensemble this density has limits that depend on the number of instrumentalists, the nature of their instruments, and the technical difficulties of performance. For a large orchestra the upper limit is of the order of 150 sounds/sec. The lower limit ($V3$) is arbitrary and positive. We choose $(V3) = 0.11$ sounds/sec. Previous experiments led us to adopt a logarithmic progression for the density sensation with a number between 2 and 3 as its base. We adopted $e = 2.71827$. Thus the densities are included between $(V3)e^0$ and $(V3)e^R$ sounds/sec., which we can draw on a line graduated logarithmically (base e).² As our purpose is total independence, we attribute to each of the sequences a_i calculated in 1. a density represented by a point drawn at random from the portion of the line mentioned above. However a certain concern for continuity leads us to temper the independence of the densities among sequences a_i ; to this end we introduce a certain "memory" from sequence to sequence in the following manner:

Let a_{i-1} be a sequence of duration a_{i-1} , $(DA)_{i-1}$ its density, and a_i the next sequence with duration a_i and density $(DA)_i$. Density $(DA)_i$ will be given by the formula:

$$(DA)_i = (DA)_{i-1}e^{\pm x},$$

in which x is a segment of line drawn at random from a line segment s of length equal to $(R - 0)$. The probability of x is given by

$$P_x = \frac{2}{s} \left(1 - \frac{x}{s}\right) dx \quad (\text{see Appendix I})$$

and finally,

$$N_{a_i} = (DA)_{i a_i}.$$

3. *Composition Q of the orchestra during sequence a_i .* First the instruments are divided into r classes of timbres, e.g., flutes and clarinets, oboes and bassoons, brasses, bowed strings, pizzicati, col legno strokes, glissandi, wood, skin, and metal percussion instruments, etc. (See the table for *Atréés*.) The composition of the orchestra is stochastically conceived, i.e., the distribution of the classes is not deterministic. Thus during a sequence of duration a_i it may happen that we have 80% pizzicati, 10% percussion, 7% keyboard, and 3% flute class. Under actual conditions the determining factor which would condition the composition of the orchestra is density. We therefore

Composition of the Orchestra for *Atréés* (ST/10-3,060962)
Timbre classes and instruments as on present input data

Class	Timbre	Instrument	Instrument No.
1	Percussion	Temple-blocks Tom-toms Maracas Susp. cymbal Gong	1—5 6—9 10 11 12
2	Horn	French horn	1
3	Flute	Flute	1
4	Clarinet	Clarinet B \flat Bass clar. B \flat	1 2
5	Glissando	Violin Cello Trombone	1 2 3
6	Tremolo or flutter-tongue	Flute Clarinet B \flat Bass clar. B \flat French horn Trumpet Trombone a Trombone b (pedal notes) Violin Cello	1 2 3 4 5 6 7 8 9
7	Plucked strings	Violin Cello	1 2
8	Struck strings*	Violin Cello	1 2
9	Vibraphone	Vibraphone	1
10	Trumpet	Trumpet	1
11	Trombone	Trombone a Trombone b (pedal notes) Violin Cello	1 2

* col legno

connect the orchestral composition with density by means of a special diagram. An example from *ST/10-1, 080262* is shown in Fig. V-2.

Fig. V-2 is expressed by the formula

$$Q_r = (n - x)(e_{n,r} - e_{n+1,r}) + e_{n,r}$$

in which r = the number of the class, $x = \log_e[(DA)_i/(V3)]$, $n = 0, 1, 2, \dots, R$, such that $n \leq x \leq n + 1$, and $e_{n,r}$ and $e_{n+1,r}$ are the probabilities of class r as a function of n . It goes without saying that the composition of this table is a precise task of great complexity and delicacy. Once these preliminaries have been completed, we can define, one after the other, the N_{a_i} sounds of sequence a_i .

4. *Definition of the moment of occurrence of the sound N within the sequence a_i .* The mean density of the points or sounds to be distributed within a_i is $k = N_{a_i}/a_i$. The formula which gives the intervals separating the sound attacks is

$$P_t = ke^{-kt} dt. \quad (\text{See Appendix I.})$$

5. *Attribution to the above sound of an instrument belonging to orchestra Q , which has already been calculated.* First class r is drawn at random with probability q_r from the orchestra ensemble calculated in 3. (Consider an urn with balls of r colors in various proportions.) Then from within class r the number of the instrument is drawn according to the probability p_n given by an arbitrary table (urn with balls of n colors). Here also the distribution of instruments within a class is delicate and complex.

6. *Attribution of a pitch as a function of the instrument.* Taking as the zero point the lowest $B\flat$ of the piano, we establish a chromatic scale in semitones of about 85 degrees. The range s of each instrument is thus expressed by a natural number (distance). But the pitch h_u of a sound is expressed by a decimal number of which the whole number part is related to a note of the chromatic scale within the instrument's range.

Just as for the density in 2., we accept a certain memory of or dependence on the preceding pitch played by the same instrument, so that we have

$$h_u = h_{u-1} \pm z,$$

where z is given by the probability formula

$$P_z = \frac{2}{s} \left(1 - \frac{z}{s}\right) dz. \quad (\text{See Appendix I.})$$

P_z is the probability of the interval z taken at random from the range s , and

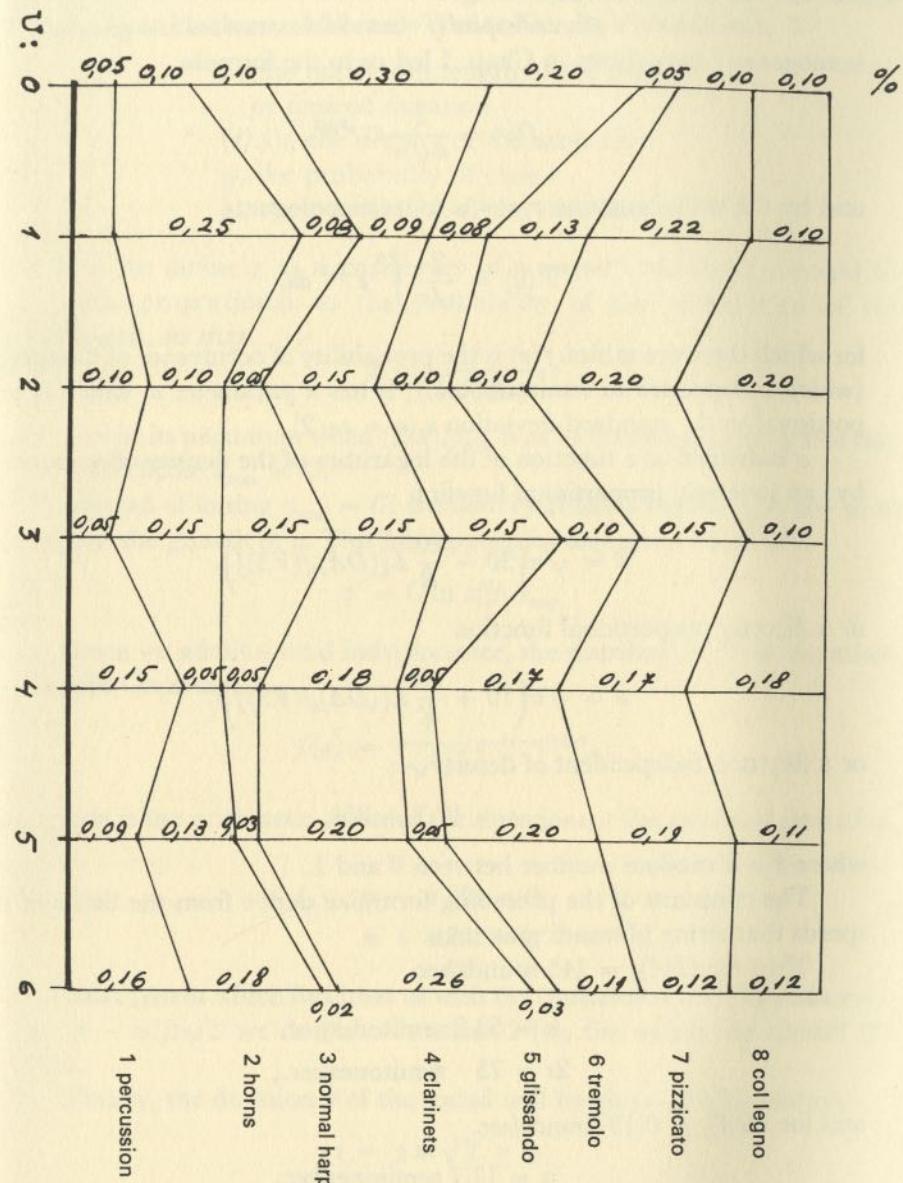


Fig. V-2. *ST/10-1, 080262, Composition of the Orchestra*

$$\text{Density} = (DA)_i = 0.11e^U, U = \log_e (DA/0.11)$$

s is expressed as the difference between the highest and lowest pitches that can be played on the instrument.

7. *Attribution of a glissando speed if class r is characterized as a glissando.* The homogeneity hypotheses in Chap. I led us to the formula .

$$f(v) = \frac{2}{a\sqrt{\pi}} e^{-v^2/a^2},$$

and by the transformation $v/a = u$ to its homologue:

$$T(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du,$$

for which there are tables. $f(v)$ is the probability of occurrence of the speed v (which is expressed in semitones/sec.); it has a parameter a , which is proportional to the standard deviation s ($a = s\sqrt{2}$).

a is defined as a function of the logarithm of the density of sequence a_i by: an inversely proportional function

$$a = \sqrt{\pi} \left(30 - \frac{20}{R} L[(DA)_i/(V3)] \right),$$

or a directly proportional function

$$a = \sqrt{\pi} \left(10 + \frac{20}{R} L[(DA)_i/(V3)] \right),$$

or a function independent of density

$$a = 17.7 + 35k,$$

where k is a random number between 0 and 1.

The constants of the preceding formulae derive from the limits of the speeds that string glissandi may take.

Thus for $(DA)_i = 145$ sounds/sec.

$$a = 53.2 \text{ semitones/sec.}$$

$$2s = 75 \text{ semitones/sec.,}$$

and for $(DA)_i = 0.13$ sounds/sec.

$$a = 17.7 \text{ semitones/sec.}$$

$$2s = 25 \text{ semitones/sec.}$$

8. *Attribution of a duration x to the sounds emitted.* To simplify we establish a mean duration for each instrument, which is independent of tessitura and

nuance. Consequently we reserve the right to modify it when transcribing into traditional notation. The following is the list of constraints that we take into account for the establishment of duration x :

G , the maximum length of respiration
or desired duration

$(DA)_i$, the density of the sequence

q_r , the probability of class r

p_n , the probability of the instrument n

Then if we define z as a parameter of a sound's duration, z could be inversely proportional to the probability of the occurrence of the instrument, so that

$$z = \frac{1}{(DA)_i p_n q_r}.$$

z will be at its maximum when $(DA)_i p_n q_r$ is at its minimum, and in this case we could choose $z_{\max} = G$.

Instead of letting $z_{\max} = G$, we shall establish a logarithmic law so as to freeze the growth of z . This law applies for any given value of z .

$$z' = G \ln z / \ln z_{\max}$$

Since we admit a total independence, the distribution of the durations x will be Gaussian:

$$f(x) = \frac{1}{s \sqrt{2\pi}} e^{-(x-m)^2/2s^2},$$

where m is the arithmetic mean of the durations, s the standard deviation, and

$$\begin{aligned} m - 4.25s &= 0 \\ m + 4.25s &= z' \end{aligned}$$

the linear system which furnishes us with the constants m and s . By assuming $u = (x - m)/s\sqrt{2}$ we find the function $T(u)$, for which we consult the tables.

Finally, the duration x of the sound will be given by the relation

$$x = \pm us\sqrt{2} + m.$$

We do not take into account incompatibilities between instruments, for this would needlessly burden the machine's program and calculation.

9. *Attribution of dynamic forms to the sounds emitted.* We define four zones of mean intensities: ppp, p, f, ff . Taken three at a time they yield $4^3 = 64$

permutations, of which 44 are different (an urn with 44 colors); for example, $\text{ppp} \leftarrow \text{f} \rightarrow \text{p}$.

10. The same operations are begun again for each sound of the cluster N_{a_i} .
11. Recalculations of the same sort are made for the other sequences.

An extract from the sequential statement was reproduced in Fig. V-1. Now we must proceed to the transcription into Fortran IV, a language "understood" by the machine (see Fig. V-3).

It is not our purpose to describe the transformation of the flow chart into Fortran. However, it would be interesting to show an example of the adaptation of a mathematical expression to machine methods.

Let us consider the elementary law of probability (density function)

$$f(x) dx = ce^{-cx} dx. \quad [20]$$

How shall we proceed in order for the computer to give us lengths x with the probability $f(x) dx$? The machine can only draw random numbers y_0 with equiprobability between 0 and 1. We shall "modulate" this probability: Assume some length x_0 ; then we have

$$\text{prob. } (0 \leq x \leq x_0) = \int_0^{x_0} f(x) dx = 1 - e^{-cx_0} = F(x_0),$$

where $F(x_0)$ is the distribution function of x . But

$$F(x_0) = \text{prob. } (0 \leq y \leq y_0) = y_0$$

then

$$1 - e^{-cx_0} = y_0$$

and

$$x_0 = -\frac{\ln(1 - y_0)}{c}$$

for all $x_0 \geq 0$.

Once the program is transcribed into language that the machine's internal organization can assimilate, a process that can take several months, we can proceed to punching the cards and setting up certain tests. Short sections are run on the machine to detect errors of logic and orthography and to determine the values of the entry parameters, which are introduced in the form of variables. This is a very important phase, for it permits us to explore all parts of the program and determine the modalities of its operation. The final phase is the decoding of the results into traditional notation, unless an automatic transcriber is available.

Table of the 44 Intensity Forms Derived from 4 Mean Intensity Values, ppp, p, f, ff

ppp	ppp	f	p
ppp	p	p	#
ppp	p	#	p
p	ppp	#	p
ppp	f	#	f
ppp	f	ppp	f
f	ppp	f	#
ppp	#	f	ppp
ppp	#	ppp	ff
f	#	ff	ppp
ppp	ff	ppp	ff
ppp	ff	ppp	f
ff	ppp	f	p
ppp	f	p	#
f	ppp	p	f
p	f	ppp	f
p	f	ppp	f
p	ppp	f	#
ppp	#	p	f
f	#	ppp	f
f	#	p	f
f	#	f	f
f	#	ppp	f
f	#	p	f
f	#	f	#
f	#	ff	f
f	#	f	#
f	#	ppp	ff
f	#	p	#
f	#	f	#

Conclusions

A large number of compositions of the same kind as *ST/10-1, 080262* is possible for a large number of orchestral combinations. Other works have already been written: *ST/48-1, 240162*, for large orchestra, commissioned by RTF (France III); *Attrées* for ten soloists; and *Morisma-Amorisma*, for four soloists.

Although this program gives a satisfactory solution to the minimal structure, it is, however, necessary to jump to the stage of pure composition by coupling a digital-to-analogue converter to the computer. The numerical calculations would then be changed into sound, whose internal organization had been conceived beforehand. At this point one could bring to fruition and generalize the concepts described in the preceding chapters.

The following are several of the advantages of using electronic computers in musical composition:

1. The long laborious calculation made by hand is reduced to nothing. The speed of a machine such as the IBM-7090 is tremendous—of the order of 500,000 elementary operations/sec.

2. Freed from tedious calculations the composer is able to devote himself to the general problems that the new musical form poses and to explore the nooks and crannies of this form while modifying the values of the input data. For example, he may test all instrumental combinations from soloists to chamber orchestras, to large orchestras. With the aid of electronic computers the composer becomes a sort of pilot: he presses the buttons, introduces coordinates, and supervises the controls of a cosmic vessel sailing in the space of sound, across sonic constellations and galaxies that he could formerly glimpse only as a distant dream. Now he can explore them at his ease, seated in an armchair.

3. The program, i.e., the list of sequential operations that constitute the new musical form, is an objective manifestation of this form. The program may consequently be dispatched to any point on the earth that possesses computers of the appropriate type, and may be exploited by any composer pilot.

4. Because of certain uncertainties introduced in the program, the composer-pilot can instill his own personality in the sonic result he obtains.

Fig. V-3. Stochastic Music Rewritten in Fortran IV

```

C   PROGRAM FREE STOCHASTIC MUSIC (FORTRAN IV)          XEN  6
C   GLOSSARY OF THE PRINCIPAL ABBREVIATIONS            XEN  7
C   XEN  8
C
C   A - DURATION OF EACH SEQUENCE IN SECONDS          XEN  9
C   A10,A20,A17,A35,A30 - NUMBERS FOR GLISSANDO CALCULATION XEN 10
C   ALEA - PARAMETER USED TO ALTER THE RESULT OF A SECOND RUN WITH THE XEN 11
C   SAME INPUT DATA                                     XEN 12
C   ALFA(3) - THREE EXPRESSIONS ENTERING INTO THE THREE SPEED VALUES XEN 13
C   OF THE SLIDING TONES ( GLISSANDI )                  XEN 14
C   ALIM - MAXIMUM LIMIT OF SEQUENCE DURATION A       XEN 15
C   (AMAX(I)+I=1,KTR) TABLE OF AN EXPRESSION ENTERING INTO THE XEN 16
C   CALCULATION OF THE NOTE LENGTH IN PART 8           XEN 17
C   BF - DYNAMIC FORM NUMBER. THE LIST IS ESTABLISHED INDEPENDENTLY XEN 18
C   OF THIS PROGRAM AND IS SUBJECT TO MODIFICATION     XEN 19
C   DELTA - THE RECIPROCAL OF THE MEAN DENSITY OF SOUND EVENTS DURING XEN 20
C   A SEQUENCE OF DURATION A                           XEN 21
C   (E(I,J),I=1,KTR,J=1,KTE) - PROBABILITIES OF THE KTR TIMBRE CLASSES XEN 22
C   INTRODUCED AS INPUT DATA, DEPENDING ON THE CLASS NUMBER I=KR AND XEN 23
C   ON THE POWER J=U OBTAINED FROM V3*EXP(F(U)=DA)          XEN 24
C   EPSI - EPSILON FOR ACCURACY IN CALCULATING PN AND E(I,J)*WHICH XEN 25
C   IT IS ADVISABLE TO RETAIN.                           XEN 26
C   (GN(I,J),I=1,KTR,J=1,KTS) - TABLE OF THE GIVEN LENGTH OF BREATH XEN 27
C   FOR EACH INSTRUMENT, DEPENDING ON CLASS I AND INSTRUMENT J XEN 28
C   GTNA - GREATEST NUMBER OF NOTES IN THE SEQUENCE OF DURATION A XEN 29
C   GTNS - GREATEST NUMBER OF NOTES IN KW LOOPS           XEN 30
C   (HAMIN(I,J),HAMAX(I,J),HBMIN(I,J),HBMAX(I,J),I=1,KTR,J=1,KTS) XEN 31
C   TABLE OF INSTRUMENT COMPASS LIMITS, DEPENDING ON TIMBRE CLASS I XEN 32
C   AND INSTRUMENT J. TEST INSTRUCTION 480 IN PART 6 DETERMINES XEN 33
C   WHETHER THE HA OR THE HB TABLE IS FOLLOWED. THE NUMBER 7 IS XEN 34
C   ARBITRARY.                                         XEN 35
C   JW - ORDINAL NUMBER OF THE SEQUENCE COMPUTED.        XEN 36
C   KNL - NUMBER OF LINES PER PAGE OF THE PRINTED RESULT. XEN 37
C   KR1 - NUMBER IN THE CLASS KR=1 USED FOR PERCUSSION OR INSTRUMENTS XEN 38
C   WITHOUT A DEFINITE PITCH.                           XEN 39
C   KTE - POWER OF THE EXPONENTIAL COEFFICIENT E SUCH THAT XEN 40
C   DA(MAX)=V3*(E**(KTE-1))                           XEN 41
C   KTR - NUMBER OF TIMBRE CLASSES                     XEN 42
C   KW - MAXIMUM NUMBER OF JW                         XEN 43
C   KTEST1,TAV1,ETC - EXPRESSIONS USEFUL IN CALCULATING HOW LONG THE XEN 44
C   VARIOUS PARTS OF THE PROGRAM WILL RUN.             XEN 45
C   KT1 - ZERO IF THE PROGRAM IS BEING RUN. NONZERO DURING DEBUGGING XEN 46
C   KT2 - NUMBER OF LOOPS, EQUAL TO 15 BY ARBITRARY DEFINITION. XEN 47
C   (MOD(I*KB)+KB=7+1) AUXILIARY FUNCTION TO INTERPOLATE VALUES IN XEN 48
C   THE TETA(256) TABLE (SEE PART 7)                  XEN 49
C   NA - NUMBER OF SOUNDS CALCULATED FOR THE SEQUENCE A(NA=DA*A) XEN 50
C   (NT(I)+I=1,KTR) NUMBER OF INSTRUMENTS ALLOCATED TO EACH OF THE XEN 51
C   KTR TIMBRE CLASSES.                                XEN 52
C   (PN(I,J),I=1,KTR,J=1,KTS),(KTS=NT(I),I=1,KTR) TABLE OF PROBABILITY XEN 53
C   OF EACH INSTRUMENT OF THE CLASS I.                 XEN 54
C   (Q(I),I=1,KTR) PROBABILITIES OF THE KTR TIMBRE CLASSES, CONSIDERED XEN 55
C   AS LINEAR FUNCTIONS OF THE DENSITY DA.            XEN 56
C   (S(I),I=1,KTR) SUM OF THE SUCCESSIVE Q(I) PROBABILITIES, USED TO XEN 57
C   CHOOSE THE CLASS KR BY COMPARING IT TO A RANDOM NUMBER XI (SEE XEN 58
C   PART 3, LOOP 380 AND PART 5, LOOP 430).           XEN 59
C   SINA - SUM OF THE COMPUTED NOTES IN THE JW CLOUDS NA, ALWAYS LESS XEN 60
C   THAN GTNS ( SEE TEST IN PART 10 ).                XEN 61
C   SQPI - SQUARE ROOT OF PI ( 3.14159... ).          XEN 62
C   TA - SOUND ATTACK TIME ABCISSA.                   XEN 63
C   TETA(256) - TABLE OF THE 256 VALUES OF THE INTEGRAL OF THE NORMAL XEN 64
C   DISTRIBUTION CURVE WHICH IS USEFUL IN CALCULATING GLISSANDO SPEED XEN 65

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C AND SOUND EVENT DURATION. XEN 66
C VIGL - GLISSANDO SPEED (VITESSE GLISSANDO), WHICH CAN VARY AS BE XEN 67
C INDEPENDENT OF, OR VARY INVERSELY AS THE DENSITY OF THE SEQUENCE. XEN 68
C THE ACTUAL MODE OF VARIATION EMPLOYED REMAINING THE SAME FOR THE XEN 69
C ENTIRE SEQUENCE (SEE PART 7). XEN 70
C VITLIM - MAXIMUM LIMITING GLISSANDO SPEED (IN SEMITONES/SEC). XEN 71
C SUBJECT TO MODIFICATION. XEN 72
C V3 - MINIMUM CLOUD DENSITY DA XEN 73
C (Z1(I)+Z2(I), I=1,8) TABLE COMPLEMENTARY TO THE TETA TABLE. XEN 74
C XEN 75
C XEN 76
C READ CONSTANTS AND TABLES XEN 78
C XEN 79
C XEN 80
C XEN 81
C XEN 82
C XEN 83
C XEN 84
C XEN 85
C XEN 86
C XEN 87
C XEN 88
C XEN 89
C XEN 90
C XEN 91
C XEN 92
C XEN 93
C XEN 94
C XEN 95
C XEN 96
C XEN 97
C XEN 98
C XEN 99
C XEN 100
C XEN 114
C XEN 115
C XEN 126
C XEN 127
C XEN 128
C XEN 129
C XEN 130
C XEN 131
C XEN 132
C XEN 133
C XEN 134
C XEN 135
C XEN 136
C XEN 141
C XEN 142
C XEN 143
C XEN 144
C XEN 145
C XEN 146
C XEN 147
C XEN 148
C XEN 149
C XEN 150
C XEN 151
C XEN 152
C XEN 153

C
I=1
DO 10 IX=1,7
IXB=8-IX
MODI(IXB)=T
10 I=I+1

C
READ 20*(TETA(I)+I=1+256)
20 FORMAT(12F6.6)
READ 30*(Z1(I)+Z2(I),I=1,8)
30 FORMAT(6(F3.2,F9.8)/F3.2+F9.8,E6.2+F9.8)
PRINT 40*TETA*Z1*Z2
40 FORMAT(*1 THE TETA TABLE = *.*21(12F10.6,/*)4F10.6,/*/*,
** THE Z1 TABLE = *.*7F6.2+E12.3,/* THE Z2 TABLE = *.*8F14.8/*XEN 97
*+1H1)
READ 50*DELTA+V3+A10+A20+A17+A30+A35+BF+SQPI+EPSI+VITLIM+ALEA+
*LIM
50 FORMAT(F3.0,F3.3,5F3.1+F2.0*F8.7+F8.8+F4.2*F8.8+F5.2)
READ 60*KT1+KT2+KW+KNL+KTR+KTE+KRI+GTNA+GTNS+(NT(I)+I=1,KTR)
60 FORMAT(513+212+2F6.0+1212)
PRINT 70*DELTA+V3+A10,A20,A17+A30+A35+BF+SQPI+EPSI+VITLIM+ALEA+
*LIM+KT1+KT2+KW+KNL+KTR+KTE+KRI+GTNA+GTNS+((NT(I))+I=1,KTR)
70 FORMAT(*1DELTA = *.*F4.0/*,* V3 = *.*F6.3/*,* A10 = *.*F4.1/*,
** A20 = *.*F4.1/*,* A17 = *.*F4.1/*,* A30 = *.*F4.1/*,* A35 = *.*F4.1/*XEN 130
*/*,* BF = *.*F3.0/*,* SQPI = *.*F11.8/*,* EPSI = *.*F12.8/*,* VITLIM = *XEN 131
*.*F5.2/*,* ALEA = *.*F12.8/*,* ALIM = *.*F6.2/*,* KT1 = *.*I3/*,
** KT2 = *.*I3/*,* KW = *.*I3/*,* KNL = *.*I3/*,* KTR = *.*I3/*,
** KTE = *.*I2/*,* KRI = *.*I2/*,* GTNA = *.*F7.0/*,* GTNS = *.*F7.0/*,
*/*12/* IN CLASS *,12/*, THERE ARE *.*12/* INSTRUMENTS.*/*)
READ 80*KTEST3+KTEST1+KTEST2
80 FORMAT(513)
PRINT 90+KTEST3+KTEST1+KTEST2
90 FORMAT(* KTEST3 = *.*I3/*,* KTEST1 = *.*I3/*,* KTEST2 = *.*I3)
C
IF(KTEST3.NE.0) PRINT 830
R=KTE-1
A10=A10*SQPI
A20=A20*SQPI/R
A30=A30*SQPI
C
IF ALEA IS NON-ZERO, THE RANDOM NUMBER IS GENERATED FROM THE TIME XEN 150
C WHEN THE FOLLOWING INSTRUCTION IS EXECUTED. IF ALEA IS NON-ZERO XEN 151
C EACH RUN OF THIS PROGRAM WILL PRODUCE DIFFERENT OUTPUT DATA. XEN 152
IF(ALEA.NE.0.0) CALL RANFSET(TIMEF(1)) XEN 153

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PRINT 830 XEN 154
DO 130 I=1,KTR XEN 155
Y=0.0 XEN 156
KTS=NT(I) XEN 157
READ 100*(HMIN(I+J),HMAX(I+J),HMIN(I+J),HMAX(I+J),GN(I+J),
*PN(I+J),J=1,KTS) XEN 158
100 FORMAT(5(F2.0,F3.3)) XEN 160
PRINT 110*I*(J,HMIN(I+J),HMAX(I+J),HMIN(I+J),HMAX(I+J),GN(I+J),
*,PN(I+J),J=1,KTS) XEN 161
110 FORMAT(//,* IN CLASS NUMBER *.*I2/*/* FOR INSTRUMENT NO. *.*I2* XEN 162
** HMIN = *.*F3.0/*,*HMAX = *.*F3.0/*,*HMIN = *.*F3.0/*,*HMAX = *.* XEN 163
* F3.0/*,*GN = *.*F3.0/*,* AND PN = *.*F6.3) XEN 164
DO 120 J=1,KTS XEN 165
Y=Y+PN(I+J) XEN 166
120 SPN(I+J)=Y XEN 167
130 IF (ABSF(Y-1.0).GE.EPSI) CALL EXIT XEN 168
XEN 169
XEN 170
XEN 171
C
DO 150 I=1,KTR XEN 172
READ 140*(E(I+J),J=1,KTE) XEN 173
140 FORMAT(12F2.2)
150 PRINT 160*I*(J,E(I+J),J=1,KTE) XEN 174
160 FORMAT(//,* CLASS NUMBER *.*I2/*/* IN DENSITY LEVEL *.*I2* XEN 175
** HAS A PROBABILITY OF *.*F6.2/*) XEN 176
DO 180 J=1,KTE XEN 177
Y=0.0 XEN 178
DO 170 I=1,KTR XEN 179
170 Y=Y+E(I+J) XEN 180
180 IF (ABSF(Y-1.0).GE.EPSI) CALL EXIT XEN 181
DO 200 I=1,KTR XEN 182
AMAX(I)=1.0/E(I+1) XEN 183
DO 200 J=2,KTE XEN 184
AJ=J-1 XEN 185
AX=1.0/(E(I+J)*EXPF(AJ)) XEN 186
IF (KT1.NE.0) PRINT 190+AX XEN 187
190 FORMAT(1H *9E12.8)
200 IF (AX.GT.AMAX(I)) AMAX(I)=AX XEN 188
IF (KT1.NE.0) PRINT 210+AMAX XEN 189
210 FORMAT(1H *9E12.8)
C
JW=1 XEN 190
SINA=0.0 XEN 191
IF (KTEST1.NE.0) TAVI=TIMEF(1) XEN 192
220 NLINE=50 XEN 193
C
C PARTS 1 AND 2: DEFINE SEQUENCE A SECONDS AND CLOUD NA DURING A XEN 194
C KNA=0 XEN 195
K1=0 XEN 196
230 XI=RANF(-1) XEN 197
A=-DELTA * LOGF(XI) XEN 198
IF(A.LE.ALIM) GO TO 250 XEN 199
IF (K1.GE.KT2) GO TO 240 XEN 200
K1=K1+1 XEN 201
GO TO 230 XEN 202
240 A=ALIM/2.0 XEN 203
X1=0.0 XEN 204
250 K2=0 XEN 205
260 X2=RANF(-1) XEN 206
IF (JW.GT.1) GO TO 280 XEN 207
270 UX=R*X2 XEN 208
GO TO 310 XEN 209
XEN 210
XEN 211
XEN 212
XEN 213
XEN 214

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280 IF (RANF(-1)*GE+0.5) GO TO 290
  UX=UPR + R * (1.0-SQRTF(X2))
  GO TO 300
290 UPR=UPR - R * ( 1.0-SQRTF(X2))
300 IF ((UX*GE+0.0)*AND.(UX,LE+R)) GO TO 310
  IF (K2*GE+KT2) GO TO 270
  K2=K2+1
  GO TO 260
310 U=UX
  DA=V3 * EXPF(U)
  NA=XINTF(A * DA + 0.5) + 1
  IF (GTNA.GT.FLOATF(NA)) GO TO 330
  IF (KNA.GE.KT2) GO TO 320
  KNA=KNA+1
  GO TO 230
320 A=DEFLTA
  GO TO 260
330 UPR=U
  IF (KT1.EQ.0) GO TO 360
  PRINT 340,JW+KNA,K1+K2+X1+X2+A,DA,NA
340 FORMAT(1H1+4I8,3X+4E18.8+3X,18)
  NA=KT1
  IF (KTEST3.NE.0) PRINT 350,JW+NA,A
350 FORMAT(1H0+2I9,F10.2)
C   PART 3. DEFINE CONSTITUTION OF ORCHESTRA DURING SEQUENCE A
360 SINA=SINA + FLOATF(NA)
  XLOGDA=U
  XALOG=A20 *XLOGDA
  M=XINTF(XLOGDA)
  IF ((M+2).GT.KTE) M=KTE-2
  SR=0.0
  M1=M+1
  M2=M+2
  DO 380 I=1,KTR
    ALFX=E(I+M1)
    BETA=E(I+M2)
    XM=M
    QR=(XLOGDA-XM) * (BETA-ALFX) + ALFX
    IF (KT1.NE.0) PRINT 370,XM,ALFX,BETA
370 FORMAT(1H +3F20.8)
  Q(I)=QR
  SR=SR+QR
380 S(I)=SR
  IF (KT1.NE.0) PRINT 390+(Q(I)+I=1+KTR)+(S(I)+I=1+KTR)
390 FORMAT(1H +12F9.4)
C   PART 4. DEFINE INSTANT TA OF EACH POINT IN SEQUENCE A
400 IF (KTEST2.NE.0) TAV2=TIMEF(1)
  N=1
  T=0.0
  TA=0.0
  GO TO 410
400 N=N+1
  X=RANF(-1)
  T=-LOGF(X)/DA
  TA=TA+T
410 IF (KT1.NE.0) PRINT 420,N+X+T+TA
420 FORMAT(1H +18+3E20.8)

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XEN 215
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XEN 275

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C   PART 5. DEFINE CLASS AND INSTRUMENT NUMBER TO EACH POINT OF A
C   X1=RANF(-1)
  DO 430 I=1+KTR
  430 IF (X1.LE.S(I)) GO TO 440
  I=KTR
  440 KTS=NT(I)
  KR=I
  X2=RANF(-1)
  DO 450 J=1+KTS
  SPIEN=SPN(KR,J)
  INSTRM=J
  450 IF (X2.LE.SPIEN) GO TO 460
  INSTRM=KTS
  460 PIEN=PN(KR+INSTRM)
  IF (KT1.NE.0) PRINT 470+X1+S(KR)+KR,X2+SPIEN+INSTRM
  470 FORMAT(1H +2E20.8+16+2E20.8+16 )
C   PART 6. DEFINE PITCH HN FOR EACH POINT OF SEQUENCE A
C   IF (KR.GT.1) GO TO 480
  IF (INSTRM.GE.KR) GO TO 490
  HX=0.0
  GO TO 560
480 IF (KR+LT.7) GO TO 490
  HSUP=HBMAX(KR+INSTRM)
  HINF=HMIN(KR+INSTRM)
  GO TO 500
490 HSUP=HAMAX(KR+INSTRM)
  HINF=HAMIN(KR+INSTRM)
500 HM=HSUP-HINF
  HPR=H(KR+INSTRM)
  K=0
  IF (HPR.LE.0.0) GO TO 520
510 X=RANF(-1)
  IF (N.GT.1) GO TO 530
520 HX=HINF+HM*X RANF(-1)
  GO TO 560
530 IF (RANF(-1)*GE+0.5) GO TO 540
  HX=HPR+HM * ( 1.0-SQRTF(X))
540 HX=HPR-HM * ( 1.0-SQRTF(X))
550 IF ((HX.GE.HINF)*AND.(HX.LE.HSUP)) GO TO 560
  IF (K.GE.KT2) GO TO 520
  K=K+1
  GO TO 510
560 H(KR+INSTRM)=HX
  IF (KT1.NE.0) PRINT 570+K+X+HX
  570 FORMAT(1H +16+2E20.8)
C   PART 7. DEFINE SPEED VIGL TO EACH POINT OF A
C   IF (KR.EQ.5) GO TO 580
  VIGL(1)=0.0
  VIGL(2)=0.0
  VIGL(3)=0.0
  X1=0.0
  X2=0.0
  XLAMBDAA=0.0
  GO TO 740
XEN 276
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XEN 335
XEN 336

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580 1X=1
590 X1=RANF(-1)
  IF (X1=0.9997) 600+650+680
600 I=128
  DO 630 IX=1+7
  IF(TETA(I)-X1) 610+640+620
610 I=I+MODI(IX)
  GO TO 630
620 I=I-MODI(IX)
630 CONTINUE
  IF(TETA(I)-X1) 670+640+660
640 XLAMBDA=FLOATF(I-1)/100.0
  GO TO (720+760)+KX
650 XLAMBDA=2.55
  GO TO (720+760)+KX
660 I=I-1
670 TX1=TETA(I)
  XLAMBDA=(FLOATF(I-1)+(X1-TX1)/(TETA(I+1)-TX1))/100.0
  GO TO (720+760)+KX
680 DO 690 I=2+7
  TX1=Z2(I)
  IF(X1-TX1) 700+710+690
690 CONTINUE
  I=8
  TX1=1.0
700 TX2=Z1(I)
  XLAMBDA=TX2-((TX1-X1)/(TX1-Z2(I-1)))*(TX2-Z1(I-1))
  GO TO (720+760)+KX
710 XLAMBDA=Z1(I)
  GO TO (720+760)+KX
720 ALFA(1)=A10+XALOG
  ALFA(3)=A30+XALOG
  X2=RANF(-1)
  ALFA(2)=A17+A35*X2
  DO 730 I=1+3
  VIGL(I)=INTF(ALFA(I)*XLAMBDA+0.5)
  IF (VIGL(I)<LT.0.0) VIGL(I)=-VIGL(I)
  IF (VIGL(I)>GT.VITLIM) VIGL(I)=VITLIM
730 IF (RANF(-1).LT.0.5) VIGL(I)=-VIGL(I)
740 IF(KT1.NE.0) PRINT 750+X1+X2+XLAMBDA+VIGL
750 FORMAT(1H +E19.8)
C
C  PART 8.DEFINE DURATION FOR EACH POINT OF A
C
  IF ((KR.EQ.7).OR.(KR.EQ.8)) GO TO 780
  ZMAX=AMAX(KR)/(V3*PIEN)
  G=GN(KR+INSTRM)
  R0=G/LOGF(ZMAX)
  QPNDA=1.0/(Q(KR)*PIEN*DA)
  GE=ABSF(R0*LOGF(QPNDA))
  XMU=GE/2.0
  SIGMA=GE*4.0
  KX=?
  GO TO 590
760 TAU=SIGMA*XLAMBDA*1.4142
  X2=RANF(-1)
  IF (X2.GE.0.5) GO TO 770
  XDUR=XMU+TAU
  GO TO 790
770 XDUR=XMU-TAU
  IF (XDUR.GE.0.0) GO TO 790

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780 XDUR=0.0
790 IF(KT1.NE.0)PRINT 800+ZMAX+XMU+SIGMA+X1+XLAMBDA+X2+XDUR
  800 FORMAT(1H +E15.8+E11.4+E15.8)
C
C  PART 9.DEFINE INTENSITY FORM TO EACH POINT OF A
C
  IFORM=XINTF(RANF(-1)*BF+0.5)
  IF (KT1.EQ.0) GO TO 840
  IF (NLIN.EQ.LT.KNL) GO TO 810
  IF (NLIN.EQ.KNL) GO TO 820
  NLIN=1
  GO TO 900
810 NLIN=NLIN+1
  GO TO 900
820 PRINT 830
830 FORMAT(1H1)
  NLIN=0
  GO TO 900
840 IF (NLIN.GE.KNL) GO TO 850
  NLIN=NLIN+1
  GO TO 880
850 PRINT 860,JW+A+NA+(Q(I)+I=1,KTR)
860 FORMAT(*1 JW=+I3+4X+*A=+FB+2+4X+*NA=+I6+4X+*Q(I)=+12(F4+2+*/*XEN
  *+//)
  PRINT 870
870 FORMAT(6X,*N*,8X,*START*+5X,*CLASS*+4X,*INSTRM*+4X,*PITCH*+6X+*GLISS1*+4X+*GLISS2*+4X+*GLISS3*+8X+*DURATION*+5X+*DYNAM*)
  NLIN=1
880 PRINT 890,N,TA,KR+INSTRM,HX+(VIGL(I)+I=1+3)+XDUR,IFORM
890 FORMAT(1H +I7,F12.2+I9+I8+F11+I1+F13+I1+2F10+I1+F14+2+I11)
C
C  PART 10.REPEAT SAME DEFINITIONS FOR ALL POINTS OF A
C
  900 IF (N.LT.NA) GO TO 400
C
C  PART 11.REPEAT SEQUENCES A
C
  IF (KTEST2.EQ.0) GO TO 910
  TAP2=TIMEF(1)-TAV2
  TAP2=TAP2/FLOATF(NA)
  PRINT 750+TAP2
C
  910 IF (JW.GE.KW) GO TO 930
  920 JW=JW+1
  IF (GTNS.GT.SINA) GO TO 220
  930 IF (KTEST1.EQ.0) CALL FXIT
  940 TAP1=TIMEF(-1)-TAV1
  TAP1=TAP1/FLOATF(KW)
  PRINT 750+TAP1
C
  END
C
C  DATA FOR ATREES (ST/10-3, 060962)
C
0000001130002260033900045100056400067600078900090100101300112500123600
13480014590015690016800017900019000020090021180022700233500244300255000
265700276300286900297400307900318300328600338900349100359300369400379400
389300399200409000418700428400438000447500456900466200475500484700493700
502700511700520500529200537900546500554900563300571600579800587900595900
60390061170061940062700634600642000649400656600663800670800677800684700
69140069810070470071120071750072380073000736100742100748000753800759500

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765100770700776100781400786700791800796900801900806800811600816300820900
 825400829900834200838500842700846800850800854800858600862400866100869800
 87330087680088020088350088680089000893100896100899100902000904800907600
 9103009130009155009181009205009229009252009275009297009310090934000936100
 938100940000941900943800945700947300949000950700952300953800955400956900
 958300959700961100962400963700964900966100967300968400969500970600971600
 97260097360097450097550097630097720097800978800979600980400981100981800
 98250098320098380098440098500985600986100986700987200987700988200988600
 989100989500989900990300990700991100991500991800992200992500992800993100
 993400993700993900994200994400994700994900995100995300995500995700995900
 996100996300996400996600996700996900997000997200997300997400997500997600
 997700997900997960998050998140998230998320998400998480998550998620998680
 998740998800998850998910998960999010999060999100999140999180999230999270
 999300999340999370999400999440999470999500999530999560999580999600999630
 999650999670999690999700
 25509997000026309998000027509999000031309999900034609999900377099999990
 406099999999100E30100000000
 0400501002001773003556317724539010000007100000000012000
 0000150500500120720001600025000120101020309020201010202

01010000100700101000010090010100010120010100010110010100010090
 010100001012001010000100800101000010080101000010120010100010080
 01010000150200101000020020
 1755000010999
 3975000015999
 29710000206001754000010400
 348500001540015630000154001953000010200
 3975000015150297100001009017540000709017550000100903363000010090
 195300001007010130001020034850000152001563000015020
 00003467005000000154800500
 00003467005000000154800500
 0000326810999
 0000336310999
 00001953108000000101307200
 00003487155000000157215500
 25080408011309
 08071602010110
 03030420010110
 02050325010112
 03350315011505
 02100302103907
 02020203150207
 02020202410207
 03090317041609
 03132003200509
 02052801030409
 45011202020106

JW= 1 A= 9.13 NA= 55
 Q(I)=0.12/0.04/0.04/0.05/0.12/0.29/0.04/0.04/0.14/0.06/0.06/0.03/

N	START	CLASS	INSTRM	PITCH	GLISS1	GLISS2	GLISS3	DURATION	DYNAM
1	0.00	7	1	34.0	0.0	0.0	0.0	0.00	3
2	0.10	10	1	43.2	0.0	0.0	0.0	0.41	50
3	0.11	6	8	81.3	0.0	0.0	0.0	0.63	21
4	0.13	6	3	47.0	0.0	0.0	0.0	0.18	10
5	0.18	1	4	0.0	0.0	0.0	0.0	1.90	29
6	0.25	9	1	48.7	0.0	0.0	0.0	0.51	35
7	0.33	6	7	11.4	0.0	0.0	0.0	0.37	42
8	0.34	9	1	38.1	0.0	0.0	0.0	0.00	59
9	0.40	1	1	0.0	0.0	0.0	0.0	2.20	45
10	0.41	6	9	55.0	0.0	0.0	0.0	1.07	0
11	0.76	6	7	11.5	0.0	0.0	0.0	0.40	7
12	0.90	8	2	23.2	0.0	0.0	0.0	0.00	19
13	1.00	7	2	26.9	0.0	0.0	0.0	0.00	6
14	1.09	10	1	46.2	0.0	0.0	0.0	0.32	57
15	1.09	6	2	68.5	0.0	0.0	0.0	0.71	25
16	1.23	6	3	46.9	0.0	0.0	0.0	0.64	32
17	1.42	6	1	44.0	0.0	0.0	0.0	0.44	1
18	1.57	10	1	36.2	0.0	0.0	0.0	0.22	21
19	1.65	4	2	32.5	0.0	0.0	0.0	1.09	13
20	1.78	6	8	72.6	0.0	0.0	0.0	0.06	60
21	1.92	6	3	38.9	0.0	0.0	0.0	0.55	60
22	1.94	5	1	74.6	71.0	-25.0	-71.0	0.80	62
23	2.18	4	1	32.6	0.0	0.0	0.0	1.50	50
24	2.18	6	6	50.9	0.0	0.0	0.0	0.60	26
25	2.19	1	12	0.0	0.0	0.0	0.0	4.58	24
26	2.20	9	1	49.3	0.0	0.0	0.0	0.02	58
27	2.23	9	1	51.0	0.0	0.0	0.0	0.22	13
28	2.32	7	1	36.9	0.0	0.0	0.0	0.00	43
29	2.33	4	1	31.8	0.0	0.0	0.0	1.38	56
30	2.54	1	6	0.0	0.0	0.0	0.0	0.28	14
31	2.57	11	2	12.2	0.0	0.0	0.0	1.69	40
32	2.71	5	1	48.5	0.0	0.0	0.0	0.37	55
33	2.80	1	5	0.0	0.0	0.0	0.0	1.50	58
34	3.28	5	2	15.4	49.0	5.0	-31.0	0.52	21
35	3.33	1	7	0.0	0.0	0.0	0.0	1.38	8
36	3.38	5	2	47.3	-71.0	-17.0	46.0	1.05	4
37	3.55	10	1	37.6	0.0	0.0	0.0	0.14	24
38	3.56	1	9	0.0	0.0	0.0	0.0	1.30	0
39	3.60	5	1	64.3	0.0	0.0	0.0	0.19	13
40	3.64	12	2	52.2	0.0	0.0	0.0	3.72	9
41	3.65	6	5	59.0	0.0	0.0	0.0	0.83	28
42	3.71	5	3	38.8	25.0	2.0	-15.0	0.00	11
43	3.80	6	8	75.6	0.0	0.0	0.0	0.43	17
44	3.87	6	2	51.5	0.0	0.0	0.0	0.77	57
45	3.89	6	7	12.1	0.0	0.0	0.0	0.39	2
46	4.15	5	2	43.0	-71.0	24.0	71.0	1.16	2
47	4.15	5	1	80.3	36.0	4.0	22.0	0.85	50
48	4.25	9	1	59.9	0.0	0.0	0.0	0.10	10
49	4.31	12	2	40.1	0.0	0.0	0.0	2.49	33
50	4.33	1	10	0.0	0.0	0.0	0.0	0.46	34

Fig. V-4. Provisional Results of One Phase of the Analysis



Fig. V-5. Bars 1-5 of ST/10-1, 080262

Chapter VI

Symbolic Music

Here we shall attack the thorny problem of the logic underlying musical composition. Logic, that queen of knowledge, monopolized by mathematics, wavers between her own name, borne through two millennia, and the name of algebra.

Let us leave the task of logically connecting the preceding chapters for the moment. We shall confine ourselves to following a path which may lead us to regions even more harmonious in the not too distant future.

A LOGICAL AND ALGEBRAIC SKETCH OF MUSICAL COMPOSITION

In this chapter we shall begin by imagining that we are suffering from a sudden amnesia. We shall thus be able to reascend to the fountain-head of the mental operations used in composition and attempt to extricate the general principles that are valid for all sorts of music. We shall not make a psycho-physiological study of perception, but shall simply try to understand more clearly the phenomenon of hearing and the thought-processes involved when listening to music. In this way we hope to forge a tool for the better comprehension of the works of the past and for the construction of new music. We shall therefore be obliged to collect, cut up, and solder scattered as well as organized entities and conceptions, while unraveling the thin thread of a logic, which will certainly present lacunae, but which will at least have the merit of existing.

CASE OF A SINGLE GENERIC ELEMENT

Let there be a sonic event which is not endless. It is seen as a whole, as

an entity, and this overall perception is sufficient for the moment. Because of our amnesia, we decide that it is neuter—neither pleasant nor unpleasant.

Postulate. We shall systematically refuse a qualitative judgment on every sonic event. What will count will be the abstract relations within the event or between several events, and the logical operations which may be imposed on them. The emission of the sonic event is thus a kind of statement, inscription, or sonic symbol, which may be notated graphically by the letter a .

If it is emitted once it means nothing more than a single existence which appears and then disappears; we simply have a .

If it is emitted several times in succession, the events are compared and we conclude that they are *identical*, and no more. Identity and tautology are therefore implied by a repetition. But simultaneously another phenomenon, subjacent to the first, is created by reason of this very repetition: *modulation of time*. If the event were a Morse sound, the temporal abscissa would take a meaning external to the sound and independent of it. In addition to the deduction of tautology, then, repetition causes the appearance of a new phenomenon, which is inscribed in time and which modulates time.

To summarize: If no account is taken of the temporal element, then a single sonic event signifies only its statement. The sign, the symbol, the generic element a have been stated. A sonic event actually or mentally repeated signifies only an identity, a tautology:

$$a \vee a \vee a \vee a \dots \vee a = a.$$

\vee is an operator that means “put side by side without regard to time.” The $=$ sign means that it is the same thing. This is all that can be done with a single sonic event.

CASE OF TWO OR MORE GENERIC ELEMENTS

Let there be two sonic events a and b such that a is not identical with b , and such that the two are distinct and easily recognizable, like the letters a and b , for example, which are only confused by a near-sighted person or when they are poorly written.

If no account is taken of the temporal element, then the two elements are considered as a pair. Consequently emitting first a then b , or first b then a , gives us no more information about these distinct events than when they are heard in isolation after long intervals of silence. And since no account is taken of the relation of similitude or of the time factor, we can

write for $a \neq b$

$$a \vee b = b \vee a,$$

which means that a and b side by side do not create a new thing, having the same meaning as before. Therefore a commutative law exists.

In the case of three distinguishable events, a, b, c , a combination of two of these sonic symbols may be considered as forming another element, an entity in relation to the third:

$$(a \vee b) \vee c.$$

But since this associational operation produces nothing more we may write

$$(a \vee b) \vee c = a \vee (b \vee c).$$

This is an associative law.

The exclusion of the time factor leads therefore to two rules of composition outside-time—the commutative and the associative. (These two rules are extensible to the case of a single event.)

On the other hand, when the manifestations of the generic events a, b, c are considered in time, then commutativity may no longer be accepted. Thus

$$a \top b \neq b \top a,$$

\top being the symbol of the law of composition which means “anterior to.”

This asymmetry is the result of our traditional experience, of our customary one-to-one correspondence between events and time instants. It is raised when we consider time by itself without events, and the consequent metric time which admits *both* the commutative and the associative properties:

$$\begin{array}{ll} a \top b = b \top a & \text{commutative law} \\ (a \top b) \top c = a \top (b \top c) & \text{associative law.} \end{array}$$

CONCEPT OF DISTANCE (INTERVAL)

The consideration of generic elements a, b, c, \dots as entities does not permit much of an advance. To exploit and clarify what has just been said, we must penetrate the internal organization of the sonic symbols.

Every sonic event is perceived as a set of qualities that is modified during its life. On a primary level we perceive pitch, duration, timbre, attack, rugosity, etc. On another level we may distinguish complexities, degrees of order, variabilities, densities, homogeneities, fluctuations, thicknesses, etc. Our study will not attempt to elucidate these questions, which are not only

difficult but at this moment secondary. They are also secondary because many of the qualities may be graduated, even if only broadly, and may be totally ordered. We shall therefore choose one quality and what will be said about it will be extensible to others.

Let us, then, consider a series of events discernible solely by pitch, such as is perceived by an observer who has lost his memory. Two elements, a, b , are not enough for him to create the notion of distance or interval. We must look for a third term, c , in order that the observer may, by successive comparisons and through his immediate sensations, form first, the concept of relative size (b compared to a and c), which is a primary expression of ranking; and then the notion of distance, of interval. This mental toil will end in the totally ordered classification not only of pitches, but also of melodic intervals. Given the set of pitch intervals

$$H = (h_a, h_b, h_c, \dots)$$

and the binary relation S (greater than or equal to), we have

1. hSh for all $h \in H$, hence reflexivity;
2. $h_a Sh_b \neq h_b Sh_a$ except for $h_a = h_b$, hence antisymmetry;
3. $h_a Sh_b$ and $h_b Sh_c$ entail $h_a Sh_c$, hence transitivity.

Thus the different aspects of the sensations produced by sonic events may eventually totally or partially constitute ordered sets according to the unit interval adopted. For example, if we adopted as the unit interval of pitch, not the relationship of the semitone (≈ 1.059) but a relationship of 1.00001, then the sets of pitches and intervals would be very vague and would not be totally ordered because the *differential sensitivity* of the human ear is inferior to this relationship. Generally for a sufficiently large unit distance, many of the qualities of sonic events can be totally ordered.

To conform with a first-degree acoustic experience, we shall suppose that the ultimate aspects of sonic events are frequency¹ (experienced as pitch), intensity, and duration, and that every sonic event may be constructed from these three when duly interwoven. In this case the number three is irreducible. For other assumptions on the microstructure of sonic events see the Preface and Chapter IX.

Structure of the Qualities of Sonic Events*

From a naive musical practice we have defined the concept of interval or distance. Now let us examine sets of intervals which are in fact isomorphic to the equivalence classes of the $N \times N$ product set of natural numbers.

1. Let there be a set H of pitch intervals (melodic). The law of internal composition states that to every couple $(h_a, h_b \in H)$ a third element may be made to correspond. This is the composite of h_a by h_b , which we shall denote $h_a + h_b = h_c$, such that $h_c \in H$. For example, let there be three sounds characterized by the pitches I, II, III, and let $h_{(I,II)}$, $h_{(II,III)}$ be the intervals in semitones separating the couples (I, II) and (II, III), respectively. The interval $h_{(I,III)}$ separating sound I and sound III will be equal to the sum of the semitones of the other two. We may therefore establish that the law of internal composition for conjuncted intervals is *addition*.

2. The law is associative:

$$h_a + (h_b + h_c) = (h_a + h_b) + h_c = h_a + h_b + h_c.$$

3. There exists a neutral element h_0 such that for every $h_a \in H$,

$$h_0 + h_a = h_a + h_0 = h_a.$$

For pitch the neutral element has a name, unison, or the zero interval; for intensity the zero interval is nameless; and for duration it is simultaneity.

4. For every h_a there exists a special element h'_a , called the inverse, such that

$$h'_a + h_a = h_a + h'_a = h_0 = 0.$$

Corresponding to an ascending melodic interval h_a , there may be a descending interval h'_a , which returns to the unison; to an increasing interval of intensity (expressed in positive decibels) may be added another diminishing interval (in negative db), such that it cancels the other's effect; corresponding to a positive time interval there may be a negative duration, such that the sum of the two is zero, or simultaneity.

5. The law is commutative:

$$h_a + h_b = h_b + h_a.$$

* Following Peano, we may state an axiomatics of pitch and construct the chromatic or whole-tone scale by means of three primary terms—origin, note, and the successor of...—and five primary propositions:

1. the origin is a note;
2. the successor of a note is a note;
3. notes having the same successor are identical;
4. the origin is not the successor of any note; and
5. if a property applies to the origin, and if when it applies to any note it also applies to its successor, then it applies to all notes (principal of induction).

See also Chap. VII, p. 194.

These five axioms have been established for pitch, outside-time. But the examples have extended them to the two other fundamental factors of sonic events, and we may state that the sets H (pitch intervals), G (intensity intervals), and U (durations) are furnished with an *Abelian additive group structure*.

To specify properly the difference and the relationship that exists between the temporal set T and the other sets examined outside-time, and in order not to confuse, for example, set U (durations characterizing a sonic event) with the time intervals chronologically separating sonic events belonging to set T , we shall summarize the successive stages of our comprehension.

SUMMARY

Let there be three events a, b, c emitted successively.

First stage: Three events are distinguished, and that is all.

Second stage: A “temporal succession” is distinguished, i.e., a correspondence between events and moments. There results from this

$$a \text{ before } b \neq b \text{ before } a \quad (\text{non-commutativity}).$$

Third stage: Three sonic events are distinguished which divide time into two sections within the events. These two sections may be compared and then expressed in multiples of a unit. Time becomes metric and the sections constitute generic elements of set T . They thus enjoy commutativity.

According to Piaget, the concept of time among children passes through these three phases (see Bibliography for Chapter VI).

Fourth stage: Three sonic events are distinguished; the time intervals are distinguished; and independence between the sonic events and the time intervals is recognized. An *algebra outside-time* is thus admitted for sonic events, and a secondary *temporal algebra* exists for temporal intervals; the two algebras are otherwise identical. (It is useless to repeat the arguments in order to show that the temporal intervals between the events constitute a set T , which is furnished with an Abelian additive group structure.) Finally, one-to-one correspondences are admitted between algebraic functions outside-time and temporal algebraic functions. They may constitute an algebra in-time.

In conclusion, most musical analysis and construction may be based on: 1. the study of an entity, the sonic event, which, according to our temporary assumption groups three characteristics, pitch, intensity, and duration, and which possesses a *structure outside-time*; 2. the study of another simpler entity,

time, which possesses a *temporal structure*; and 3. the correspondence between the structure outside-time and the temporal structure: the *structure in-time*.

Vector Space

Sets H (melodic intervals), G (intensity intervals), U (time intervals), and T (intervals of time separating the sonic events, and independent of them) are totally ordered. We also assume that they may be isomorphic under certain conditions with set R of the real numbers, and that an external law of composition for each of them may be established with set R . For every $a \in E$ (E is any one of the above sets) and for every element $A \in R$, there exists an element $b = Aa$ such that $b \in E$. For another approach to vector space, see the discussion of sets of intervals as a product of a group times a field, Chap. VIII, p. 210.

Let \bar{X} be a sequence of three numbers x_1, x_2, x_3 , corresponding to the elements of the sets H, G, U , respectively, and arranged in a certain order: $\bar{X} = (x_1, x_2, x_3)$. This sequence is a vector and x_1, x_2, x_3 are its *components*. The particular case of the vector in which all the components are zero is a zero vector, \bar{O} . It may also be called the origin of the coordinates, and by analogy with elementary geometry, the vector with the numbers (x_1, x_2, x_3) as components will be called point M of coordinates (x_1, x_2, x_3) . Two points or vectors are said to be equal if they are defined by the same sequence: $x_i = y_i$.

The set of these sequences constitutes a vector space in three dimensions, E_3 . There exist two laws of composition relative to E_3 : 1. An *internal* law of composition, addition: If $\bar{X} = (x_1, x_2, x_3)$ and $\bar{Y} = (y_1, y_2, y_3)$, then

$$\bar{X} + \bar{Y} = (x_1 + y_1, x_2 + y_2, x_3 + y_3).$$

The following properties are verified: a. $\bar{X} + \bar{Y} = \bar{Y} + \bar{X}$ (commutative); b. $\bar{X} + (\bar{Y} + \bar{Z}) = (\bar{X} + \bar{Y}) + \bar{Z}$ (associative); and c. Given two vectors \bar{X} and \bar{Y} , there exists a single vector $\bar{Z} = (z_1, z_2, z_3)$ such that $\bar{X} = \bar{Y} + \bar{Z}$. We have $z_i = x_i - y_i$; \bar{Z} is called the difference of \bar{X} and \bar{Y} and is denoted $\bar{Z} = \bar{X} - \bar{Y}$. In particular $\bar{X} + \bar{O} = \bar{O} + \bar{X} = \bar{X}$; and each vector \bar{X} may be associated with the opposite vector $(-\bar{X})$, with components $(-x_1, -x_2, -x_3)$, such that $\bar{X} + (-\bar{X}) = \bar{O}$.

2. An *external* law of composition, multiplication by a scalar: If $p \in R$ and $\bar{X} \in E$, then

$$p\bar{X} = (px_1, px_2, px_3) \in E_3.$$

The following properties are verified for $(p, q) \in R$: a. $1 \cdot \bar{X} = \bar{X}$; b. $p(q\bar{X}) =$

information on the linear relations of a vectorial multiplicity so as to obtain the timbres of existing instruments or of other kinds of sonic events.

The following is an analysis of a fragment of *Sonata*, Op. 57 (Appassionata), by Beethoven (see Fig. VI-1). We do not take the timbre into account since the piano is considered to have only one timbre, homogeneous over the register of this fragment.

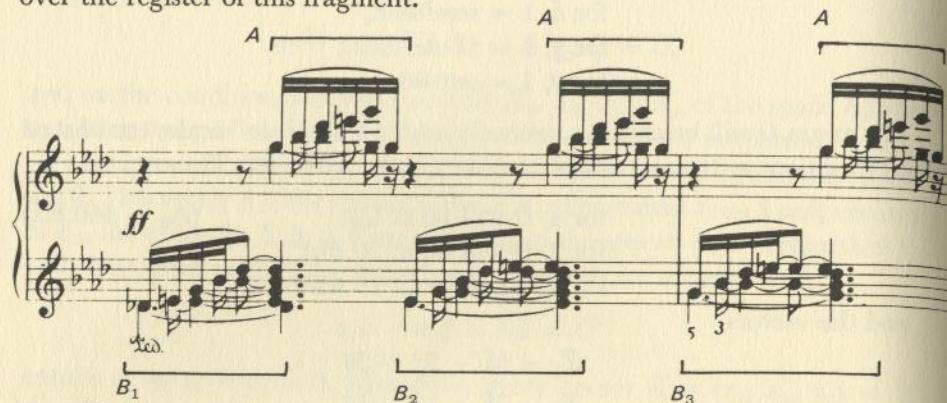
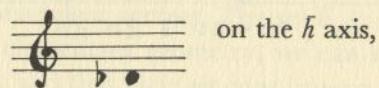


Fig. VI-1

Assume as unit vectors: \bar{h} , for which $1 \hat{=} \text{semitone}$; \bar{g} , for which $1 \hat{=} 10 \text{ db}$; and \bar{u} , for which $1 \hat{=} \text{dot}$. Assume for the origins



$ff = 60 \text{ db}$ (invariable) on the \bar{g} axis, and
 5dot on the \bar{u} axis.

ALGEBRA OUTSIDE-TIME (OPERATIONS AND RELATIONS IN SET A)

The vector $X_0 = 18\bar{h} + 0\bar{g} + 5\bar{u}$ corresponds to G .

The vector $X_1 = (18 + 3)\bar{h} + 0\bar{g} + 4\bar{u}$ corresponds to $B\flat$.

The vector $X_2 = (18 + 6)\bar{h} + 0\bar{g} + 3\bar{u}$ corresponds to $D\flat$.

The vector $X_3 = (18 + 9)\bar{h} + 0\bar{g} + 2\bar{u}$ corresponds to E .

The vector $X_4 = (18 + 12)\bar{h} + 0\bar{g} + 1\bar{u}$ corresponds to G .

The vector $X_5 = (18 + 0)\bar{h} + 0\bar{g} + 1\bar{u}$ corresponds to G .

(See Fig. VI-2.)

Let us also admit the free vector $\bar{v} = 3\bar{h} + 0\bar{g} - 1\bar{u}$; then the vectors X_i (for $i = 0, 1, 2, 3, 4$) are of the form $X_i = X_0 + \bar{v}i$.

We notice that set A consists of two vector families, X_i and $i\bar{v}$, combined by means of addition.

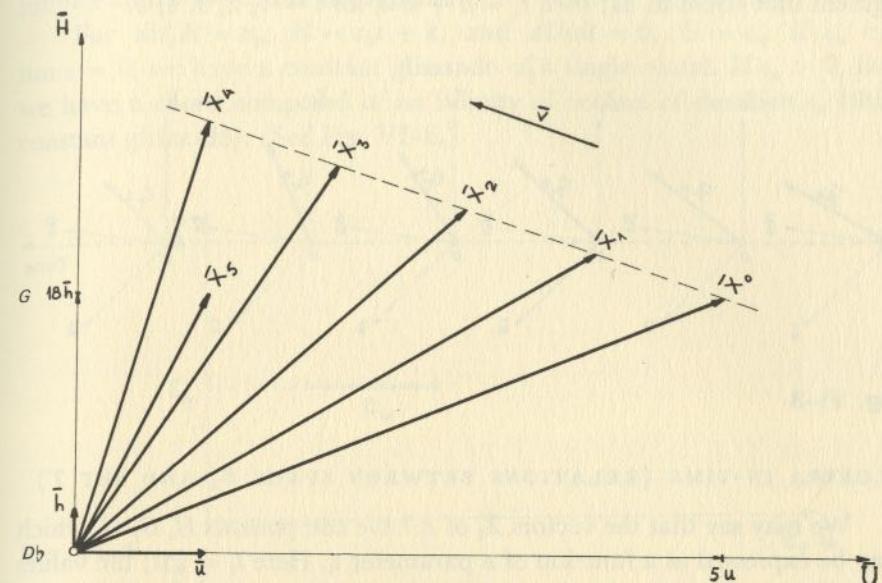


Fig. VI-2

A second law of composition exists in the set ($i = 0, 1, 2, 3, 4$); it is an arithmetic progression.

Finally, the scalar i leads to an antisymmetric variation of the components \bar{h} and \bar{u} of X_i , the second \bar{g} remaining invariant.

TEMPORAL ALGEBRA (IN SET T)

The sonic statement of the vectors X_i of set A is successive:

$$X_0 \top X_1 \top X_2 \top \dots$$

\top being the operator "before."

This boils down to saying that the origin O of the base of $A \hat{=} E_3 \hat{=} V$ is displaced on the axis of time, a shifting that has nothing to do with the change of the base, which is in fact an operation within space E_3 of base $\bar{h}, \bar{g}, \bar{u}$. Thus in the case of a simultaneity (a chord) of the attacks of the six vectors described for set A , the displacement would be zero.

In Fig. VI-3 the segments designated on the axis of time by the origins O of X_i are equal and obey the function $\Delta t_i = \Delta t_j$, which is an internal law

of composition in set T ; or consider an origin O' on the axis of time and a segment unit equal to Δt ; then $t_i = a + i\Delta t$, for $i = 1, 2, 3, 4, 5$.

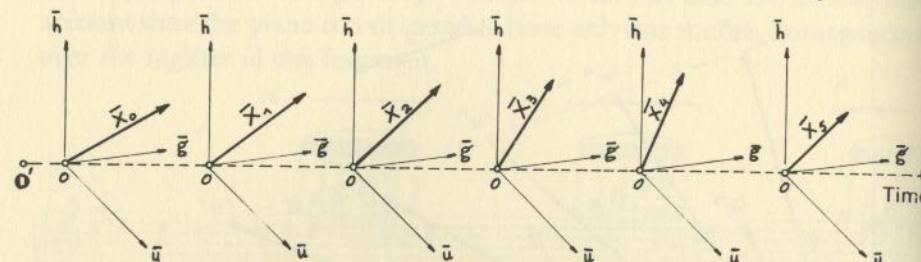


Fig. VI-3

ALGEBRA IN-TIME (RELATIONS BETWEEN SPACE E_3 AND SET T)

We may say that the vectors X_i of A have components H, G, U , which may be expressed as a function of a parameter t_i . Here $t_i = i\Delta t$; the values are lexicographically ordered and defined by the increasing order $i = 1, 2, 3, 4, 5$. This constitutes an association of each of the components with the ordered set T . It is therefore an algebra of sonic events that is independent of time (algebra outside-time), as well as an algebra of sonic events as a function of time (algebra in-time).

In general we admit that a vector X is a function of the parameter of time t if its components are also a function of t . This is written

$$X(t) = H(t)\bar{h} + G(t)\bar{g} + U(t)\bar{u}.$$

When these functions are continuous they have differentials. What is the meaning of the variations of X as a function of time t ? Suppose

$$\frac{dX}{dt} = \frac{dH}{dt} \bar{h} + \frac{dG}{dt} \bar{g} + \frac{dU}{dt} \bar{u}.$$

If we neglect the variation of the component G , we will have the following conditions: For $dH/dt = 0$, $H = c_h$, and $dU/dt = 0$, $U = c_u$, H and U will be independent of the variation of t ; and for c_h and $c_u \neq 0$, the sonic event will be of invariable pitch and duration. If c_h and $c_u = 0$, there is no sound (silence). (See Fig. VI-4.)

For $dH/dt = 0$, $H = c_h$, and $dU/dt = c_u$, $U = c_u t + k$, if c_h and $c_u \neq 0$, we have an infinity of vectors at the unison. If $c_u = 0$, then we have a single vector of constant pitch c_h and duration $U = k$. (See Fig. VI-5).

For $dH/dt = 0$, $H = c_h$, and $dU/dt = f(t)$, $U = F(t)$, we have an infinite family of vectors at the unison.

For $dH/dt = c_h$, $H = c_h t + k$, and $dU/dt = 0$, $U = c_u$, if $c_u < \varepsilon$, $\lim \varepsilon = 0$, we have a constant glissando of a single sound. If $c_u > 0$, then we have a chord composed of an infinity of vectors of duration c_u (thick constant glissando). (See Fig. VI-6.)

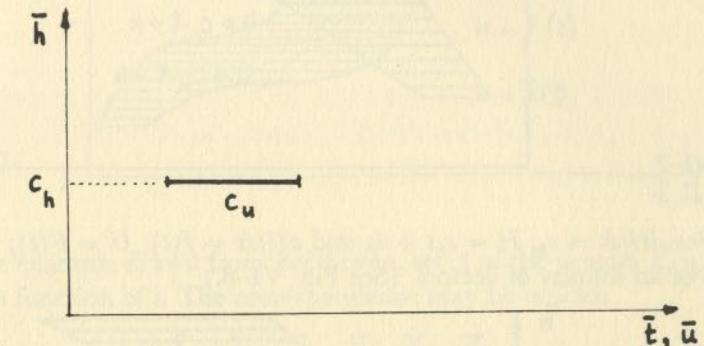


Fig. VI-4

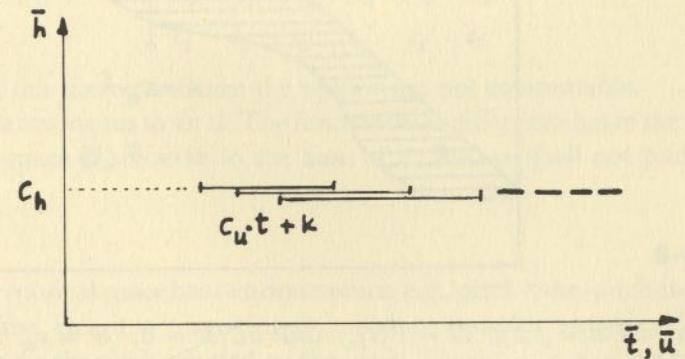


Fig. VI-5

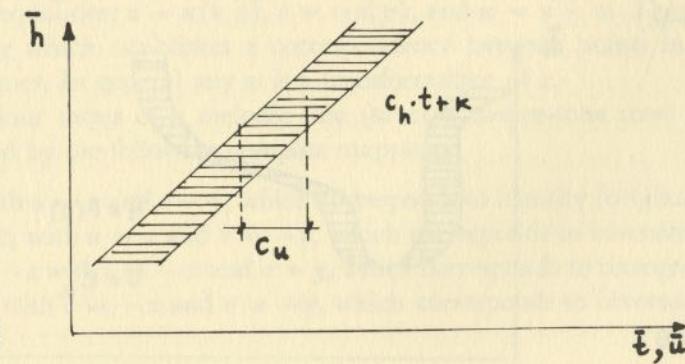


Fig. VI-6

For $dH/dt = c_h$, $H = c_h t + k$, and $dU/dt = c_u$, $U = c_u t + r$, we have a chord of an infinity of vectors of variable durations and pitches. (See Fig. VI-7.)

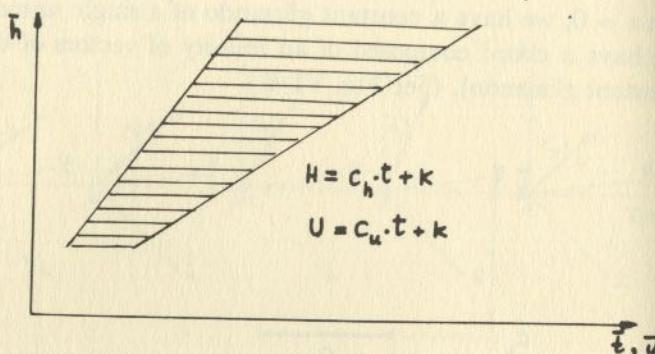


Fig. VI-7

For $dH/dt = c_h$, $H = c_h t + k$, and $dU/dt = f(t)$, $U = F(t)$, we have a chord of an infinity of vectors. (See Fig. VI-8.)

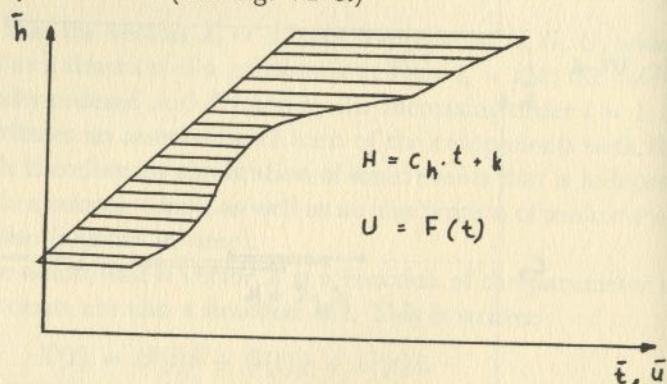


Fig. VI-8

For $dH/dt = f(t)$, $H = F(t)$, and $dU/dt = 0$, $U = c_u$, if $c_u < \epsilon$, $\lim \epsilon = 0$, we have a thin variable glissando. If $c_u > 0$, then we have a chord of an infinity of vectors of duration c_u (thick variable glissando). (See Fig. VI-9.)

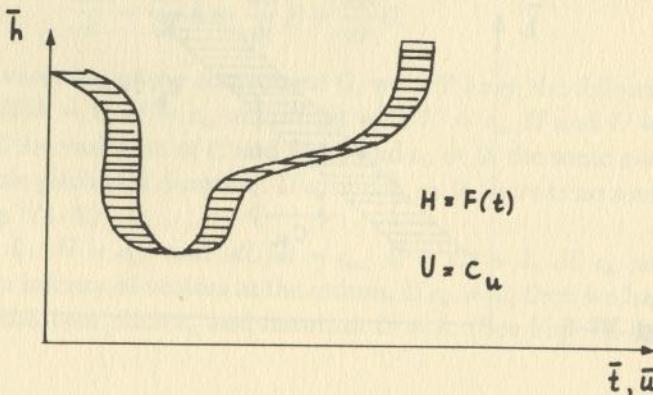


Fig. VI-9

For $dH/dt = f(t)$, $H = F(t)$, and $dU/dt = s(t)$, $U = S(t)$, we have a chord of an infinity of vectors. (See Fig. VI-10.)

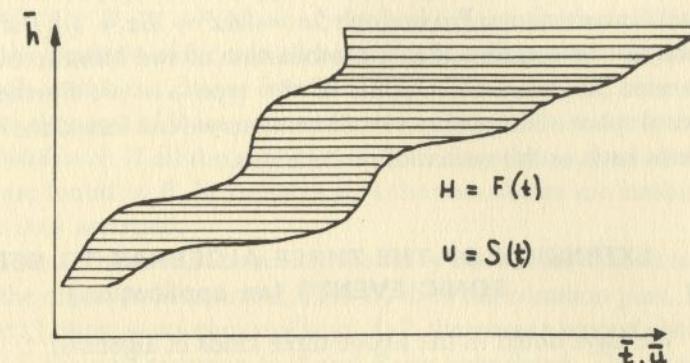


Fig. VI-10

In the example drawn from Beethoven, set A of the vectors X_i is not a continuous function of t . The correspondence may be written

$$\begin{array}{ccccccc} X_0 & X_1 & X_2 & X_3 & X_4 & X_5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ t_0 & t_1 & t_2 & t_3 & t_4 & t_5 \end{array}$$

Because of this correspondence the vectors are not commutable.

Set B is analogous to set A . The fundamental difference lies in the change of base in space E_3 relative to the base of A . But we shall not pursue the analysis.

Remark

If our musical space has two dimensions, e.g., pitch-time, pitch-intensity, pressure-time, etc., it is interesting to introduce complex variables. Let x be the time and y the pitch, plotted on the i axis. Then $z = x + yi$ is a sound of pitch y with the attack at the instant x . Let there be a plane uv with the following equalities: $u = u(x, y)$, $v = v(x, y)$, and $w = u + vi$. They define a mapping which establishes a correspondence between points in the uv and xy planes. In general any w is a transformation of z .

The four forms of a melodic line (or of a twelve-tone row) can be represented by the following complex mappings:

$w = z$, with $u = x$ and $v = y$, which corresponds to identity (original form)
 $w = |z|^2/z$, with $u = x$ and $v = -y$, which corresponds to inversion
 $w = |z|^2/z$ with $u = -x$ and $v = y$, which corresponds to retrogradation
 $w = -z$, with $u = -x$ and $v = -y$, which corresponds to inverted retrogradation.

These transformations form the Klein group.²

Other transformations, as yet unknown, even to present-day musicians, could be envisaged. They could be applied to any product of two sets of sound characteristics. For example, $w = (Az^2 + Bz + c)/(Dz^2 + Ez + F)$, which can be considered as a combination of two bilinear transformations separated by a transformation of the type $\rho = \sigma^2$. Furthermore, for a musical space of more than two dimensions we can introduce hypercomplex systems such as the system of quaternions.

EXTENSION OF THE THREE ALGEBRAS TO SETS OF SONIC EVENTS (an application)

We have noted in the above three kinds of algebras:

1. The algebra of the components of a sonic event, with its vector language, independent of the procession of time, therefore an *algebra outside-time*.
2. A *temporal algebra*, which the sonic events create on the axis of metric time, and which is independent of the vector space.
3. An *algebra in-time*, issuing from the correspondences and functional relations between the elements of the set of vectors X and of the set of metric time, T , independent of the set of X .

All that has been said about sonic events themselves, their components, and about time can be generalized for sets of sonic events X and for sets T .

In this chapter we have assumed that the reader is familiar with the concept of the set, and in particular with the concept of the class as it is interpreted in Boolean algebra. We shall adopt this specific algebra, which is isomorphic with the theory of sets.

To simplify the exposition, we shall first take a concrete example by considering the *referential* or *universal set* R , consisting of all the sounds of a piano. We shall consider only the pitches; timbres, attacks, intensities, and durations will be utilized in order to clarify the exposition of the logical operations and relations which we shall impose on the set of pitches.

Suppose, then, a set A of keys that have a characteristic property. This will be set A , a subset of set R , which consists of all the keys of the piano. This subset is chosen a priori and the characteristic property is the particular choice of a certain number of keys.

For the amnesic observer this class may be presented by playing the keys one after the other, with a period of silence in between. He will deduce from this that he has heard a collection of sounds, or a listing of elements.

Another class, B , consisting of a certain number of keys, is chosen in the same way. It is stated after class A by causing the elements of B to sound.

The observer hearing the two classes, A and B , will note the temporal fact: A before B ; $A \tau B$, (τ = before). Next he begins to notice relationships between the elements of the two classes. If certain elements or keys are common to both classes the classes *intersect*. If none are common, they are *disjoint*. If all the elements of B are common to one part of A he deduces that B is a class *included* in A . If all the elements of B are found in A , and all the elements of A are found in B , he deduces that the two classes are indistinguishable, that they are *equal*.

Let us choose A and B in such a way that they have some elements in common. Let the observer hear first A , then B , then the common part. He will deduce that: 1. there was a choice of keys, A ; 2. there was a second choice of keys, B ; and 3. the part common to A and B was considered. The operation of *intersection* (conjunction) has therefore been used:

$$A \cdot B \text{ or } B \cdot A.$$

This operation has therefore engendered a new class, which was symbolized by the sonic enumeration of the part common to A and B .

If the observer, having heard A and B , hears a mixture of all the elements of A and B , he will deduce that a new class is being considered, and that a logical summation has been performed on the first two classes. This operation is the *union* (disjunction) and is written

$$A + B \text{ or } B + A.$$

If class A has been symbolized or played to him and he is made to hear all the sounds of R except those of A , he will deduce that the complement of A with respect to R has been chosen. This is a new operation, *negation*, which is written \bar{A} .

Hitherto we have shown by an imaginary experiment that we can define and *state classes of sonic events* (while taking precautions for clarity in the symbolization); and effect three operations of fundamental importance: intersection, union, and negation.

On the other hand, an observer must undertake an intellectual task in order to deduce from this both classes and operations. On our plane of immediate comprehension, we replaced graphic signs by sonic events. We consider these sonic events as symbols of abstract entities furnished with abstract logical relations on which we may effect at least the fundamental operations of the logic of classes. We have not allowed special symbols for the statement of the classes; only the sonic enumeration of the generic

elements was allowed (though in certain cases, if the classes are already known and if there is no ambiguity, shortcuts may be taken in the statement to admit a sort of mnemotechnical or even psychophysiological stenosymbolization).

We have not allowed special sonic symbols for the three operations which are expressed graphically by \cdot , $+$, $-$; only the classes resulting from these operations are expressed, and the operations are consequently deduced mentally by the observer. In the same way the observer must deduce the relation of equality of the two classes, and the relation of implication based on the concept of inclusion. The empty class, however, may be symbolized by a duly presented silence. In sum, then, we can only state classes, not the operations. The following is a list of correspondences between the sonic symbolization and the graphical symbolization as we have just defined it:

Graphic symbols	Sonic symbols
Classes A, B, C, \dots	Sonic enumeration of the generic elements having the properties A, B, C, \dots (with possible shortcuts)
Intersection (\cdot)	_____
Union ($+$)	_____
Negation ($-$)	_____
Implication (\rightarrow)	_____
Membership (\in)	_____
\bar{A}	Sonic enumeration of the elements of R not included in A
$A \cdot B$	Sonic enumeration of the elements of $A \cdot B$
$A + B$	Sonic enumeration of the elements of $A + B$
$A \supset B$	_____
$A = B$	_____

This table shows that we can reason by pinning down our thoughts by means of sound. This is true even in the present case where, because of a concern for economy of means, and in order to remain close to that immediate intuition from which all sciences are built, we do not yet wish to propose sonic conventions symbolizing the operations \cdot , $+$, $-$, and the relations $=$, \rightarrow . Thus propositions of the form A, E, I, O may not be symbolized by sounds, nor may theorems. Syllogisms and demonstrations of theorems may only be inferred.

Besides these logical relations and operations outside-time, we have seen that we may obtain temporal classes (T classes) issuing from the sonic symbolization that defines distances or intervals on the axis of time. The role of time is again defined in a new way. It serves primarily as a crucible, mold, or space in which are inscribed the classes whose relations one must *decipher*. Time is in some ways equivalent to the area of a sheet of paper or a blackboard. It is only in a secondary sense that it may be considered as carrying generic elements (temporal distances) and relations or operations between these elements (temporal algebra).

Relations and correspondences may be established between these temporal classes and the outside-time classes, and we may recognize in-time operations and relations on the class level.

After these general considerations, we shall give an example of musical composition constructed with the aid of the algebra of classes. For this we must search out a necessity, a knot of interest.

Construction

Every Boolean expression or function $F(A, B, C)$, for example, of the three classes A, B, C can be expressed in the form called *disjunctive canonic*:

$$\sum_{i=1}^8 \sigma_i k_i$$

where $\sigma_i = 0, 1$ and $k_i = A \cdot B \cdot C, A \cdot B \cdot \bar{C}, A \cdot \bar{B} \cdot C, A \cdot \bar{B} \cdot \bar{C}, \bar{A} \cdot B \cdot C, \bar{A} \cdot B \cdot \bar{C}, \bar{A} \cdot \bar{B} \cdot C, \bar{A} \cdot \bar{B} \cdot \bar{C}$.

A Boolean function with n variables can always be written in such a way as to bring in a maximum of operations $+$, \cdot , $-$, equal to $3n \cdot 2^{n-2} - 1$. For $n = 3$ this number is 17, and is found in the function

$$F = A \cdot B \cdot C + A \cdot B \cdot \bar{C} + \bar{A} \cdot B \cdot C + \bar{A} \cdot B \cdot \bar{C}. \quad (1)$$

For three classes, each of which intersects with the other two, function (1) can be represented by the Venn diagram in Fig. VI-11. The flow chart of the operations is shown in Fig. VI-12.

This same function F can be obtained with only ten operations:

$$F = (A \cdot B + \bar{A} \cdot \bar{B}) \cdot C + (\bar{A} \cdot B + \bar{A} \cdot \bar{B}) \cdot \bar{C}. \quad (2)$$

Its flow chart is given in Fig. VI-13.

If we compare the two expressions of F , each of which defines a different procedure in the composition of classes A, B, C , we notice a more elegant

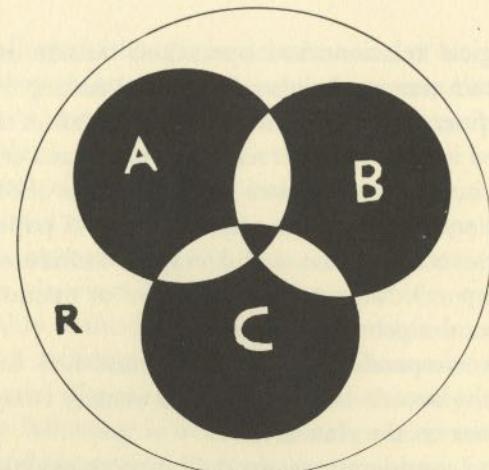


Fig. VI-11

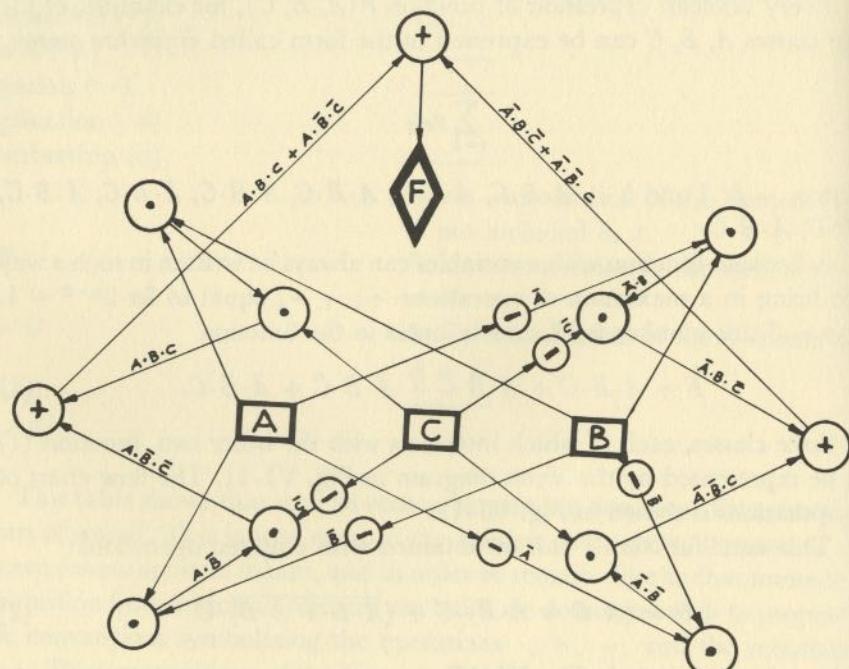


Fig. VI-12

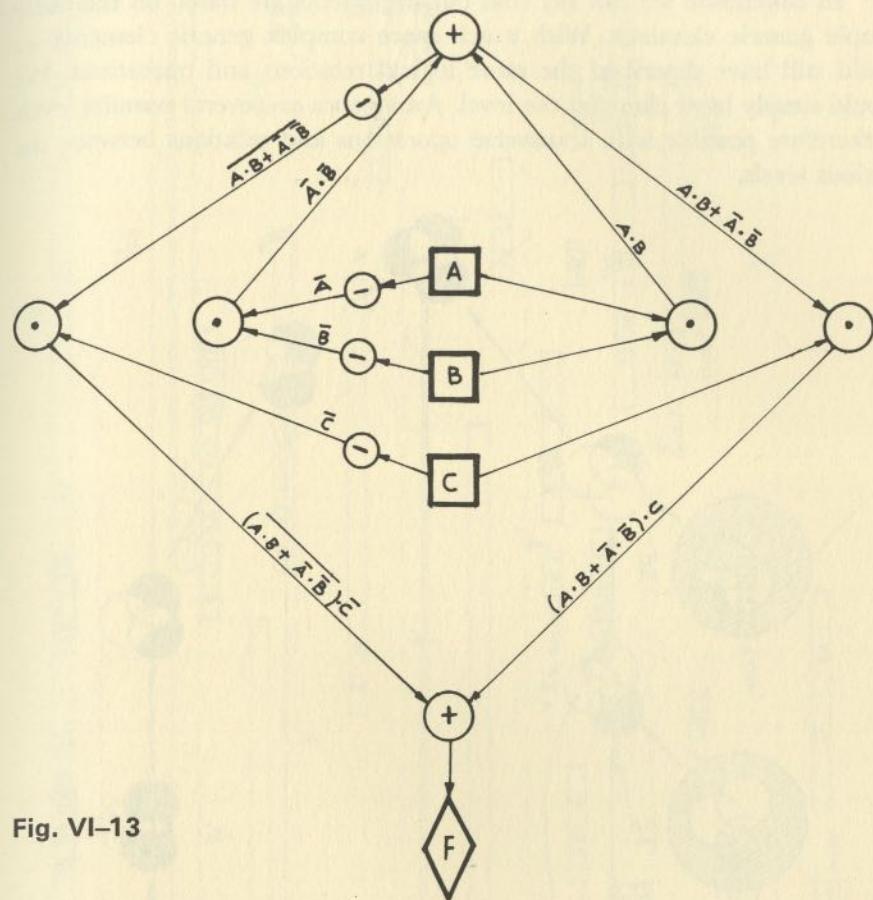


Fig. VI-13

symmetry in (1) than in (2). On the other hand (2) is more economical (ten operations as against seventeen). It is this comparison that was chosen for the realization of *Herma*, a work for piano. Fig. VI-14 shows the flow chart that directs the operations of (1) and (2) on two parallel planes, and Fig. VI-15 shows the precise plan of the construction of *Herma*.

The three classes *A*, *B*, *C* result in an appropriate set of keys of the piano. There exists a stochastic correspondence between the pitch components and the moments of occurrence in set *T*, which themselves follow a stochastic law. The intensities and densities (number of vectors/sec.), as well as the silences, help clarify the levels of the composition. This work was composed in 1960-61, and was first performed by the extraordinary Japanese pianist Yuji Takahashi in Tokyo in February 1962.

In conclusion we can say that our arguments are based on relatively simple generic elements. With much more complex generic elements we could still have described the same logical relations and operations. We would simply have changed the level. An algebra on several parallel levels is therefore possible with transverse operations and relations between the various levels.

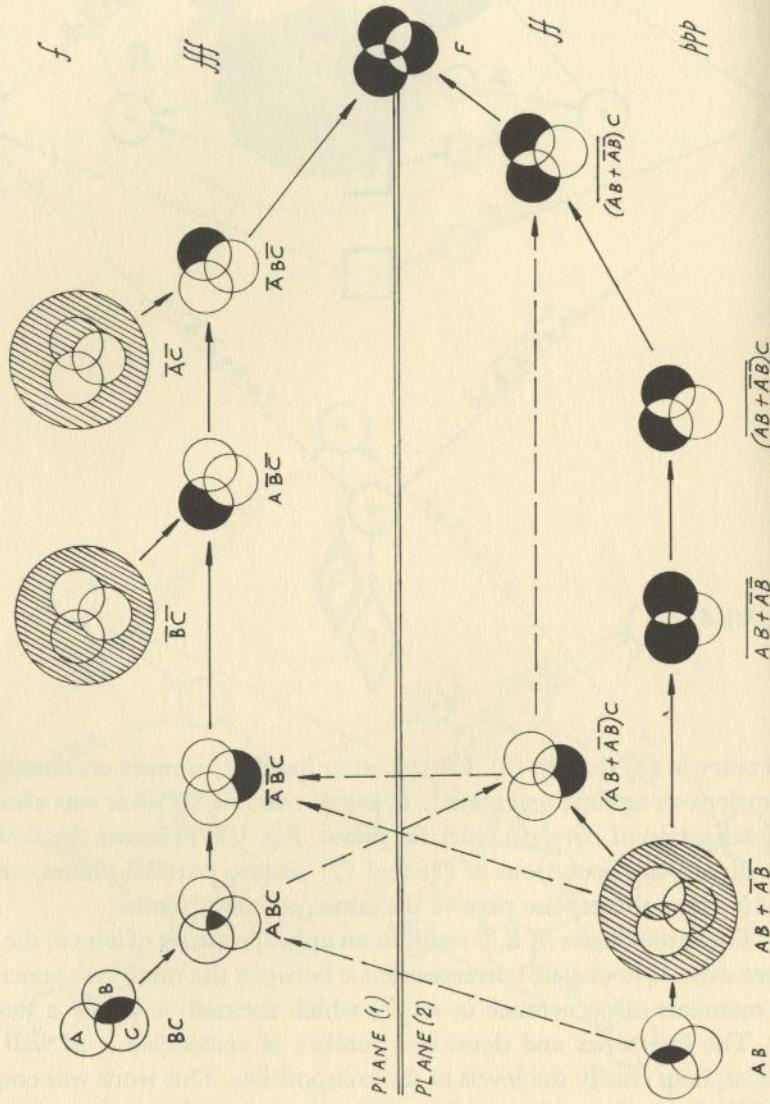


Fig. VI-14

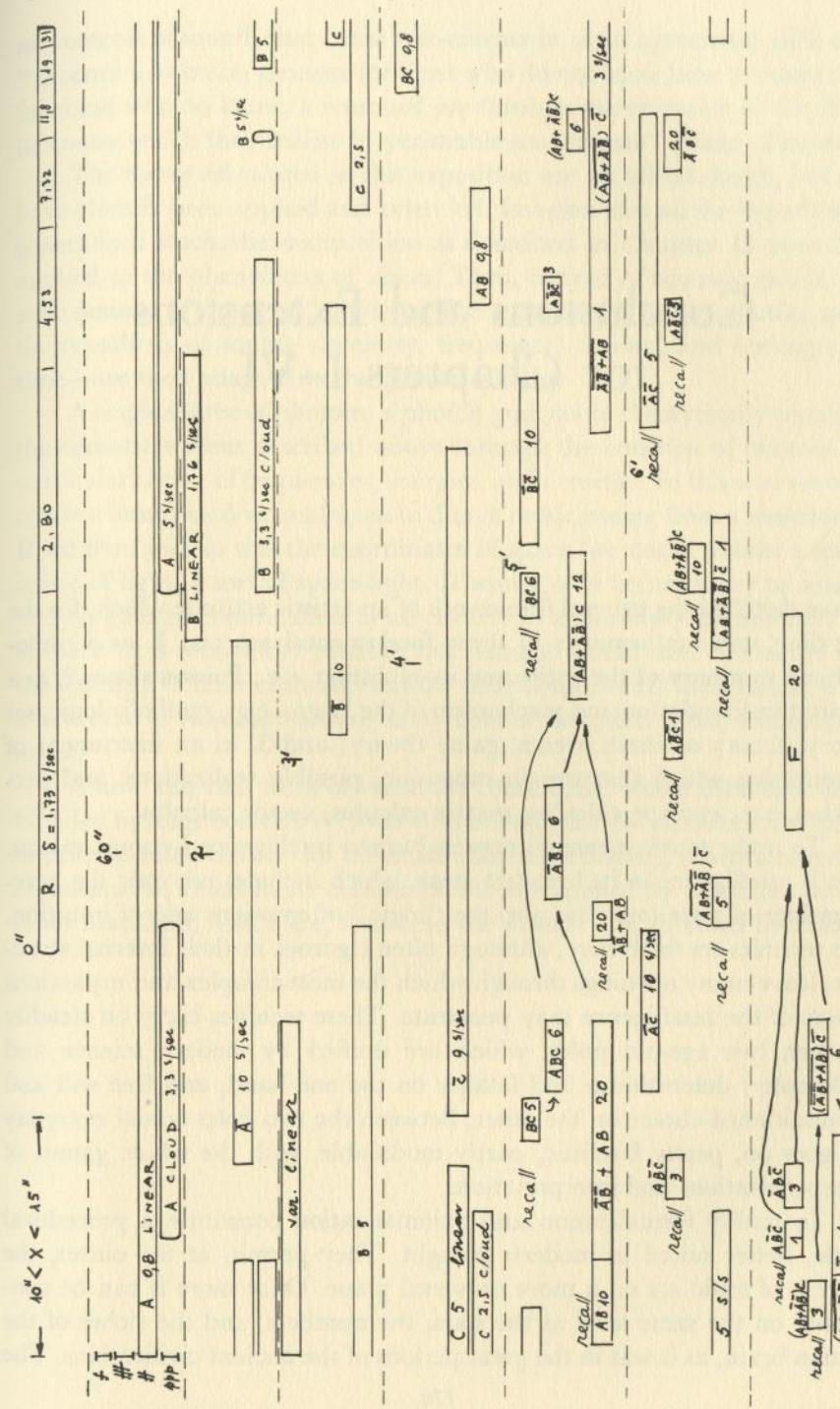


Fig. VI-15. Herma for Solo Piano

Conclusions and Extensions for Chapters I–VI

I have sketched the general framework of an artistic attitude which, for the first time, uses mathematics in three fundamental aspects: 1. as a philosophical summary of the entity and its evolution, e.g., Poisson's law; 2. as a qualitative foundation and mechanism of the Logos, e.g., symbolic logic, set theory, theory of chain events, game theory; and 3. as an instrument of mensuration which sharpens investigation, possible realizations, and perception, e.g., entropy calculus, matrix calculus, vector calculus.

To make music means to express human intelligence by sonic means. This is intelligence in its broadest sense, which includes not only the peregrinations of pure logic but also the "logic" of emotions and of intuition. The technics set forth here, although often rigorous in their internal structure, leave many openings through which the most complex and mysterious factors of the intelligence may penetrate. These technics carry on steadily between two age-old poles, which are unified by modern science and philosophy: determinism and fatality on the one hand, and free will and unconditioned choice on the other. Between the two poles actual everyday life goes on, partly fatalistic, partly modifiable, with the whole gamut of interpenetrations and interpretations.

In reality formalization and axiomatization constitute a procedural guide, better suited to modern thought. They permit, at the outset, the placing of sonic art on a more universal plane. Once more it can be considered on the same level as the stars, the numbers, and the riches of the human brain, as it was in the great periods of the ancient civilizations. The

movements of sounds that cause movements in us in agreement with them "procure a common pleasure for those who do not know how to reason; and for those who do know, a reasoned joy through the imitation of the divine harmony which they realize in perishable movements" (Plato, *Timaeus*).

The theses advocated in this exposition are an initial sketch, but they have already been applied and extended. Imagine that all the hypotheses of generalized stochastic composition as described in Chapter II were to be applied to the phenomena of vision. Then, instead of acoustic grains, suppose quanta of light, i.e., photons. The components in the atomic, quantic hypothesis of sound—intensity, frequency, density, and lexicographic time—are then adapted to the quanta of light.

A single source of photons, a photon gun, could theoretically reproduce the acoustic screens described above through the emission of photons of a particular choice of frequencies, energies, and densities. In this way we could create a luminous flow analogous to that of music issuing from a sonic source. If we then join to this the coordinates of space, we could obtain a spatial music of light, a sort of space-light. It would only be necessary to activate photon guns in combination at all corners in a gloriously illuminated area of space. It is technically possible, but painters would have to emerge from the lethargy of their craft and forsake their brushes and their hands, unless a new type of visual artist were to lay hold of these new ideas, technics, and needs.

A new and rich work of visual art could arise, whose evolution would be ruled by huge computers (tools vital not only for the calculation of bombs or price indexes, but also for the artistic life of the future), a total audiovisual manifestation ruled in its compositional intelligence by machines serving other machines, which are, thanks to the *scientific arts*, directed by man.

Towards a Metamusic

Today's technocrats and their followers treat music as a message which the composer (source) sends to a listener (receiver). In this way they believe that the solution to the problem of the nature of music and of the arts in general lies in formulae taken from information theory. Drawing up an account of bits or quanta of information transmitted and received would thus seem to provide them with "objective" and scientific criteria of aesthetic value. Yet apart from elementary statistical recipes this theory—which is valuable for technological communications—has proved incapable of giving the characteristics of aesthetic value even for a simple melody of J. S. Bach. Identifications of music with message, with communication, and with language are schematizations whose tendency is towards absurdities and desiccations. Certain African tom-toms cannot be included in this criticism, but they are an exception. Hazy music cannot be forced into too precise a theoretical mold. Perhaps, it will be possible later when present theories have been refined and new ones invented.

The followers of information theory or of cybernetics represent one extreme. At the other end there are the intuitionists, who may be broadly divided into two groups:

1. The "graphists," who exalt the graphic symbol above the sound of the music and make a kind of fetish of it. In this group it is the fashionable thing not to write notes, but to create any sort of design. The "music" is judged according to the beauty of the drawing. Related to this is the so-called aleatory music, which is an abuse of language, for the true term should be

English translation of Chapter VII by G. W. Hopkins.

the "improvised" music our grandfathers knew. This group is ignorant of the fact that graphical writing, whether it be symbolic, as in traditional notation, geometric, or numerical, should be no more than an image that is as *faithful* as possible to all the instructions the composer gives to the orchestra or to the machine.¹ This group is taking music outside itself.

2. Those who add a spectacle in the form of extra-musical scenic action to accompany the musical performance. Influenced by the "happenings" which express the confusion of certain artists, these composers take refuge in mimetics and disparate occurrences and thus betray their very limited confidence in pure music. In fact they concede certain defeat for their music in particular.

The two groups share a romantic attitude. They believe in immediate action and are not much concerned about its control by the mind. But since musical action, unless it is to risk falling into trivial improvisation, imprecision, and irresponsibility, imperiously demands reflection, these groups are in fact denying music and take it outside itself.

Linear Thought

I shall not say, like Aristotle, that the mean path is the best, for in music—as in politics—the middle means compromise. Rather lucidity and harshness of critical thought—in other words, action, reflection, and self-transformation by the sounds themselves—is the path to follow. Thus when scientific and mathematical thought serve music, or any human creative activity, it should amalgamate dialectically with intuition. Man is one, indivisible, and total. He thinks with his belly and feels with his mind. I would like to propose what, to my mind, covers the term "music":

1. It is a sort of comportment necessary for whoever thinks it and makes it.
2. It is an individual pleroma, a realization.
3. It is a fixing in sound of imagined virtualities (cosmological, philosophical, . . ., arguments).
4. It is normative, that is, unconsciously it is a model for being or for doing by sympathetic drive.
5. It is catalytic: its mere presence permits internal psychic or mental transformations in the same way as the crystal ball of the hypnotist.
6. It is the gratuitous play of a child.
7. It is a mystical (but atheistic) asceticism. Consequently expressions of sadness, joy, love, and dramatic situations are only very limited particular instances.

Musical syntax has undergone considerable upheaval and today it seems that innumerable possibilities coexist in a state of chaos. We have an abundance of theories, of (sometimes) individual styles, of more or less ancient "schools." But how does one make music? What can be communicated by oral teaching? (A burning question, if one is to reform musical education—a reform that is necessary in the entire world.)

It cannot be said that the informationists or the cyberneticians—much less the intuitionists—have posed the question of an ideological purge of the dross accumulated over the centuries as well as by present-day developments. In general they all remain ignorant of the substratum on which they found this theory or that action. Yet this substratum exists, and it will allow us to establish for the first time an axiomatic system, and to bring forth a formalization which will unify the ancient past, the present, and the future; moreover it will do so on a planetary scale, comprising the still separate universes of sound in Asia, Africa, etc.

In 1954² I denounced *linear thought* (polyphony), and demonstrated the contradictions of serial music. In its place I proposed a world of sound-masses, vast groups of sound-events, clouds, and galaxies governed by new characteristics such as density, degree of order, and rate of change, which required definitions and realizations using probability theory. Thus stochastic music was born. In fact this new, mass-conception with large numbers was more general than linear polyphony, for it could embrace it as a particular instance (by reducing the density of the clouds). General harmony? No, not yet.

Today these ideas and the realizations which accompany them have been around the world, and the exploration seems to be closed for all intents and purposes. However the tempered diatonic system—our musical *terra firma* on which all our music is founded—seems not to have been breached either by reflection or by music itself.³ This is where the next stage will come. The exploration and transformations of this system will herald a new and immensely promising era. In order to understand its determinative importance we must look at its pre-Christian origins and at its subsequent development. Thus I shall point out the structure of the music of ancient Greece; and then that of Byzantine music, which has best preserved it while developing it, and has done so with greater fidelity than its sister, the occidental plainchant. After demonstrating their abstract logical construction in a modern way, I shall try to express in a simple but universal mathematical and logical language what was and what might be valid in time (transverse musicology) and in space (comparative musicology).

In order to do this I propose to make a distinction in musical architectures or categories between *outside-time*,⁴ *in-time*, and *temporal*. A given pitch scale, for example, is an outside-time architecture, for no horizontal or vertical combination of its elements can alter it. The event in itself, that is, its actual occurrence, belongs to the temporal category. Finally, a melody or a chord on a given scale is produced by relating the outside-time category to the temporal category. Both are realizations in-time of outside-time constructions. I have dealt with this distinction already, but here I shall show how ancient and Byzantine music can be analyzed with the aid of these categories. This approach is very general since it permits both a universal axiomatization and a formalization of many of the aspects of the various kinds of music of our planet.

Structure of Ancient Music

Originally the Gregorian chant was founded on the structure of ancient music, *pace* Combarieu and the others who accused Hucbald of being behind the times. The rapid evolution of the music of Western Europe after the ninth century simplified and smoothed out the plainchant, and theory was left behind by practice. But shreds of the ancient theory can still be found in the secular music of the fifteenth and sixteenth centuries, witness the *Terminorum Musicae diffinitorium* of Johannis Tinctoris.⁵ To look at antiquity scholars have been looking through the lens of the Gregorian chant and its modes, which have long ceased to be understood. We are only beginning to glimpse other directions in which the modes of the plainchant can be explained. Nowadays the specialists are saying that the modes are not in fact proto-scales, but that they are rather characterized by melodic formulae. To the best of my knowledge only Jacques Chailley⁶ has introduced other concepts complementary to that of the scale, and he would seem to be correct. I believe we can go further and affirm that ancient music, at least up to the first centuries of Christianity, was not based at all on scales and modes related to the octave, but on tetrachords and *systems*.

Experts on ancient music (with the above exception) have ignored this fundamental reality, clouded as their minds have been by the tonal construction of post-medieval music. However, this is what the Greeks used in their music: a hierarchic structure whose complexity proceeded by successive "nesting," and by inclusions and intersections from the particular to the general; we can trace its main outline if we follow the writings of Aristoxenos:⁷

A. The *primary order* consists of the tone and its subdivisions. The whole

tone is defined as the amount by which the interval of a fifth (the pentachord, or dia pente) exceeds the interval of a fourth (the tetrachord, or dia tessaron). The tone is divided into halves, called *semitones*; thirds, called *chromatic dieseis*; and quarters, the extremely small *enharmonic dieseis*. No interval smaller than the quarter-tone was used.

B. The *secondary order* consists of the tetrachord. It is bounded by the interval of the *dia tessaron*, which is equal to two and a half tones, or thirty twelfth-tones, which we shall call Aristoxenean segments. The two outer notes always maintain the same interval, the fourth, while the two inner notes are mobile. The positions of the inner notes determine the three genera of the tetrachord (the intervals of the fifth and the octave play no part in it). The position of the notes in the tetrachord are always counted from the lowest note up:

1. The enharmonic genus contains two enharmonic dieseis, or $3 + 3 + 24 = 30$ segments. If X equals the value of a tone, we can express the enharmonic as $X^{1/4} \cdot X^{1/4} \cdot X^2 = X^{5/2}$.

2. The chromatic genus consists of three types: a. soft, containing two chromatic dieseis, $4 + 4 + 22 = 30$, or $X^{1/3} \cdot X^{1/3} \cdot X^{(1/3+3/2)} = X^{5/2}$; b. hemiolon (sesquialterus), containing two hemioloi dieseis, $4.5 + 4.5 + 21 = 30$ segments, or $X^{(3/2)(1/4)} \cdot X^{(3/2)(1/4)} \cdot X^{7/4} = X^{5/2}$; and c. "toniaion," consisting of two semitones and a trihemitone, $6 + 6 + 18 = 30$ segments, or $X^{1/2} \cdot X^{1/2} \cdot X^{3/2} = X^{5/2}$.

3. The diatonic consists of: a. soft, containing a semitone, then three enharmonic dieseis, then five enharmonic dieseis, $6 + 9 + 15 = 30$ segments, or $X^{1/2} \cdot X^{3/4} \cdot X^{5/4} = X^{5/2}$; b. syntomon, containing a semitone, a whole tone, and another whole tone, $6 + 12 + 12 = 30$ segments, or $X^{1/2} \cdot X \cdot X = X^{5/2}$.

C. The *tertiary order*, or the *system*, is essentially a combination of the elements of the first two—tones and tetrachords either conjuncted or separated by a tone. Thus we get the pentachord (outer interval the perfect fifth) and the octochord (outer interval the octave, sometimes perfect). The subdivisions of the system follow exactly those of the tetrachord. They are also a function of connexity and of consonance.

D. The *quaternary order* consists of the tropes, the keys, or the modes, which were probably just particularizations of the systems, derived by means of cadential, melodic, dominant, registral, and other formulae, as in Byzantine music, ragas, etc.

These orders account for the outside-time structure of Hellenic music. After Aristoxenos all the ancient texts one can consult on this matter give

this same hierarchical procedure. Seemingly Aristoxenos was used as a model. But later, traditions parallel to Aristoxenos, defective interpretations, and sediments distorted this hierarchy, even in ancient times. Moreover, it seems that theoreticians like Aristides Quintilianos and Claudio Ptolemaeos had but little acquaintance with music.

This hierarchical "tree" was completed by transition algorithms—the metabolae—from one genus to another, from one system to another, or from one mode to another. This is a far cry from the simple modulations or transpositions of post-medieval tonal music.

Pentachords are subdivided into the same genera as the tetrachord they contain. They are derived from tetrachords, but nonetheless are used as primary concepts, on the same footing as the tetrachord, in order to define the interval of a tone. This vicious circle is accounted for by Aristoxenos' determination to remain faithful to musical experience (on which he insists), which alone defines the structure of tetrachords and of the entire harmonic edifice which results combinatorially from them. His whole axiomatics proceeds from there and his text is an example of a method to be followed. Yet the absolute (physical) value of the interval *dia tessaron* is left undefined, whereas the Pythagoreans defined it by the ratio 3/4 of the lengths of the strings. I believe this to be a sign of Aristoxenos' wisdom; the ratio 3/4 could in fact be a mean value.

Two Languages

Attention must be drawn to the fact that he makes use of the additive operation for the intervals, thus foreshadowing logarithms before their time; this contrasts with the practice of the Pythagoreans, who used the geometrical (exponential) language, which is multiplicative. Here, the method of Aristoxenos is fundamental since: 1. it constitutes one of the two ways in which musical theory has been expressed over the millennia; 2. by using addition it institutes a means of "calculation" that is more economical, simpler, and better suited to music; and 3. it lays the foundation of the tempered scale nearly twenty centuries before it was applied in Western Europe.

Over the centuries the two languages—arithmetic (operating by addition) and geometric (derived from the ratios of string lengths, and operating by multiplication)—have always intermingled and interpenetrated so as to create much useless confusion in the reckoning of intervals and consonances, and consequently in theories. In fact they are both expressions of group structure, having two non-identical operations; thus they have a formal equivalence.⁸

There is a hare-brained notion that has been sanctimoniously repeated by musicologists in recent times. "The Greeks," they say, "had descending scales instead of the ascending ones we have today." Yet there is no trace of this in either Aristoxenos or his successors, including Quintilianos⁹ and Alypios, who give a new and fuller version of the steps of many of the tropes. On the contrary, the ancient writers always begin their theoretical explanations and nomenclature of the steps from the bottom. Another bit of foolishness is the supposed Aristoxenean scale, of which no trace is to be found in his text.¹⁰

Structure of Byzantine Music

Now we shall look at the structure of Byzantine music. It can contribute to an infinitely better understanding of ancient music, occidental plainchant, non-European musical traditions, and the dialectics of recent European music, with its wrong turns and dead-ends. It can also serve to foresee and construct the future from a view commanding the remote landscapes of the past as well as the electronic future. Thus new directions of research would acquire their full value. By contrast the deficiencies of serial music in certain domains and the damage it has done to musical evolution by its ignorant dogmatism will be indirectly exposed.

Byzantine music amalgamates the two means of calculation, the Pythagorean and the Aristoxenean, the multiplicative and the additive.¹¹ The fourth is expressed by the ratio 3/4 of the monochord, or by the 30 tempered segments (72 to the octave).¹² It contains three kinds of tones: major (9/8 or 12 segments), minor (10/9 or 10 segments), and minimal (16/15 or 8 segments). But smaller and larger intervals are constructed and the elementary units of the primary order are more complex than in Aristoxenos. Byzantine music gives a preponderant role to the *natural diatonic scale* (the supposed Aristoxenean scale) whose steps are in the following ratios to the first note: 1, 9/8, 5/4, 4/3, 27/16, 15/8, 2 (in segments 0, 12, 22, 30, 42, 54, 64, 72; or 0, 12, 23, 30, 42, 54, 65, 72). The degrees of this scale bear the alphabetical names *A*, *B*, Γ , Δ , *E*, *Z*, and *H*. Δ is the lowest note and corresponds roughly to *G*₂. This scale was propounded at least as far back as the first century by Didymos, and in the second century by Ptolemy, who permuted one term and recorded the shift of the tetrachord (tone-tone-semitone), which has remained unchanged ever since.¹³ But apart from this dia pason (octave) attraction, the musical architecture is hierarchical and "nested" as in Aristoxenos, as follows:

A. The *primary order* is based on the three tones 9/8, 10/9, 16/15, a

supermajor tone 7/6, the trihemitone 6/5, another major tone 15/14, the semitone or leima 256/243, the apotome of the minor tone 135/128, and finally the comma 81/80. This complexity results from the mixture of the two means of calculation.

B. The *secondary order* consists of the tetrachords, as defined in Aristoxenos, and similarly the pentachords and the octochords. The tetrachords are divided into three genera:

1. Diatonic, subdivided into: first scheme, $12 + 11 + 7 = 30$ segments, or $(9/8)(10/9)(16/15) = 4/3$, starting on Δ , *H*, etc; second scheme, $11 + 7 + 12 = 30$ segments, or $(10/9)(16/15)(9/8) = 4/3$, starting on *E*, *A*, etc; third scheme, $7 + 12 + 11 = 30$ segments, or $(16/15)(9/8)(10/9) = 4/3$, starting on *Z*, etc. Here we notice a developed combinatorial method that is not evident in Aristoxenos; only three of the six possible permutations of the three notes are used.

2. Chromatic, subdivided into:¹⁴ a. soft chromatic, derived from the diatonic tetrachords of the first scheme, $7 + 16 + 7 = 30$ segments, or $(16/15)(7/6)(15/14) = 4/3$, starting on Δ , *H*, etc.; b. syntomon, or hard chromatic, derived from the diatonic tetrachords of the second scheme, $5 + 19 + 6 = 30$ segments, or $(256/243)(6/5)(135/128) = 4/3$, starting on *E*, *A*, etc.

3. Enharmonic, derived from the diatonic by alteration of the mobile notes and subdivided into: first scheme, $12 + 12 + 6 = 30$ segments, or $(9/8)(9/8)(256/243) = 4/3$, starting on *Z*, *H*, Γ , etc.; second scheme, $12 + 6 + 12 = 30$ segments, or $(9/8)(256/243)(9/8) = 4/3$, starting on Δ , *H*, *A*, etc.; third scheme, $6 + 12 + 12 = 30$ segments, or $(256/243)(9/8)(9/8) = 4/3$, starting on *E*, *A*, *B*, etc.

PARENTHESIS

We can see a phenomenon of absorption of the ancient enharmonic by the diatonic. This must have taken place during the first centuries of Christianity, as part of the Church fathers' struggle against paganism and certain of its manifestations in the arts. The diatonic had always been considered sober, severe, and noble, unlike the other types. In fact the chromatic genus, and especially the enharmonic, demanded a more advanced musical culture, as Aristoxenos and the other theoreticians had already pointed out, and such a culture was even scarcer among the masses of the Roman period. Consequently combinatorial speculations on the one hand and practical usage on the other must have caused the specific characteristics of the enharmonic to disappear in favor of the chromatic, a subdivision of which fell

away in Byzantine music, and of the syntonon diatonic. This phenomenon of absorption is comparable to that of the scales (or modes) of the Renaissance by the major diatonic scale, which perpetuates the ancient syntonon diatonic.

However, this simplification is curious and it would be interesting to study the exact circumstances and causes. Apart from differences, or rather variants of ancient intervals, Byzantine typology is built strictly on the ancient. It builds up the next stage with tetrachords, using definitions which singularly shed light on the theory of the Aristoxenean systems; this was expounded in some detail by Ptolemy.¹⁵

THE SCALES

C. The *tertiary order* consists of the scales constructed with the help of systems having the same ancient rules of consonance, dissonance, and assonance (paraphonia). In Byzantine music the principle of iteration and juxtaposition of the system leads very clearly to scales, a development which is still fairly obscure in Aristoxenos and his successors, except for Ptolemy. Aristoxenos seems to have seen the system as a category and end in itself, and the concept of the scale did not emerge independently from the method which gave rise to it. In Byzantine music, on the other hand, the system was called a method of constructing scales. It is a sort of iterative operator, which starts from the lower category of tetrachords and their derivatives, the pentachord and the octochord, and builds up a chain of more complex organisms, in the same manner as chromosomes based on genes. From this point of view, *system-scale* coupling reached a stage of fulfillment that had been unknown in ancient times. The Byzantines defined the system as the simple or multiple repetition of two, several, or all the notes of a scale. "Scale" here means a succession of notes that is already organized, such as the tetrachord or its derivatives. Three systems are used in Byzantine music:

- the octachord or dia pason
- the pentachord or wheel (trochos)
- the tetrachord or triphony.

The system can unite elements by conjunct (synimenon) or disjunct (diazugmenon) juxtaposition. The disjunct juxtaposition of two tetrachords one tone apart form the dia pason scale spanning a perfect octave. The conjunct juxtaposition of several of these perfect octave dia pason leads to the scales and modes with which we are familiar. The conjunct juxtaposition of several tetrachords (triphony) produces a scale in which the

octave is no longer a fixed sound in the tetrachord but one of its mobile sounds. The same applies to the conjunct juxtaposition of several pentachords (trochos).

The system can be applied to the three genera of tetrachords and to each of their subdivisions, thus creating a very rich collection of scales. Finally one may even mix the genera of tetrachords in the same scale (as in the selidia of Ptolemy), which will result in a vast variety. Thus the scale order is the product of a combinatorial method—indeed, of a gigantic montage (harmony)—by iterative juxtapositions of organisms that are already strongly differentiated, the tetrachords and their derivatives. The scale as it is defined here is a richer and more universal conception than all the impoverished conceptions of medieval and modern times. From this point of view, it is not the tempered scale so much as the absorption by the diatonic tetrachord (and its corresponding scale) of all the other combinations or montages (harmonies) of the other tetrachords that represents a vast loss of potential. (The diatonic scale is derived from a disjunct system of two diatonic tetrachords separated by a whole tone, and is represented by the white keys on the piano.) It is this potential, as much sensorial as abstract, that we are seeking here to reinstate, albeit in a modern way, as will be seen.

The following are examples of scales in segments of Byzantine tempering (or Aristoxenean, since the perfect fourth is equal to 30 segments):

Diatonic scales. Diatonic tetrachords: system by disjunct tetrachords, 12, 11, 7; 12; 11, 7, 12, starting on the lower Δ , 12, 11, 7; 12; 12, 11, 7, starting on the lower H or A ; system by tetrachord and pentachord, 7, 12, 11; 7, 12, 12, 11, starting on the lower Z ; wheel system (trochos), 11, 7, 12, 12; 11, 7, 12, 12; 11, 7, 12, 12; etc.

Chromatic scales. Soft chromatic tetrachords: wheel system starting on H , 7, 16, 7, 12; 7, 16, 7, 12; 7, 16, 7, 12; etc.

Enharmonic scales. Enharmonic tetrachords, second scheme: system by disjunct tetrachords, starting on Δ , 12, 6, 12; 12; 6, 12, corresponding to the mode produced by all the white keys starting with D . The enharmonic scales produced by the disjunct system form all the ecclesiastical scales or modes of the West, and others, for example: chromatic tetrachord, first scheme, by the triphonic system, starting on low H : 12, 12, 6; 12, 12, 6; 12, 6; 12, 12, 6.

Mixed scales. Diatonic tetrachords, first scheme + soft chromatic; disjunct system, starting on low H , 12, 11, 7; 12; 7, 16, 7. Hard chromatic tetrachord + soft chromatic; disjunct system, starting on low H , 5, 19, 6; 12; 7, 16, 7; etc. All the montages are not used, and one can observe the

phenomenon of the absorption of imperfect octaves by the perfect octave by virtue of the basic rules of consonance. This is a limiting condition.

D. The *quaternary order* consists of the tropes or echoi (ichi). The echoi is defined by:

- the genera of tetrachords (or derivatives) constituting it
- the system of juxtaposition
- the attractions
- the bases or fundamental notes
- the dominant notes
- the termini or cadences (katalixis)
- the apichima or melodies introducing the mode
- the ethos, which follows ancient definitions.

We shall not concern ourselves with the details of this quaternary order.

Thus we have succinctly expounded our analysis of the outside-time structure of Byzantine music.

THE METABOLAE

But this outside-time structure could not be satisfied with a compartmentalized hierarchy. It was necessary to have free circulation between the notes and their subdivisions, between the kinds of tetrachords, between the genera, between the systems, and between the echoi—hence the need for a sketch of the in-time structure, which we will now look at briefly. There exist operative signs which allow alterations, transpositions, modulations, and other transformations (metabolae). These signs are the phthorai and the chroai of notes, tetrachords, systems (or scales), and echoi.

Note metabolae

The metathesis: transition from a tetrachord of 30 segments (perfect fourth) to another tetrachord of 30 segments.

The parachordi: distortion of the interval corresponding to the 30 segments of a tetrachord into a larger interval and vice versa; or again, transition from one distorted tetrachord to another distorted tetrachord.

Genus Metabolae

Phthora characteristic of the genus, not changing note names

Changing note names

Using the parachordi

Using the chroai.

System metabolae

Transition from one system to another using the above metabolae.

Echos metabolae using special signs, the martyrikai phthorai or alterations of the mode initialization.

Because of the complexity of the metabolae, pedal notes (isokratima) cannot be “trusted to the ignorant.” Isokratima constitutes an art in itself, for its function is to emphasize and pick out all the in-time fluctuations of the outside-time structure that marks the music.

First Comments

It can easily be seen that the consummation of this outside-time structure is the most complex and most refined thing that could be invented by monody. What could not be developed in polyphony has been brought to such luxuriant fruition that to become familiar with it requires many years of practical studies, such as those followed by the vocalists and instrumentalists of the high cultures of Asia. It seems, however, that none of the specialists in Byzantine music recognize the importance of this structure. It would appear that interpreting ancient systems of notation has claimed their attention to such an extent that they have ignored the living tradition of the Byzantine Church and have put their names to incorrect assertions. Thus it was only a few years ago that one of them¹⁶ took the line of the Gregorian specialists in attributing to the echoi characteristics other than those of the oriental scales which had been taught them in the conformist schools. They have finally discovered that the echoi contained certain characteristic melodic formulae, though of a sedimentary nature. But they have not been able or willing to go further and abandon their soft refuge among the manuscripts.

Lack of understanding of ancient music,¹⁷ of both Byzantine and Gregorian origin, is doubtless caused by the blindness resulting from the growth of polyphony, a highly original invention of the barbarous and uncultivated Occident following the schism of the churches. The passing of centuries and the disappearance of the Byzantine state have sanctioned this neglect and this severance. Thus the effort to feel a “harmonic” language that is much more refined and complex than that of the syntomon diatonic and its scales in octaves is perhaps beyond the usual ability of a Western music specialist, even though the music of our own day may have been able to liberate him partly from the overwhelming dominance of diatonic thinking. The only exceptions are the specialists in the music of the Far East,¹⁸ who have always remained in close contact with musical practice and, dealing as they were with living music, have been able to look for a harmony other than the tonal harmony with twelve semitones. The height of error is to be found in the transcriptions of Byzantine melodies¹⁹ into Western notation using the tempered system. Thus, thousands of transcribed melodies are completely

wrong! But the real criticism one must level at the Byzantinists is that in remaining aloof from the great musical tradition of the eastern church, they have ignored the existence of this abstract and sensual architecture, both complex and remarkably interlocking (harmonious), this developed remnant and genuine achievement of the Hellenic tradition. In this way they have retarded the progress of musicological research in the areas of:

antiquity
plainchant

folk music of European lands, notably in the East²⁰

musical cultures of the civilizations of other continents

better understanding of the musical evolution of Western Europe from the middle ages up to the modern period

the syntactical prospects for tomorrow's music, its enrichment, and its survival.

Second Comments

I am motivated to present this architecture, which is linked to antiquity and doubtless to other cultures, because it is an elegant and lively witness to what I have tried to define as an outside-time category, algebra, or structure of music, as opposed to its other two categories, in-time and temporal. It has often been said (by Stravinsky, Messiaen, and others) that in music time is *everything*. Those who express this view forget the basic structures on which personal languages, such as "pre- or post-Webernian" serial music, rest, however simplified they may be. In order to understand the universal past and present, as well as prepare the future, it is necessary to distinguish structures, architectures, and sound organisms from their temporal manifestations. It is therefore necessary to take "snapshots," to make a series of veritable tomographies over time, to compare them and bring to light their relations and architectures, and vice versa. In addition, thanks to the metrical nature of time, one can furnish it too with an outside-time structure, leaving its true, unadorned nature, that of immediate reality, of instantaneous becoming, in the final analysis, to the temporal category alone.

In this way, time could be considered as a blank blackboard, on which symbols and relationships, architectures and abstract organisms are inscribed. The clash between organisms and architectures and instantaneous immediate reality gives rise to the primordial quality of the living consciousness.

The architectures of Greece and Byzantium are concerned with the pitches (the dominant character of the simple sound) of sound entities.

Here rhythms are also subjected to an organization, but a much simpler one. Therefore we shall not refer to it. Certainly these ancient and Byzantine models cannot serve as examples to be imitated or copied, but rather to exhibit a fundamental outside-time architecture which has been thwarted by the temporal architectures of modern (post-medieval) polyphonic music. These systems, including those of serial music, are still a somewhat confused magma of temporal and outside-time structures, for no one has yet thought of unravelling them. However we cannot do this here.

Progressive Degradation of Outside-Time Structures

The tonal organization that has resulted from venturing into polyphony and neglecting the ancients has leaned strongly, by virtue of its very nature, on the temporal category, and defined the hierarchies of its harmonic functions as the in-time category. Outside-time is appreciably poorer, its "harmonics" being reduced to a single octave scale (*C* major on the two bases *C* and *A*), corresponding to the synteton diatonic of the Pythagorean tradition or to the Byzantine enharmonic scales based on two disjunct tetrachords of the first scheme (for *C*) and on two disjunct tetrachords of the second and third scheme (for *A*). Two metabolae have been preserved: that of transposition (shifting of the scale) and that of modulation, which consists of transferring the base onto steps of the same scale. Another loss occurred with the adoption of the crude tempering of the semitone, the twelfth root of two. The consonances have been enriched by the interval of the third, which, until Debussy, had nearly ousted the traditional perfect fourths and fifths. The final stage of the evolution, atonality, prepared by the theory and music of the romantics at the end of the nineteenth and the beginning of the twentieth centuries, practically abandoned all outside-time structure. This was endorsed by the dogmatic suppression of the Viennese school, who accepted only the ultimate total time ordering of the tempered chromatic scale. Of the four forms of the series, only the inversion of the intervals is related to an outside-time structure. Naturally the loss was felt, consciously or not, and symmetric relations between intervals were grafted onto the chromatic total in the choice of the notes of the series, but these always remained in the in-time category. Since then the situation has barely changed in the music of the post-Webernians. This degradation of the outside-time structures of music since late medieval times is perhaps the most characteristic fact about the evolution of Western European music, and it has led to an unparalleled excrescence of temporal and in-time structures. In this lies its originality and its contribution to the universal culture. But herein also lies its impoverishment, its loss of vitality, and also an apparent

risk of reaching an impasse. For as it has thus far developed, European music is ill-suited to providing the world with a field of expression on a planetary scale, as a universality, and risks isolating and severing itself from historical necessities. We must open our eyes and try to build bridges towards other cultures, as well as towards the immediate future of musical thought, before we perish suffocating from electronic technology, either at the instrumental level or at the level of composition by computers.

Reintroduction of the Outside-Time Structure by Stochastics

By the introduction of the calculation of probability (stochastic music) the present small horizon of outside-time structures and asymmetries was completely explored and enclosed. But by the very fact of its introduction, stochastics gave an impetus to musical thought that carried it over this enclosure towards the clouds of sound events and towards the plasticity of large numbers articulated statistically. There was no longer any distinction between the vertical and the horizontal, and the indeterminism of in-time structures made a dignified entry into the musical edifice. And, to crown the Herakleitean dialectic, indeterminism, by means of particular stochastic functions, took on color and structure, giving rise to generous possibilities of organization. It was able to include in its scope determinism and, still somewhat vaguely, the outside-time structures of the past. The categories outside-time, in-time, and temporal, unequally amalgamated in the history of music, have suddenly taken on all their fundamental significance and for the first time can build a coherent and universal synthesis in the past, present, and future. This is, I insist, not only a possibility, but even a direction having priority. But as yet we have not managed to proceed beyond this stage. To do so we must add to our arsenal sharper tools, trenchant axiomatics and formalization.

SIEVE THEORY

It is necessary to give an axiomatization for the totally ordered structure (additive group structure = additive Aristoxenean structure) of the tempered chromatic scale.²¹ The axiomatics of the tempered chromatic scale is based on Peano's axiomatics of numbers:

Preliminary terms. O = the stop at the origin; n = a stop; n' = a stop resulting from elementary displacement of n ; D = the set of values of the particular sound characteristic (pitch, density, intensity, instant, speed, disorder . . .). The values are identical with the stops of the displacements.

First propositions (axioms).

1. Stop O is an element of D .
2. If stop n is an element of D then the new stop n' is an element of D .
3. If stops n and m are elements of D then the new stops n' and m' are identical if, and only if, stops n and m are identical.
4. If stop n is an element of D , it will be different from stop O at the origin.
5. If elements belonging to D have a special property P , such that stop O also has it, and if, for every element n of D having this property the element n' has it also, all the elements of D will have the property P .

We have just defined axiomatically a tempered chromatic scale not only of pitch, but also of all the sound properties or characteristics referred to above in D (density, intensity . . .). Moreover, this abstract scale, as Bertrand Russell has rightly observed, à propos the axiomatics of numbers of Peano, has no unitary displacement that is either predetermined or related to an absolute size. Thus it may be constructed with tempered semitones, with Aristoxenean segments (twelfth-tones), with the commas of Didymos (81/80), with quarter-tones, with whole tones, thirds, fourths, fifths, octaves, etc. or with any other unit that is not a factor of a perfect octave.

Now let us define another equivalent scale based on this one but having a unitary displacement which is a multiple of the first. It can be expressed by the concept of *congruence modulo m*.

Definition. Two integers x and n are said to be *congruent modulo m* when m is a factor of $x - n$. It may be expressed as follows: $x \equiv n \pmod{m}$. Thus, two integers are congruent modulo m when and only when they differ by an exact (positive or negative) multiple of m ; e.g., $4 \equiv 19 \pmod{5}$, $3 \equiv 13 \pmod{8}$, $14 \equiv 0 \pmod{7}$.

Consequently, every integer is congruent modulo m with one and with only one value of n :

$$n = (0, 1, 2, \dots, m-2, m-1).$$

Of each of these numbers it is said that it forms a *residual class modulo m*; they are, in fact, the smallest non-negative residues modulo m . $x \equiv n \pmod{m}$ is thus equivalent to $x = n + km$, where k is an integer.

$$k \in \mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \dots\}.$$

For a given n and for any $k \in \mathbb{Z}$, the numbers x will belong by definition to the residual class n modulo m . This class can be denoted m_n .

In order to grasp these ideas in terms of music, let us take the tempered

semitone of our present-day scale as the unit of displacement. To this we shall again apply the above axiomatics, with say a value of 4 semitones (major third) as the elementary displacement.²² We shall define a new chromatic scale. If the stop at the origin of the first scale is a $D\sharp$, the second scale will give us all the multiples of 4 semitones, in other words a "scale" of major thirds: $D\sharp, G, B, D'\sharp, G', B'$; these are the notes of the first scale whose order numbers are congruent with 0 modulo 4. They all belong to the residual class 0 modulo 4. The residual classes 1, 2, and 3 modulo 4 will use up all the notes of this chromatic total. These classes may be represented in the following manner:

residual class 0 modulo 4: 4_0
 residual class 1 modulo 4: 4_1
 residual class 2 modulo 4: 4_2
 residual class 3 modulo 4: 4_3
 residual class 4 modulo 4: 4_0 , etc.

Since we are dealing with a sieving of the basic scale (elementary displacement by one semitone), each residual class forms a sieve allowing certain elements of the chromatic continuity to pass through. By extension the chromatic total will be represented as sieve 1_0 . The scale of fourths will be given by sieve 5_n , in which $n = 0, 1, 2, 3, 4$. Every change of the index n will entail a transposition of this gamut. Thus the Debussian whole-tone scale, 2_n with $n = 0, 1$, has two transpositions:

$$2_0 \rightarrow C, D, E, F\sharp, G\sharp, A\sharp, C \dots$$

$$2_1 \rightarrow C\sharp, D\sharp, F, G, A, B, C\sharp \dots$$

Starting from these elementary sieves we can build more complex scales—all the scales we can imagine—with the help of the three operations of the Logic of Classes: union (disjunction) expressed as \vee , intersection (conjunction) expressed as \wedge , and complementation (negation) expressed as a bar inscribed over the modulo of the sieve. Thus

$$2_0 \vee 2_1 = \text{chromatic total (also expressible as } 1_0)$$

$$2_0 \wedge 2_1 = \text{no notes, or empty sieve, expressed as } \emptyset$$

$$\bar{2}_0 = 2_1 \text{ and } \bar{2}_1 = 2_0.$$

The major scale can be written as follows:

$$(\bar{3}_2 \wedge 4_0) \vee (\bar{3}_1 \wedge 4_1) \vee (3_2 \wedge 4_2) \vee (\bar{3}_0 \wedge 4_3).$$

By definition, this notation does not distinguish between all the modes on the white keys of the piano, for what we are defining here is the scale; modes are the architectures founded on these scales. Thus the white-key mode D , starting on D , will have the same notation as the C mode. But in order to distinguish the modes it would be possible to introduce non-commutativity in the logical expressions. On the other hand each of the 12 transpositions of this scale will be a combination of the cyclic permutations of the indices of sieves modulo 3 and 4. Thus the major scale transposed a semitone higher (shift to the right) will be written

$$(\bar{3}_0 \wedge 4_1) \vee (\bar{3}_2 \wedge 4_2) \vee (3_0 \wedge 4_3) \vee (\bar{3}_1 \wedge 4_0),$$

and in general

$$(\bar{3}_{n+2} \wedge 4_n) \vee (\bar{3}_{n+1} \wedge 4_{n+1}) \vee (3_{n+2} \wedge 4_{n+2}) \vee (\bar{3}_n \wedge 4_{n+3}),$$

where n can assume any value from 0 to 11, but reduced after the addition of the constant index of each of the sieves (moduli), modulo the corresponding sieve. The scale of D transposed onto C is written

$$(3_n \wedge 4_n) \vee (\bar{3}_{n+1} \wedge 4_{n+1}) \vee (\bar{3}_n \wedge 4_{n+2}) \vee (\bar{3}_{n+2} \wedge 4_{n+3}).$$

Musicology

Now let us change the basic unit (elementary displacement ELD) of the sieves and use the quarter-tone. The major scale will be written

$$(8_n \wedge \bar{3}_{n+1}) \vee (8_{n+2} \wedge \bar{3}_{n+2}) \vee (8_{n+4} \wedge 3_{n+1}) \vee (8_{n+6} \wedge \bar{3}_n),$$

with $n = 0, 1, 2, \dots, 23$ (modulo 3 or 8). The same scale with still finer sieving (one octave = 72 Aristoxenean segments) will be written

$$(8_n \wedge (9_n \vee 9_{n+6})) \vee (8_{n+2} \wedge (9_{n+3} \vee 9_{n+6})) \vee (8_{n+4} \wedge 9_{n+3})$$

$$\vee (8_{n+6} \wedge (9_n \vee 9_{n+3})),$$

with $n = 0, 1, 2, \dots, 71$ (modulo 8 or 9).

One of the mixed Byzantine scales, a disjunct system consisting of a chromatic tetrachord and a diatonic tetrachord, second scheme, separated by a major tone, is notated in Aristoxenean segments as 5, 19, 6; 12; 11, 7, 12, and will be transcribed logically as

$$(8_n \wedge (9_n \vee 9_{n+6})) \vee (9_{n+6} \wedge (8_{n+2} \vee 8_{n+4}))$$

$$\vee (8_{n+5} \wedge (9_{n+5} \vee 9_{n+8})) \vee (8_{n+6} \vee 9_{n+3}),$$

with $n = 0, 1, 2, \dots, 71$ (modulo 8 or 9).

The Raga Bhairavi of the Andara-Sampurna type (pentatonic ascending, heptatonic descending),²³ expressed in terms of an Aristoxenean basic sieve (comprising an octave, periodicity 72), will be written as:
Pentatonic scale:

$$(8_n \wedge (9_n \vee 9_{n+3})) \vee (8_{n+2} \wedge (9_n \vee 9_{n+6})) \vee (8_{n+6} \wedge 9_{n+3})$$

Heptatonic scale:

$$(8_n \wedge (9_n \vee 9_{n+3})) \vee (8_{n+2} \wedge (9_n \vee 9_{n+6})) \vee (8_{n+4} \wedge (9_{n+4} \vee 9_{n+8})) \\ \vee (8_{n+6} \wedge (9_{n+3} \vee 9_{n+6}))$$

with $n = 0, 1, 2, \dots, 71$ (modulo 8 or 9).

These two scales expressed in terms of a sieve having as its elementary displacement, ELD, the comma of Didymos, $ELD = 81/80$ ($81/80$ to the power $55.8 = 2$), thus having an octave periodicity of 56, will be written as:
Pentatonic scale:

$$(7_n \wedge (8_n \vee 8_{n+6})) \vee (7_{n+2} \wedge (8_{n+5} \vee 8_{n+7})) \vee (7_{n+5} \wedge 8_{n+1})$$

Heptatonic scale:

$$(7_n \wedge (8_n \vee 8_{n+6})) \vee (7_{n+2} \wedge (8_{n+5} \vee 8_{n+7})) \vee (7_{n+3} \wedge 8_{n+3}) \\ \vee (7_{n+4} \wedge (8_{n+4} \vee 8_{n+6})) \vee (7_{n+5} \wedge 8_{n+1})$$

for $n = 0, 1, 2, \dots, 55$ (modulo 7 or 8).

We have just seen how the sieve theory allows us to express any scale in terms of logical (hence mechanizable) functions, and thus unify our study of the structures of superior range with that of the total order. It can be useful in entirely new constructions. To this end let us imagine complex, non-octave-forming sieves.²⁴ Let us take as our sieve unit a tempered quarter-tone. An octave contains 24 quarter-tones. Thus we have to construct a compound sieve with a periodicity other than 24 or a multiple of 24, thus a periodicity non-congruent with $k \cdot 24$ modulo 24 (for $k = 0, 1, 2, \dots$). An example would be any logical function of the sieve of moduli 11 and 7 (periodicity $11 \times 7 = 77 \neq k \cdot 24$), $(11_n \vee 11_{n+1}) \wedge 7_{n+6}$. This establishes an asymmetric distribution of the steps of the chromatic quarter-tone scale. One can even use a compound sieve which throws periodicity outside the limits of the audible area; for example, any logical function of modules 17 and 18 ($f[17, 18]$), for $17 \times 18 = 306 > (11 \times 24)$.

Suprastructures

One can apply a stricter structure to a compound sieve or simply leave the choice of elements to a stochastic function. We shall obtain a statistical

coloration of the chromatic total which has a higher level of complexity.

Using metabolae. We know that at every cyclic combination of the sieve indices (transpositions) and at every change in the module or moduli of the sieve (modulation) we obtain a metabola. As examples of metabolic transformations let us take the smallest residues that are prime to a positive number r . They will form an Abelian (commutative) group when the composition law for these residues is defined as multiplication with reduction to the least positive residue with regard to r . For a numerical example let $r = 18$; the residues 1, 5, 7, 11, 13, 17 are primes to it, and their products after reduction modulo 18 will remain within this group (closure). The finite commutative group they form can be exemplified by the following fragment:

$$5 \times 7 = 35; 35 - 18 = 17; \\ 11 \times 11 = 121; 121 - (6 \times 18) = 13; \text{etc.}$$

Modules 1, 7, 13 form a cyclic sub-group of order 3. The following is a logical expression of the two sieves having modules 5 and 13:

$$L(5, 13) = \overline{(13_{n+4} \vee 13_{n+5} \vee 13_{n+7} \vee 13_{n+9})} \\ \wedge 5_{n+1} \vee \overline{(5_{n+2} \vee 5_{n+4})} \wedge 13_{n+9} \vee 13_{n+6}.$$

One can imagine a transformation of modules in pairs, starting from the Abelian group defined above. Thus the cinematic diagram (in-time) will be

$$L(5, 13) \rightarrow L(11, 17) \rightarrow L(7, 11) \rightarrow L(5, 1) \rightarrow L(5, 5) \rightarrow \dots \rightarrow L(5, 13)$$

so as to return to the initial term (closure).²⁵

This sieve theory can be put into many kinds of architecture, so as to create included or successively intersecting classes, thus stages of increasing complexity; in other words, orientations towards increased determinisms in selection, and in topological textures of neighborhood.

Subsequently we can put into in-time practice this veritable histology of outside-time music by means of temporal functions, for instance by giving functions of change—of indices, moduli, or unitary displacement—in other words, encased logical functions parametric with time.

Sieve theory is very general and consequently is applicable to any other sound characteristics that may be provided with a totally ordered structure, such as intensity, instants, density, degrees of order, speed, etc. I have already said this elsewhere, as in the axiomatics of sieves. But this method can be applied equally to visual scales and to the optical arts of the future.

Moreover, in the immediate future we shall witness the exploration of

this theory and its widespread use with the help of computers, for it is entirely mechanizable. Then, in a subsequent stage, there will be a study of partially ordered structures, such as are to be found in the classification of timbres, for example, by means of lattice or graph techniques.

Conclusion

I believe that music today could surpass itself by research into the outside-time category, which has been atrophied and dominated by the temporal category. Moreover this method can unify the expression of fundamental structures of all Asian, African, and European music. It has a considerable advantage: its mechanization—hence tests and models of all sorts can be fed into computers, which will effect great progress in the musical sciences.

In fact, what we are witnessing is an industrialization of music which has already started, whether we like it or not. It already floods our ears in many public places, shops, radio, TV, and airlines, the world over. It permits a consumption of music on a fantastic scale, never before approached. But this music is of the lowest kind, made from a collection of outdated clichés from the dregs of the musical mind. Now it is not a matter of stopping this invasion, which, after all, increases participation in music, even if only passively. It is rather a question of effecting a qualitative conversion of this music by exercising a radical but constructive critique of our ways of thinking and of making music. Only in this way, as I have tried to show in the present study, will the musician succeed in dominating and transforming this poison that is discharged into our ears, and only if he sets about it without further ado. But one must also envisage, and in the same way, a radical conversion of musical education, from primary studies onwards, throughout the entire world (all national councils for music take note). Non-decimal systems and the logic of classes are already taught in certain countries, so why not their application to a new musical theory, such as is sketched out here?

Chapter VIII

Towards a Philosophy of Music

PRELIMINARIES

We are going to attempt briefly: 1. an “unveiling of the historical tradition” of music,¹ and 2. to construct a music.

“Reasoning” about phenomena and their explanation was the greatest step accomplished by man in the course of his liberation and growth. This is why the Ionian pioneers—Thales, Anaximander, Anaximenes—must be considered as the starting point of our truest culture, that of “reason.” When I say “reason,” it is not in the sense of a logical sequence of arguments, syllogisms, or logico-technical mechanisms, but that very extraordinary quality of feeling an uneasiness, a curiosity, then of applying the question, *ελεγχός*. It is, in fact, impossible to imagine this advance, which, in Ionia, created cosmology from nothing, in spite of religions and powerful mystiques, which were early forms of “reasoning.” For example, Orphism, which so influenced Pythagorism, taught that the human soul is a fallen god, that only *ek-stasis*, the departure from self, can reveal its true nature, and that with the aid of purifications (*καθαρμοί*) and sacraments (*օρπυα*) it can regain its lost position and escape the *Wheel of Birth* (*τροχός γενέσεως, bhavachakra*) that is to say, the fate of reincarnations as an animal or vegetable. I am citing this mystique because it seems to be a very old and widespread form of thought, which existed independently about the same time in the Hinduism of India.²

Above all, we must note that the opening taken by the Ionians has finally surpassed all mystiques and all religions, including Christianity.

English translation of Chapter VIII by John and Amber Challifour.

Never has the spirit of this philosophy been as universal as today: The U.S., China, U.S.S.R., and Europe, the present principal protagonists, restate it with a homogeneity and a uniformity that I would even dare to qualify as disturbing.

Having been established, the question (*έλεγχος*) embodied a Wheel of Birth *sui generis*, and the various pre-Socratic schools flourished by conditioning all further development of philosophy until our time. Two are in my opinion the high points of this period: the Pythagorean concept of numbers and the Parmenidean dialectics—both unique expressions of the same preoccupation.

As it went through its phases of adaptation, up to the fourth century B.C., the Pythagorean concept of numbers affirmed that things are numbers, or that all things are furnished with numbers, or that things are similar to numbers. This thesis developed (and this in particular interests the musician) from the study of musical intervals in order to obtain the orphic catharsis, for according to Aristoxenos, the Pythagoreans used music to cleanse the soul as they used medicine to cleanse the body. This method is found in other *orgia*, like that of Korybantes, as confirmed by Plato in the *Laws*. In every way, Pythagorism has permeated all occidental thought, first of all, Greek, then Byzantine, which transmitted it to Western Europe and to the Arabs.

All musical theorists, from Aristoxenos to Hucbald, Zarlino, and Rameau, have returned to the same theses colored by expressions of the moment. But the most incredible is that all intellectual activity, including the arts, is actually immersed in the world of numbers (I am omitting the few backward-looking or obscurantist movements). We are not far from the day when genetics, thanks to the geometric and combinatorial structure of DNA, will be able to metamorphise the Wheel of Birth at will, as we wish it, and as preconceived by Pythagoras. It will not be the *ek-stasis* (Orphic, Hindu, or Taoist) that will have arrived at one of the supreme goals of all time, that of controlling the quality of reincarnations (hereditary rebirths *παλιγγενεσία*) but the very force of the “theory,” of the question, which is the essence of human action, and whose most striking expression is Pythagorism. We are all Pythagoreans.³

On the other hand, Parmenides was able to go to the heart of the question of change by denying it, in contrast to Herakleitos. He discovered the principle of the excluded middle and logical tautology, and this created such a dazzlement that he used them as a means of cutting out, in the evanescent change of senses, the notion of Being, of that which is, one, motionless, filling the universe, without birth and indestructible; the

not-Being, not existing, circumscribed, and spherical (which Melissos had not understood).

[F]or it will be forever impossible to prove that things that are not are; but restrain your thought from this route of inquiry. . . . Only one way remains for us to speak of, namely, *that it is*; on this route there are many signs indicating that it is uncreated and indestructible, for it is complete, undisturbed, and without end; it never was, nor will it be, for now it is all at once complete, one, continuous; for what kind of birth are you seeking for it? How and from where could it grow? I will neither let you say nor think that it came from what is not; for it is unutterable and unthinkable that a thing is not. And what need would have led it to be created sooner or later if it came from nothing? Therefore it must be, absolutely, or not at all.

—Fragments 7 and 8 of *Poem*, by Parmenides⁴

Besides the abrupt and compact style of the thought, the method of the question is absolute. It leads to denial of the sensible world, which is only made of contradictory appearances that “two-faced” mortals accept as valid without turning a hair, and to stating that the only truth is the notion of reality itself. But this notion, substantiated with the help of abstract logical rules, needs no other concept than that of its opposite, the not-Being, the nothing that is immediately rendered impossible to formulate and to conceive.

This concision and this axiomatics, which surpasses the deities and cosmogonies fundamental to the first elements,⁵ had a tremendous influence on Parmenides’ contemporaries. This was the first absolute and complete materialism. Immediate repercussions were, in the main, the continuity of Anaxagoras and the atomic discontinuity of Leukippos. Thus, all intellectual action until our time has been profoundly imbued with this strict axiomatics. The principle of the conservation of energy in physics is remarkable. Energy is that which fills the universe in electromagnetic, kinetic, or material form by virtue of the equivalence matter—energy. It has become *that which is* “par excellence.” Conservation implies that it does not vary by a single photon in the entire universe and that it has been thus throughout eternity. On the other hand, by the same reasoning, the logical truth is tautological: All that which is affirmed is a truth to which no alternative is conceivable (Wittgenstein). Modern knowledge accepts the void, but is it truly a non-Being? Or simply the designation of an unclarified complement?

After the failures of the nineteenth century, scientific thought became rather skeptical and pragmatic. It is this fact that has allowed it to adapt

and develop to the utmost. "All happens as if . . ." implies this doubt, which is positive and optimistic. We place a provisional confidence in new theories, but we abandon them readily for more efficacious ones provided that the procedures of action have a suitable explanation which agrees with the whole. In fact, this attitude represents a retreat, a sort of fatalism. This is why today's Pythagorism is relative (exactly like the Parmenidean axiomatics) in all areas, including the arts.

Throughout the centuries, the arts have undergone transformations that paralleled two essential creations of human thought: the hierarchical principle and the principle of numbers. In fact, these principles have dominated music, particularly since the Renaissance, down to present-day procedures of composition. In school we emphasize unity and recommend the unity of themes and of their development; but the serial system imposes another hierarchy, with its own tautological unity embodied in the tone row and in the principle of perpetual variation, which is founded on this tautology . . . —in short, all these axiomatic principles that mark our lives agree perfectly with the inquiry of Being introduced twenty-five centuries ago by Parmenides.

It is not my intention to show that everything has already been discovered and that we are only plagiarists. This would be obvious nonsense. There is never repetition, but a sort of tautological identity throughout the vicissitudes of Being that might have mounted the Wheel of Birth. It would seem that some areas are less mutable than others, and that some regions of the world change very slowly indeed.

The *Poem* of Parmenides implicitly admits that necessity, need, causality, and justice identify with logic; since Being is born from this logic, pure chance is as impossible as not-Being. This is particularly clear in the phrase, "And what need would have led it to be born sooner or later, if it came from nothing?" This contradiction has dominated thought throughout the millennia. Here we approach another aspect of the dialectics, perhaps the most important in the practical plan of action—determinism. If logic indeed implies the absence of chance, then one can know all and even construct everything with logic. The problem of choice, of decision, and of the future, is resolved.

We know, moreover, that if an element of chance enters a deterministic construction all is undone. This is why religions and philosophies everywhere have always driven chance back to the limits of the universe. And what they utilized of chance in divination practices was absolutely not considered as such but as a mysterious web of signs, sent by the divinities (who were often contradictory but who knew well what they wanted), and which

could be read by elect soothsayers. This web of signs can take many forms—the Chinese system of I-Ching, auguries predicting the future from the flight of birds and the entrails of sacrificed animals, even telling fortunes from tea leaves. This inability to admit pure chance has even persisted in modern mathematical probability theory, which has succeeded in incorporating it into some deterministic logical laws, so that *pure chance* and *pure determinism* are only two facets of one entity, as I shall soon demonstrate with an example.

To my knowledge, there is only one "unveiling" of pure chance in all of the history of thought, and it was Epicurus who dared to do it. Epicurus struggled against the deterministic networks of the atomists, Platonists, Aristoteleans, and Stoics, who finally arrived at the negation of free will and believed that man is subject to nature's will. For if all is logically ordered in the universe as well as in our bodies, which are products of it, then our will is subject to this logic and our freedom is nil. The Stoics admitted, for example, that no matter how small, every action on earth had a repercussion on the most distant star in the universe; today we would say that the network of connections is compact, sensitive, and without loss of information.

This period is unjustly slighted, for it was in this time that all kinds of sophisms were debated, beginning with the logical calculus of the Megarians, and it was the time in which the Stoics created the logic called modal, which was distinct from the Aristotelian logic of classes. Moreover, Stoicism, by its moral thesis, its fullness, and its scope, is without doubt basic to the formation of Christianity, to which it has yielded its place, thanks to the substitution of punishment in the person of Christ and to the myth of eternal reward at the Last Judgment—regal solace for mortals.

In order to give an axiomatic and cosmogonical foundation to the proposition of man's free will, Epicurus started with the atomic hypothesis and admitted that "in the straight line fall that transports the atoms across the void, . . . at an undetermined moment the atoms deviate ever so little from the vertical . . . but the deviation is so slight, the least possible, that we could not conceive of even seemingly oblique movements."⁶ This is the theory of *ekklisis* (Lat. *clinamen*) set forth by Lucretius. A senseless principle is introduced into the grand deterministic atomic structure. Epicurus thus based the structure of the universe on determinism (the inexorable and parallel fall of atome) and, at the same time, on indeterminism (*ekklisis*). It is striking to compare his theory with the kinetic theory of gases first proposed by Daniel Bernoulli. It is founded on the corpuscular nature of matter and, at the same time, on determinism and indeterminism. No one but Epicurus had ever thought of utilizing chance as a principle or as a type of behavior.

It was not until 1654 that a doctrine on the use and understanding of chance appeared. Pascal, and especially Fermat, formulated it by studying "games of chance"—dice, cards, etc. Fermat stated the two primary rules of probabilities using multiplication and addition. In 1713 *Ars Conjectandi* by Jacques Bernoulli was published.⁷ In this fundamental work Bernoulli enunciated a universal law, that of Large Numbers. Here it is as stated by E. Borel: "Let p be the probability of the favorable outcome and q the probability of the unfavorable outcome, and let ϵ be a small positive number. The probability that the difference between the observed ratio of favorable events to unfavorable events and the theoretical ratio p/q is larger in absolute value than ϵ will approach zero when the number of trials n becomes infinitely large."⁸ Consider the example of the game of heads and tails. If the coin is perfectly symmetric, that is to say, absolutely true, we know that the probability p of heads (favorable outcome) and the probability q of tails (unfavorable outcome) are each equal to $1/2$, and the ratio p/q to 1. If we toss the coin n times, we will get heads P times and tails Q times, and the ratio P/Q will generally be different from 1. The Law of Large Numbers states that the more we play, that is to say the larger the number n becomes, the closer the ratio P/Q will approach 1.

Thus, Epicurus, *who admits the necessity of birth at an undetermined moment*, in exact contradiction to all thought, even modern, remains an isolated case;* for the aleatory, and truly stochastic event, is the result of an accepted ignorance, as H. Poincaré has perfectly defined it. If probability theory admits an uncertainty about the outcome of each toss, it encompasses this uncertainty in two ways. The first is hypothetical: ignorance of the trajectory produces the uncertainty; the other is deterministic: the Law of Large Numbers removes the uncertainty with the help of time (or of space). However, by examining the coin tossing closely, we will see how the symmetry is strictly bound to the unpredictability. If the coin is perfectly symmetrical, that is, perfectly homogeneous and with its mass uniformly distributed, then the uncertainty⁹ at each toss will be a maximum and the probability for each side will be $1/2$. If we now alter the coin by redistributing the matter unsymmetrically, or by replacing a little aluminum with platinum, which has a specific weight eight times that of aluminum, the coin will tend to land with the heavier side down. The uncertainty will decrease and the probabilities for the two faces will be unequal. When the substitution of material is pushed to the limit, for example, if the aluminum is replaced with a slip of paper and the other side is entirely of platinum, then the uncertainty will approach zero, that is, towards the certainty that

* Except perhaps for Heisenberg.

the coin will land with the lighter side up. Here we have shown the inverse relation between uncertainty and symmetry. This remark seems to be a tautology, but it is nothing more than the mathematical definition of probability: *probability* is the ratio of the number of favorable outcomes to the number of possible outcomes when all outcomes are regarded as equally *likely*. Today, the axiomatic definition of probability does not remove this difficulty, it circumvents it.

MUSICAL STRUCTURES *EX NIHILO*

Thus we are, at this point in the exposition, still immersed in the lines of force introduced twenty-five centuries ago and which continue to regulate the basis of human activity with the greatest efficacy, or so it seems. It is the source of those problems about which we, in the darkness of our ignorance, concern ourselves: determinism or chance,¹⁰ unity of style or eclecticism, calculated or not, intuition or constructivism, a priori or not, a metaphysics of music or music simply as a means of entertainment.

Actually, these are the questions that we should ask ourselves: 1. What consequence does the awareness of the Pythagorean-Parmenidean field have for musical composition? 2. In what ways? To which the answers are: 1. Reflection on *that which is* leads us directly to the reconstruction, as much as possible *ex nihilo*, of the ideas basic to musical composition, and above all to the rejection of every idea that does not undergo the inquiry (*ἐλεγχος, διληψία*). 2. This reconstruction will be prompted by modern axiomatic methods.

Starting from certain premises we should be able to construct the most general musical edifice in which the utterances of Bach, Beethoven, or Schönberg, for example, would be unique realizations of a gigantic virtuality, rendered possible by this axiomatic removal and reconstruction.

It is necessary to divide musical construction into two parts (see Chapters VI and VII): 1. that which pertains to time, a mapping of entities or structures onto the ordered structure of time; and 2. that which is independent of temporal becomingness. There are, therefore, two categories: *in-time* and *outside-time*. Included in the category outside-time are the durations and constructions (relations and operations) that refer to elements (points, distances, functions) that belong to and that can be expressed on the time axis. The temporal is then reserved to the instantaneous creation.

In Chapter VII I made a survey of the structure of monophonic music,

with its rich outside-time combinatory capability, based on the original texts of Aristoxenos of Tarentum and the manuals of actual Byzantine music. This structure illustrates in a remarkable way that which I understand by the category outside-time.

Polyphony has driven this category back into the subconscious of musicians of the European occident, but has not completely removed it; that would have been impossible. For about three centuries after Monteverdi, in-time architectures, expressed chiefly by the tonal (or modal) functions, dominated everywhere in central and occidental Europe. However, it is in France that the rebirth of outside-time preoccupations occurred, with Debussy and his invention of the whole-tone scale. Contact with three of the more conservative traditions of the Orientals was the cause of it: the plainchant, which had vanished, but which had been rediscovered by the abbots at Solesmes; one of the Byzantine traditions, experienced through Moussorgsky; and the Far East.

This rebirth continues magnificently through Messiaen, with his "modes of limited transpositions" and "non-retrogradable rhythms," but it never imposes itself as a general necessity and never goes beyond the framework of the scales. However Messiaen himself abandoned this vein, yielding to the pressure of serial music.

In order to put things in their proper historical perspective, it is necessary to prevail upon more powerful tools such as mathematics and logic and go to the bottom of things, to the structure of musical thought and composition. This is what I have tried to do in Chapters VI and VII and what I am going to develop in the analysis of *Nomos alpha*.

Here, however, I wish to emphasize the fact that it was Debussy and Messiaen¹¹ in France who reintroduced the category outside-time in the face of the general evolution that resulted in its own atrophy, to the advantage of structures in-time.¹² In effect, atonality does away with scales and accepts the outside-time neutrality of the half-tone scale.¹³ (This situation, furthermore, has scarcely changed for fifty years.) The introduction of in-time order by Schönberg made up for this impoverishment. Later, with the stochastic processes that I introduced into musical composition, the hypertrophy of the category in-time became overwhelming and arrived at a dead end. It is in this cul-de-sac that music, abusively called aleatory, improvised, or graphic, is still stirring today.

Questions of choice in the category outside-time are disregarded by musicians as though they were unable to hear, and especially unable to think. In fact, they drift along unconscious, carried away by the agitations of superficial musical fashions which they undergo heedlessly. In depth,

however, the outside-time structures do exist and it is the privilege of man not only to sustain them, but to construct them and to go beyond them.

Sustain them? Certainly; there are basic evidences of this order which will permit us to inscribe our names in the Pythagorean-Parmenidean field and to lay the platform from which our ideas will build bridges of understanding and insight into the past (we are after all products of millions of years of the past), into the future (we are equally products of the future), and into other sonic civilizations, so badly explained by the present-day musicologies, for want of the original tools that we so graciously set up for them.

Two axiomatics will open new doors, as we shall see in the analysis of *Nomos alpha*. We shall start from a naive position concerning the perception of sounds, naive in Europe as well as in Africa, Asia, or America. The inhabitants of all these countries learned tens or hundreds of thousands of years ago to distinguish (if the sounds were neither too long nor too short) such characteristics as pitch, instants, loudness, roughness, rate of change, color, timbre. They are even able to speak of the first three characteristics in terms of intervals.

The first axiomatics leads us to the construction of all possible scales. We will speak of pitch since it is more familiar, but the following arguments will relate to all characteristics which are of the same nature (instants, loudness, roughness, density, degree of disorder, rate of change).

We will start from the obvious assumption that within certain limits men are able to recognize whether two modifications or displacements of pitch are identical. For example, going from *C* to *D* is the same as going from *F* to *G*. We will call this modification elementary displacement, ELD. (It can be a comma, a half tone, an octave, etc.) It permits us to define any *Equally Tempered Chromatic Gamut* as an ETCHG sieve.¹⁴ By modifying the displacement step ELD, we engender a new ETCHG sieve with the same axiomatics. With this material we can go no farther. Here we introduce the three logical operations (Aristotelean logic as seen by Boole) of *conjunction* ("and," intersection, notated \wedge), *disjunction* ("or," union, notated \vee), and *negation* ("no," complement, notated $-$), and use them to create classes of pitch (various ETCHG sieves).

The following is the logical expression with the conventions as indicated in Chapter VII:

The major scale (ELD = $\frac{1}{4}$ tone):

$$(8_n \wedge \bar{3}_{n+1}) \vee (8_{n+2} \wedge 3_{n+2}) \vee (8_{n+4} \wedge 3_{n+1}) \vee (8_{n+6} \wedge \bar{3}_n)$$

where $n = 0, 1, 2, \dots, 23$, modulo 3 or 8.

(It is possible to modify the step ELD by a "rational metabola." Thus the logical function of the major scale with an ELD equal to a quarter-tone can be based on an $ELD = 1/3$ tone or on any other portion of a tone. These two sieves, in turn, could be combined with the three logical operations to provide more complex scales. Finally, "irrational metabolae" of ELD may be introduced, which can only be applied in non-instrumental music. Accordingly, the ELD can be taken from the field of real numbers).

The scale of limited transposition n° 4 of Olivier Messiaen¹⁵ (ELD = 1/2 tone):

$$\begin{aligned} \bar{3}_n \wedge (4_{n+1} \vee 4_{n+3}) \vee \bar{3}_{n+1} \wedge (4_n \vee 4_{n+2}) \\ 4_{n+1} \vee 4_{n+3} \vee \bar{3}_{n+1} \wedge (4_n \vee 4_{n+2}) \end{aligned}$$

where $n = 0, 1, \dots$, modulo 3 or 4.

The second axiomatics leads us to vector spaces and graphic and numerical representations.¹⁶

Two conjunct intervals a and b can be combined by a musical operation to produce a new interval c . This operation is called addition. To either an ascending or a descending interval we may add a second conjunct interval such that the result will be a unison; this second interval is the symmetric interval of the first. Unison is a neutral interval; that is, when it is added to any other interval, it does not modify it. We may also create intervals by association without changing the result. Finally, in composing intervals we can invert the orders of the intervals without changing the result. We have just shown that the naive experience of musicians since antiquity (cf. Aristoxenos) all over the earth attributes the structure of a commutative group to intervals.

Now we are able to combine this group with a field structure. At least two fields are possible: the set of real numbers, R , and the isomorphic set of points on a straight line. It is moreover possible to combine the Abelian group of intervals with the field C of complex numbers or with a field of characteristic P . By definition the combination of the group of intervals with a field forms a vector space in the following manner: As we have just said, interval group G possesses an internal law of composition, addition. Let a and b be two elements of the group. Thus we have:

1. $a + b = c, c \in G$
2. $a + b + c = (a + b) + c = a + (b + c)$ associativity
3. $a + o = o + a$, with $o \in G$ the neutral element (unison)
4. $a + a' = o$, with $a' = -a$ = the symmetric interval of a
5. $a + b = b + a$ commutativity

We note the external composition of elements in G with those in the field C by a dot \cdot . If $\lambda, \mu \in C$ (where C = the field of real numbers) then we have the following properties:

6. $\lambda \cdot a, \mu \cdot a \in G$
7. $1 \cdot a = a \cdot 1 = a$ (1 is the neutral element in C with respect to multiplication)
8. $\lambda \cdot (\mu a) = (\lambda \cdot \mu) \cdot a$ associativity of λ, μ
9. $(\lambda + \mu) \cdot a = \lambda \cdot a + \mu \cdot a$
- $\lambda \cdot (a + b) = \lambda \cdot a + \lambda \cdot b$ distributivity

MUSICAL NOTATIONS AND ENCODINGS

The vector space structure of intervals of certain sound characteristics permits us to treat their elements mathematically and to express them by the set of numbers, which is indispensable for dialogue with computers, or by the set of points on a straight line, graphic expression often being very convenient.

The two preceding axiomatics may be applied to all sound characteristics that possess the same structure. For example, at the moment it would not make sense to speak of a scale of timbre which might be universally accepted as the scales of pitch, instants, and intensity are. On the other hand, time, intensity, density (number of events per unit of time), the quantity of order or disorder (measured by entropy), etc., could be put into one-to-one correspondence with the set of real numbers R and the set of points on a straight line. (See Fig. VIII-1.)

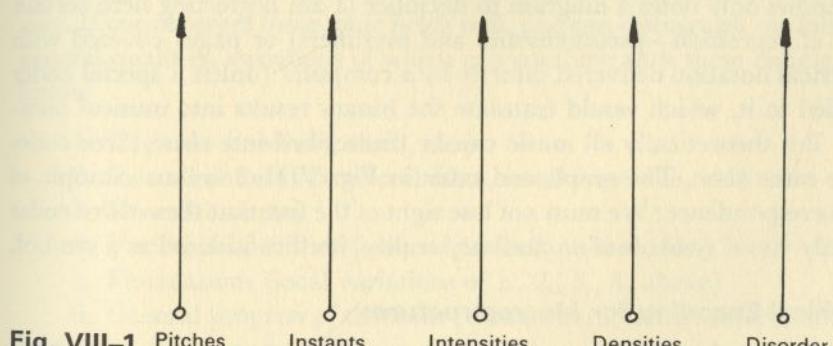


Fig. VIII-1 Pitches Instants Intensities Densities Disorder

Moreover, the phenomenon of sound is a correspondence of sound characteristics and therefore a correspondence of these axes. The simplest

correspondence may be shown by Cartesian coordinates; for example, the two axes in Fig. VIII-2. The unique point (H, T) corresponds to the sound that has a pitch H at the instant T .

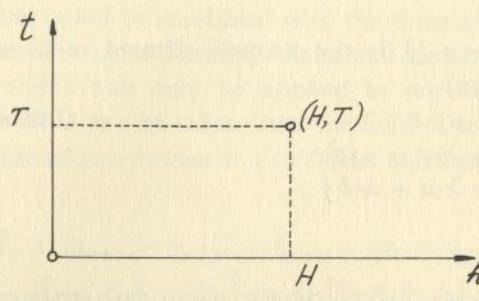


Fig. VIII-2

I must insist here on some facts that trouble many people and that are used by others as false guides. We are all acquainted with the traditional notation, perfected by thousands of years of effort, and which goes back to Ancient Greece. Here we have just represented sounds by two new methods: algebraically by a collection of numbers, and geometrically (or graphically by sketches).

These three types of notation are nothing more than three codes, and indeed there is no more reason to be dismayed by a page of figures than by a full musical score, just as there is no reason to be totally amazed by a nicely elaborated graph. Each code has its advantages and disadvantages, and the code of classical musical notation is very refined and precise, a synthesis of the other two. It is absurd to think of giving an instrumentalist who knows only notes a diagram to decipher (I am neglecting here certain forms of regression—pseudomystics and mystifiers) or pages covered with numerical notation delivered directly by a computer (unless a special coder is added to it, which would translate the binary results into musical notation). But theoretically all music can be transcribed into these three codes at the same time. The graph and table in Fig. VIII-3 are an example of this correspondence: We must not lose sight of the fact that these three codes are only visual symbols of an auditory reality, itself considered as a symbol.

Graphical Encoding for Macrostructures

At this point of this exposition, the unveiling of history as well as the axiomatic reconstruction have been realized in part, and it would be useless to continue. However, before concluding, I would like to give an example of the advantage of a diagram in studying cases of great complexity.

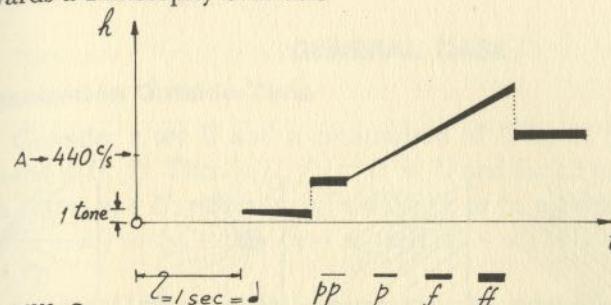


Fig. VIII-3

<i>N</i>	<i>T</i>	<i>H</i>	<i>V</i>	<i>D</i>	<i>I</i>
1	1.00	1	0	0.66	3
2	1.66	6	0	0.33	5
3	2.00	6	+17.5	0.80	6
4	2.80	13	0	?	5

N = note number

H = pitch in half tones with $+10 \leq A \leq 440$ Hz

V = slope of glissando (if it exists) in semitones/sec, positive if ascending, negative if descending

D = duration in seconds

I = number corresponding to a list of intensity forms

Let us imagine some forms constructed with straight lines, using string glissandi, for example.¹⁷ Is it possible to distinguish some elementary forms? Several of these elementary ruled fields are shown in Fig. VIII-4. In fact, they can constitute elements incorporated into larger configurations. Moreover it would be interesting to define and use in sequence the intermediary steps (continuous or discontinuous) from one element to another, especially to pass from the first to the last element in a more or less violent way. If one observes these sonic fields well, one can distinguish the following general qualities, variations of which can combine with these basic general forms:

1. Registers (medium, shrill, etc.)
2. Overall density (large orchestra, small ensemble, etc.)
3. Overall intensity
4. Variation of timbre (*arco*, *sul ponticello*, *tremolo*, etc.)
5. Fluctuations (local variations of 1., 2., 3., 4. above)
6. General progress of the form (transformation into other elementary forms)
7. Degree of order. (Total disorder can only make sense if it is calculated according to the kinetic theory of gases. Graphic representation is the most convenient for this study.)

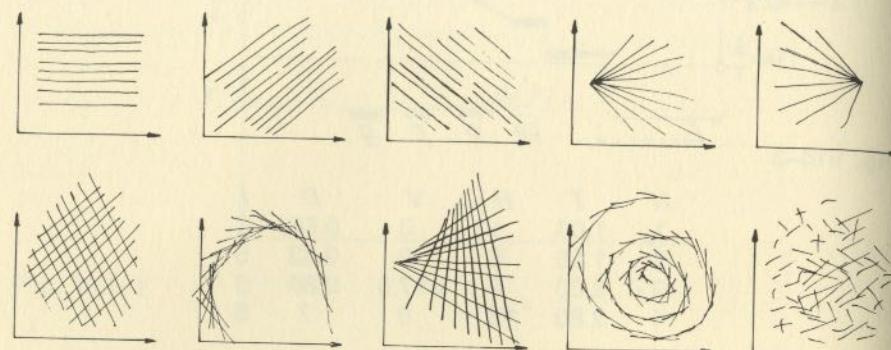


Fig. VIII-4

Let us now suppose the inverse, forms constructed by means of discontinuity, by sound-points; for example, string pizzicati. Our previous remarks about continuity can be transferred to this case (see Fig. VIII-5). Points 1.-7. are identical, so very broad is the abstraction. Besides, a mixture of discontinuity and continuity gives us a new dimension.

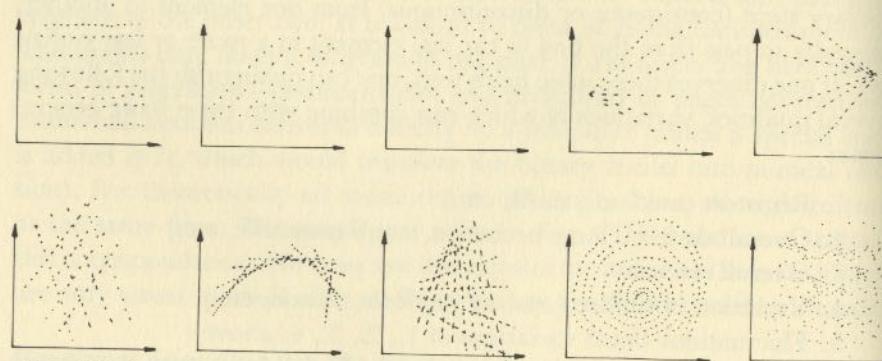


Fig. VIII-5

GENERAL CASE

Organization Outside-Time

Consider a set U and a comparison of U by U (a product $U \times U$) denoted $\psi(U, f)$. Then $\psi(U, f) \subset U \times U$ and for all pairs $(u, u_f) \in U \times U$ such that $u, u_f \in U$, either $(u, u_f) \in \psi(U, f)$, or $(u, u_f) \notin \psi(U, f)$. It is reflexive and $(u \sim u_f) \Rightarrow (u_f \sim u)$; $(u \sim u_f$ and $u_f \sim u') \Rightarrow u \sim u'$ for $u, u', u_f \in \psi(U, f)$.

Thus $\psi(U, f)$ is an equivalence class. In particular if U is isomorphic to the set Q of rational numbers, then $u \sim u_f$ if $|u - u_f| \leq \Delta u_f$ for arbitrary Δu_f .

Now we define $\psi(U, f)$ as the set of weak values of U , $\psi(U, m)$ as the set of average values, and $\psi(U, p)$ as the strong values. We then have

$$\psi = \psi(U, f) \cup \psi(U, m) \cup \psi(U, p) \subseteq U \times U$$

where ψ is the quotient set of U by ψ . The subsets of ψ may intersect or be disjoint, and may or may not form a partition of $U \times U$. Here

$$\psi(U, f) \rightarrow \psi(U, m) \rightarrow \psi(U, p)$$

are ordered by the relation \rightarrow in such a way that the elements of $\psi(U, f)$ are smaller than those of $\psi(U, m)$ and those of $\psi(U, m)$ are smaller than those of $\psi(U, p)$. Then

$$\psi(U, f) \cap \psi(U, m) = \emptyset, \psi(U, m) \cap \psi(U, p) = \emptyset.$$

In each of these subsets we define four new equivalence relations and therefore four sub-classes:

$$\psi^i(U, f) \text{ with } u_f^i \sim (u_f^i)'$$

if and only if

$$|u_f^i - (u_f^i)'| \leq \Delta u_f^i \text{ with } u_f^i, (u_f^i)' \in \psi(U, f)$$

for $i = 1, 2, 3, 4$, with $\psi^i(U, f) \subset \psi(U, f)$ and $\psi^1(U, f) \rightarrow \psi^2(U, f) \rightarrow \psi^3(U, f) \rightarrow \psi^4(U, f)$ ordered by the same relation \rightarrow . The same equivalence relations and sub-classes are defined for $\psi(U, m)$ and $\psi(U, p)$.

For simplification we write

$$u_f^i = \{u: u \in \psi^i(U, f)\},$$

and the same for u_m^m and u_p^p .

In the same way, equivalence sub-classes are created in two other sets, G and D . Here U represents the set of time values, G the set of intensity values, and D the set of density values with

$$\begin{aligned} U &= \{u_i^f, u_j^m, u_k^p\} \\ G &= \{g_i^f, g_j^m, g_k^p\} \\ D &= \{d_i^f, d_j^m, d_k^p\} \end{aligned}$$

for $i, j, k = 1, 2, 3, 4$.

Take part of the triple product $U \times G \times D$ composed of the points (u_i^f, g_j^m, d_k^p) . Consider the paths $V1: \{u_i^p, g_i^m, d_i^f\}$, $V2: \{u_i^f, g_i^p, d_i^m\}$, ..., $VS: \{(u_1^p, u_2^f, u_3^m, u_4^p), (g_1^f, g_2^m, g_3^p, g_4^p), (d_1^m, d_2^f, d_3^p, d_4^m)\}$ for $i = 1, 2, 3, 4$. VS will be a subset of the triple product $U \times G \times D$ split into $4^3 = 64$ different points.

In each of these subsets choose a new subset K_j^λ defined by the n points K_j^λ ($j = 1, 2, \dots, n$ and $\lambda = V1, V2, \dots, VS$). These n points are considered as the n vertices of a regular polyhedron. Consider the transformations which leave the polyhedron unchanged, that is, its corresponding group.

To sum up, we have the following chain of inclusions:

$$\begin{array}{ccccccc} \omega & \in & S_s^{K^\lambda} & \subset & K_j^\lambda & \subset & \lambda \\ \text{element} & & \text{vertex of} & & \text{set of} & & \text{path } \lambda \\ \text{of} & & \text{the poly-} & & \text{vertices} & & \text{(subset of} \\ U \times G \times D & & \text{hedron } K, & & \text{of the} & & U \times G \times D) \\ & & & & \text{polyhedron} & & \end{array}$$

Consider the two other sets H (pitch) and X (sonic material, way of playing, etc.). Form the product $H \times X \times C$ in which C is the set of n forms or complexes or sound types C_i ($i = 1, 2, \dots, n$); for example, a cloud of sound-points or a cloud of glissandi. Map the product $H \times X \times C$ onto the vertices of the polyhedron K_j^λ .

1. The complexes C_i traverse the fixed vertices and thus produce group transformations; we call this operation θ_0 .
2. The complexes C_i are attached to corresponding vertices which remain fixed, but the $H \times X$ traverse the vertices, also producing group transformations; this operation is called θ_1 .
3. The product $H \times X \times C$ traverses the vertices thus producing the group transformations of the polyhedron; we call this operation θ_u because the product can change definition at each transformation of the polyhedron.

Organization In-Time

The last mapping will be inscribed in time in two possible ways in order to manifest the peculiarities of this polyhedral group or the symmetric group

to which it is isomorphic: operation t_0 —the vertices of the polyhedron are expressed successively (model of the symmetric group); operation t_1 —the vertices are expressed simultaneously (n simultaneous voices).

Product $t_0 \times \theta_0$:

The vertices K_j^λ are expressed successively with:

1. only one sonic complex C_r , always the same one, for example, a cloud of sound-points only,
2. several sonic complexes, at most n , in one-to-one attachment with indices of vertices K_j^λ ,
3. several sonic complexes whose successive appearances express the operations of the polyhedral group, the vertices i (defined by $U \times G \times D$) always appearing in the same order,
4. several sonic complexes always in the same order while the order of the vertices i reproduces the group transformations,
5. several sonic complexes transforming independently from the vertices of the polyhedron.

Product $t_0 \times \theta_1$:

The list which this product generates may be obtained from the preceding one by substituting $H \times X$ in place of c_i .

Product $t_0 \times \theta$:

This list may be readily established.

Case t_1 and θ , is obtained from the preceding ones by analogy.

To these in-time operational products one ought to be able to add *in-space* operations when, for example, the sonic sources are distributed in space in significant manner, as in *Terrétektork* or *Nomos gamma*.

Organization Outside-Time

The three sets, D (densities), G (intensities), U (durations), are mapped onto three vector spaces or onto a single three-dimensional vector space. The following selection (subset) of equivalence classes, called path $V1$, is made: D (densities) strong, G (intensities) strong, U (durations) weak. Precise and ordered values have been given to these classes:

Set D	a	b	c	Set G		Set U	sec
	(Elements/sec)						
d_1	1.0	0.5	1	g_1	<i>mf</i>	u_1	2
d_2	1.5	1.08	2	g_2	<i>f</i>	u_2	3
d_3	2.0	2.32	3	g_3	<i>ff</i>	u_3	4
d_4	2.5	5.00	4	g_4	<i>fff</i>	u_4	5

A second selection (subset), called path $V2$, is formed in the following manner: D strong, G average, U strong, with ordered and precise values:

Set D	Elements/sec	Set G	Set U	sec	
d_1	0.5	g_1	<i>p</i>	u_1	10
d_2	1	g_2	<i>mp</i>	u_2	17
d_3	2	g_3	<i>mf</i>	u_3	21
d_4	3	g_4	<i>f</i>	u_4	30

Eight "points" of the triple product $D \times G \times U$ are selected.
For path $V1$:

$$K_1^r = d_1 g_1 u_1; \quad K_2^r = d_1 g_4 u_4; \quad K_3^r = d_4 g_4 a_4; \quad K_4^r = d_4 g_1 u_1; \\ K_5^r = d_2 g_2 u_2; \quad K_6^r = d_2 g_3 u_3; \quad K_7^r = d_3 g_3 u_3; \quad K_8^r = d_3 g_2 u_2.$$

r is the column (sub-class) of the table of set D . ($r = a, b, c$)

For path $V2$:

$$K_1 = d_4 g_3 u_2; \quad K_2 = d_3 g_2 u_1; \quad K_3 = d_2 g_4 u_4; \quad K_4 = d_1 g_2 u_3; \\ K_5 = d_4 g_1 u_4; \quad K_6 = d_3 g_2 u_3; \quad K_7 = d_2 g_3 u_2; \quad K_8 = d_1 g_4 u_1.$$

I. These eight points are regarded as solidly connected to each other so as to form a cube (a mapping of these eight points onto the vertices of a cube). The group formed by substitutions among these eight points, isomorphic to the symmetric group P_4 , is taken as the organizer principle. (See Fig. VIII-6.)

ANALYSIS OF NOMOS ALPHA

Organization In-Time

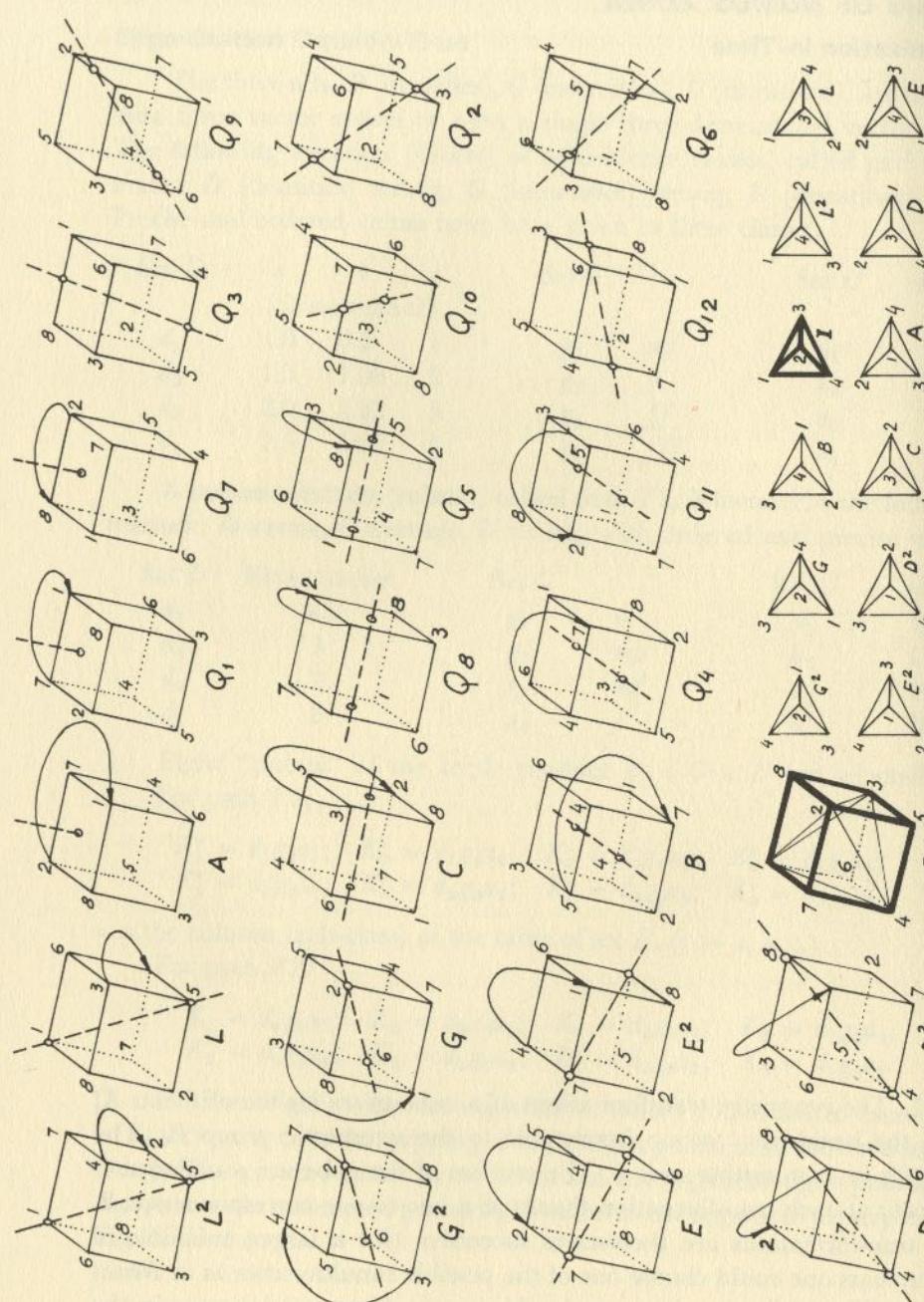


Fig. VIII-6. Hexahedral (Octahedral) Group

\downarrow	I	A	B	C	D	D^2	E	E^2	G	G^2	L	L^2	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_{12}		
I	I	A	B	C	D	D^2	E	E^2	G	G^2	L	L^2	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9	Q_{10}	Q_{11}	Q_{12}		
A	A	I	C	B	G	L	G^2	L^2	D	E	D^2	E^2	Q_7	Q_4	Q_5	Q_2	Q_3	Q_{12}	Q_1	Q_{10}	Q_{11}	Q_8	Q_9	Q_6		
B	B	C	I	A	L^2	E	D^2	G	E^2	L	G^2	D	Q_6	Q_9	Q_8	Q_{11}	Q_{10}	Q_1	Q_{12}	Q_3	Q_2	Q_5	Q_4	Q_7		
C	C	B	A	I	E^2	G^2	L	D	L^2	D^2	E	G	Q_{12}	Q_1	Q_{10}	Q_9	Q_8	Q_7	Q_6	Q_5	Q_4	Q_3	Q_2	Q_1		
D	D	L^2	E^2	G	D^2	I	C	L	E	A	B	G^2	Q_3	Q_6	Q_4	Q_1	Q_{10}	Q_8	Q_9	Q_7	Q_2	Q_{12}	Q_5	Q_3		
D^2	D^2	G^2	L	E	I	D	G	B	C	L^2	E^2	A	Q_4	Q_{10}	Q_1	Q_3	Q_{12}	Q_2	Q_9	Q_7	Q_8	Q_6	Q_5	Q_4		
E	E	L	G^2	D^2	B	L^2	E^2	I	A	D	G	C	Q_{11}	Q_5	Q_6	Q_8	Q_7	Q_9	Q_2	Q_3	Q_{12}	Q_4	Q_6	Q_1	Q_8	
E^2	E^2	G	D	L^2	G^2	C	I	E	L	B	A	D^2	Q_{10}	Q_7	Q_9	Q_{12}	Q_2	Q_3	Q_5	Q_4	Q_6	Q_{11}	Q_1	Q_8	Q_3	
G	G	E^2	L^2	D	L	A	B	D^2	G^2	I	C	E	Q_{15}	Q_2	Q_7	Q_9	Q_8	Q_{10}	Q_{11}	Q_1	Q_4	Q_6	Q_5	Q_3	Q_2	
G^2	G^2	D^2	E	L	C	E^2	L^2	A	I	G	D	B	Q_9	Q_3	Q_{12}	Q_{10}	Q_1	Q_{14}	Q_4	Q_6	Q_5	Q_3	Q_8	Q_2		
L	L	L	E^2	G^2	A	G	D	C	B	E^2	L^2	I	Q_2	Q_8	Q_9	Q_5	Q_6	Q_{10}	Q_4	Q_{11}	Q_1	Q_3	Q_{12}	Q_9	Q_7	Q_{10}
L^2	L^2	D	G	E^2	E	B	A	G^2	D^2	C	I	L	Q_8	Q_1	Q_{11}	Q_6	Q_4	Q_5	Q_3	Q_2	Q_{12}	Q_9	Q_7	Q_{10}	Q_1	
Q_1	Q_1	Q_3	Q_{12}	Q_6	Q_9	Q_5	Q_8	Q_2	Q_{11}	Q_0	Q_3	Q_4	A	L^2	D^2	E^2	L	B	I	G^2	G	E	D	C		
Q_2	Q_2	Q_{11}	Q_9	Q_4	Q_{10}	Q_6	Q_1	Q_8	Q_3	Q_{12}	Q_7	Q_5	E	I	G	C	L^2	D^2	L	E^2	B	D	A	G^2		
Q_3	Q_3	Q_8	Q_5	Q_{10}	Q_7	Q_4	Q_9	Q_6	Q_{12}	Q_2	Q_4	Q_1	L^2	G^2	I	L	B	E^2	D	A	E	C	D^2	G		
Q_4	Q_4	Q_9	Q_{11}	Q_2	Q_8	Q_{12}	Q_7	Q_{10}	Q_5	Q_6	Q_1	Q_3	G^2	A	D	B	E^2	L	D^2	L^2	C	G	I	E		
Q_5	Q_5	Q_{10}	Q_3	Q_8	Q_1	Q_9	Q_{11}	Q_{12}	Q_6	Q_4	Q_2	Q_7	E^2	E	A	D^2	C	L^2	G	I	G^2	B	L	D		
Q_6	Q_6	Q_2	Q_7	Q_1	Q_2	Q_{10}	Q_3	Q_9	Q_4	Q_5	Q_8	Q_{11}	C	D	E	G	S^2	I	B	L	E^2	D^2	L^2	A		
Q_7	Q_7	Q_4	Q_6	Q_{12}	Q_{11}	Q_3	Q_{10}	Q_4	Q_9	Q_8	Q_5	Q_2	I	E^2	L	L^2	D^2	C	A	E	D	G^2	G	B		
Q_8	Q_8	Q_3	Q_{10}	Q_5	Q_2	Q_4	Q_7	Q_1	Q_6	Q_9	Q_4	Q_{11}	D	L	B	G^2	I	G	L^2	C	D^2	A	E	E^2		
Q_9	Q_9	Q_1	Q_2	Q_4	Q_5	Q_6	Q_8	Q_3	Q_7	Q_{10}	Q_1	Q_9	D^2	B	E^2	A	D	E	G^2	I	L^2	C	L			
Q_{10}	Q_{10}	Q_5	Q_8	Q_3	Q_6	Q_2	Q_4	Q_1	Q_9	Q_8	Q_2	Q_{12}	G^2	D	C	E	A	D^2	B	L	I	G^2	L^2			
Q_{11}	Q_{11}	Q_2	Q_4	Q_9	Q_3	Q_7	Q_{12}	Q_5	Q_{10}	Q_1	Q_8	Q_6	L	C	L^2	Z	G	G^2	E	D	A	E^2	B	D^2		
Q_{12}	Q_{12}	Q_6	Q_1	Q_7	Q_4	Q_8	Q_5	Q_{11}	Q_2	Q_3	Q_{10}	Q_9	B	G	G^2	D	E	A	C	D^2	L^2	L	E^2	I		

Fig. VIII-7

Example: DA = G on D the transformation of A. (Columns \rightarrow rows)Fig. VIII-6. Symmetric Group P_4 : (1, 2, 3, 4)

I	12345678	G^2	32417685	Q_8	68572413
A	21436587	G	42138657	Q_6	65782134
B	34127856	L^2	13425786	Q_1	87564312
C	43218765	L	14235867	Q_5	75863142
D^2	23146758	Q_7	78653421	Q_9	58761432
D	31247568	Q_2	76583214	Q_{10}	57681324
E^2	24316875	Q_3	86754231	Q_4	85674123
E	41328576	Q_{11}	67852341	Q_{12}	56871243

The numbers in roman type
also correspond to Group $P_4 = 4!$

Organization Outside-Time

II. Eight elements from the macroscopic sound complexes are mapped onto the letters C_i in three ways, α, β, γ :

$$\begin{matrix} \alpha & \beta & \gamma \end{matrix}$$

$C_1 \ C_1 \ C_1$ = ataxic cloud of sound-points

$C_7 \ C_2 \ C_5$ = relatively ordered ascending or descending cloud of sound-points

$C_3 \ C_3 \ C_6$ = relatively ordered cloud of sound-points, neither ascending nor descending

$C_5 \ C_5 \ C_2$ = ataxic field of sliding sounds

$C_6 \ C_6 \ C_3$ = relatively ordered ascending or descending field of sliding sounds

$C_2 \ C_7 \ C_4$ = relatively ordered field of sliding sounds, neither ascending nor descending

$C_8 \ C_8 \ C_8$ = atom represented on a cello by interferences of a quasi-unison

$C_4 \ C_4 \ C_7$ = ionized atom represented on a cello by interferences, accompanied by pizzicati

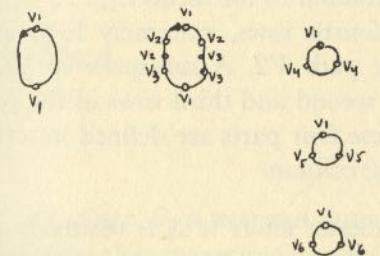
III. These letters are mapped one-to-one onto the eight vertices of a second cube. Thus a second hexahedral group is taken as the organizer principle.

Organization In-Time

II. The mapping of the eight forms onto the letters C_i change cyclically in the order $\alpha, \beta, \gamma, \alpha, \dots$ after each three substitutions of the cube.

A	$A \ Q$	A_4	$V_1 \ V_2 \ V_3 \ V_4 \ V_5 \ V_6$
A	$A \ Q$	A_4	$V_1 \ V_1 \ V_2 \ V_3 \ V_4 \ V_5 \ V_6$
Q	$Q \ A$	Q	$V_2 \ V_2 \ V_3 \ V_1 \ V_6 \ V_4 \ V_5$
			$V_3 \ V_3 \ V_1 \ V_2 \ V_5 \ V_6 \ V_4 \ V_6$
			$V_4 \ V_4 \ V_5 \ V_6 \ V_1 \ V_2 \ V_3 \ V_3$
			$V_5 \ V_5 \ V_6 \ V_4 \ V_3 \ V_1 \ V_2$
			$V_6 \ V_6 \ V_4 \ V_5 \ V_2 \ V_3 \ V_1$

$A_4 \ Q$



A_4

Q

$$\begin{aligned} V_1 &= I + A + B + C \\ V_2 &= D + E^2 + G + L^2 \\ V_3 &= D^2 + E + G^2 + L \\ V_4 &= Q_6 + Q_{12} + Q_2 + Q_1 \\ V_5 &= Q_{10} + Q_8 + Q_3 + Q_5 \\ V_6 &= Q_2 + Q_4 + Q_{11} + Q_9 \end{aligned}$$

Fig. VIII-8

Organization Outside-Time

IV. Take the products $K_i^r \times C_j$ and $K_l \times C_m$. Then take the product set $H \times X$. Set H is the vector space of pitch, while set X is the set of ways of playing the C_i . This product is given by a table of double entries:

Extremely High								
Medium High								
Medium Low								
Extremely Low								

pizz.	f.c.l.	an	pizz. gl.	an	a trem.	harm.	hr trem.	asp.	asp. trem.	an	a interf.
C_1, C_2, C_3				C_4, C_5, C_6						C_7, C_8	

pizz. = pizzicati

f.c.l. = struck with the wood of the

bow

an = normal arco

pizz. gl. = pizzicato-glissando

a trem. = normal arco with tremolo

harm. = harmonic sound

hr trem. = harmonic sound with

tremolo

asp = arco sul ponticello

asp trem. = arco sul ponticello

tremolo

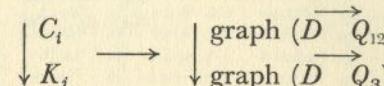
a interf. = arco with interferences

Various methods of playing are attributed to the forms C_1, \dots, C_8 , as indicated in the table. The first and fourth rows, extremely high and extremely low pitches, are reserved for path $V2$. A sub-space of H' is attributed to path $V1$. It consists of the second and third rows of the preceding table, each divided into two. These four parts are defined in terms of the playing range of the corresponding column.

V. The mapping of C_i onto the product set $H \times X$ is relatively independent and will be determined by a kinematic diagram of operations at the moment of the in-time setting.

Organization In-Time

IV. The products $K_i^r \times C_j$ and $K_l \times C_m$ are the result of the product of two graphs of closed transformations of the cube in itself. The mapping of the graphs is one-to-one and sounded successively; for example:



(See Figs. VIII-9, 10.)

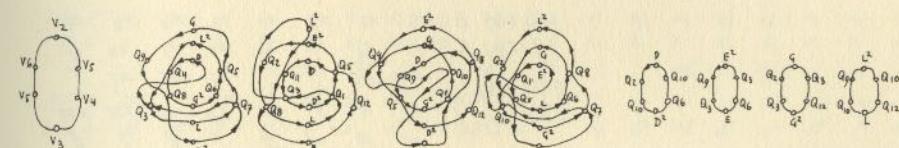
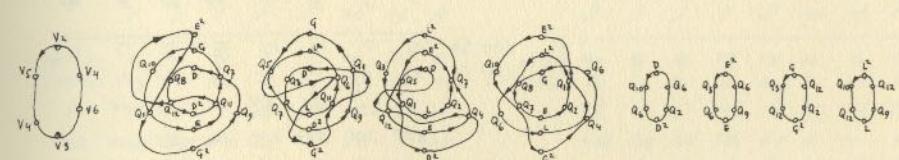
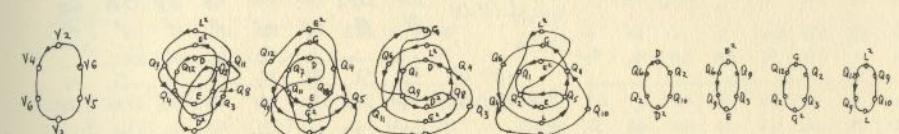
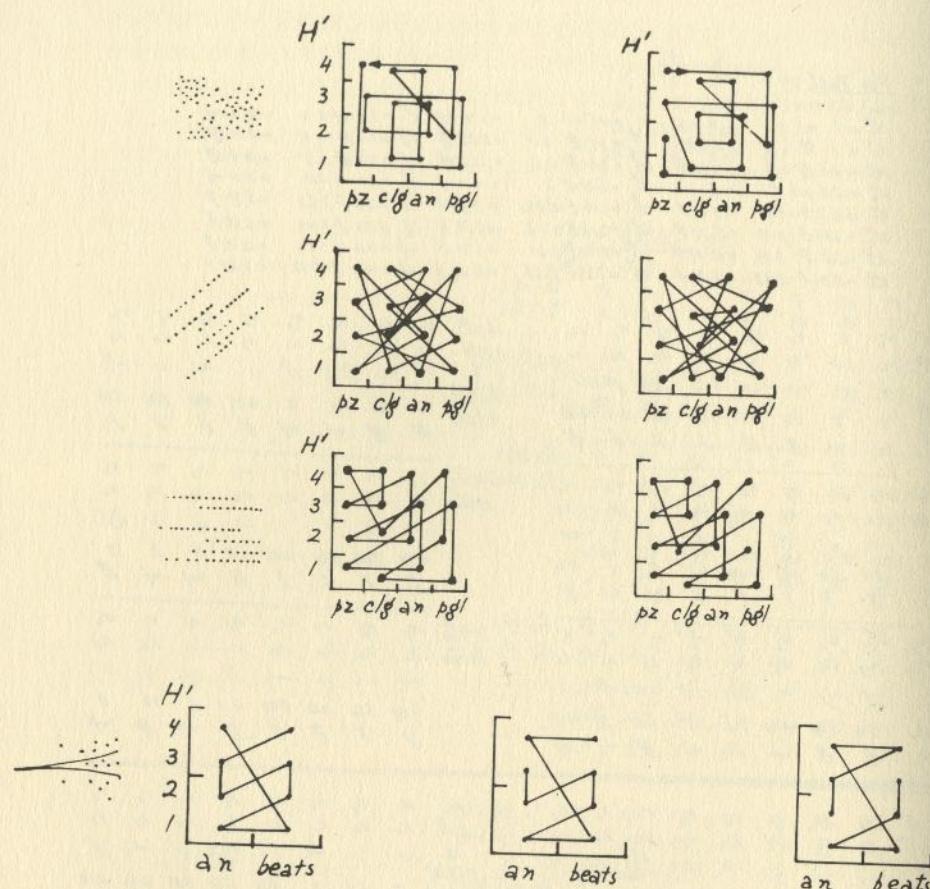


Fig. VIII-9

V. Each C_i is mapped onto one of the cells of $H \times X$ according to two principles: maximum expansion (minimum repetition), and maximum contrast or maximum resemblance. (See Fig. VIII-11.)

Fig. VIII-10

For Path Y

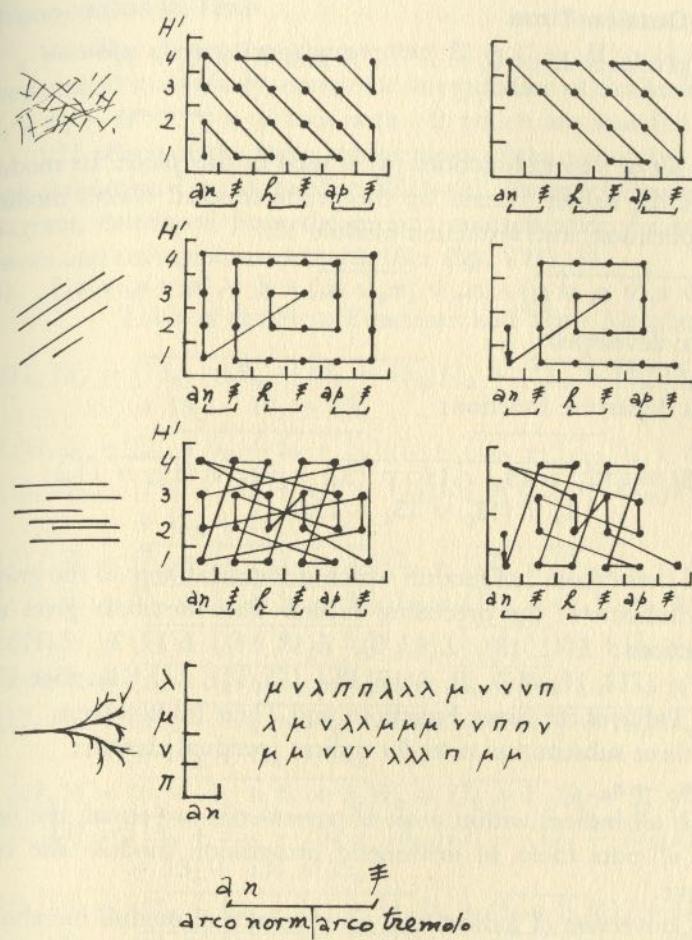


pz = pizzicati

c/g = battuto col legno

an = arco normal

pgf = pizzicati glissandi



an $\overbrace{\text{f h f ap f}}$

h $\overbrace{\text{harmonic tremolo}}$

ap $\overbrace{\text{arco s. pont. arco s. pont. tremolo}}$

Fig. VIII-11. Products $H' \times X$ in the Set of the C_i

Organization Outside-Time

VI. The products $K_i' \times C_i \times H' \times X$ and $K_j \times C_j \times H^{\text{extremes}} \times X$ are formed.

VII. The set of logical functions (a) is used in this piece. Its moduli are taken from the subset formed by the prime residual classes modulo 18, with multiplication, and reduction modulo 18.

$$L(m, n) = (\overline{n_i \vee n_j \vee n_k \vee n_l}) \wedge m_p \vee (\overline{m_q \vee m_r}) \wedge n_s \vee (n_t \vee n_u \vee n_w) \quad (a)$$

Its elements are developed:

1. From a departure function:

$$L(11, 13) = (\overline{13_3 \vee 13_5 \vee 13_7 \vee 13_9}) \wedge 11_2 \vee (\overline{11_4 \vee 11_8}) \\ \wedge 13_9 \vee (13_0 \vee 13_1 \vee 13_6)$$

2. From a "metabola" of moduli which is identical here to the graph coupling the elements of the preceding subset. This metabola gives the following functions: $L(11, 13)$, $L(17, 5)$, $L(13, 11)$, $L(17, 7)$, $L(11, 5)$, $L(1, 5)$, $L(5, 7)$, $L(17, 11)$, $L(7, 5)$, $L(17, 13)$, $L(5, 11)$, $L(1, 11)$. (See Fig. VIII-12, and Table of the Sieve Functions and Their Metabolae.)

3. From three substitution rules for indices (residual classes):

Rule a: $m_0 \rightarrow n_{0+1}$

Rule b: If all indices within a set of parentheses are equal, the next function $L(m, n)$ puts them in arithmetic progression modulo the corresponding sieve.

Rule c: Conversion of indices as a consequence of moduli metabolae (see Rule c. Table):

$$m_j \rightarrow n_x, \quad x = j(n/m); \quad \text{for example, } 7_4 \rightarrow 11_x, \quad x = 4(11/7) \sim 6.$$

4. From a metabola of ELD (elementary displacement: one quarter-tone for path $V1$, three-quarters of a tone for path $V2$).

The two types of metabolae which generate the elements of set $L(m, n)$ can be used outside-time or inscribed in-time. In the first case, they give us the totality of the elements; in the second case, these elements appear in a temporal order. Nevertheless a *structure* of temporal order is subjacent even in the first case.

5. From a special metabola that would simultaneously attribute different notes to the origins of the sieves constituting the function $L(m, n)$.

Organization In-Time

VI. The elements of the product $K_i' \times C_j \times H' \times X$ of the path $V1$ are sounded successively, except for interpolation of elements of the product $K_i \times C_i \times H^{\text{extremes}} \times X$ from path $V2$, which are sounded intermittently.

VII. Each of the three substitutions of the two cubes K_i and C_j , the logical function $L(m, n)$ (see Fig. VIII-11), changes following its kinematic diagram, developed from the group: multiplication by pairs of residual classes and reduction modulo 18. (See Fig. VIII-10.)

Table of the Sieve Functions and Their Metabolae

$$\begin{aligned} L(11, 13) &= (\overline{13_3 + 13_5 + 13_7 + 13_9})11_2 + (\overline{11_4 + 11_8})13_9 \\ &\quad + 13_0 + 13_1 + 13_6 \\ L(17, 5) &= (\overline{5_1 + 5_2 + 5_3 + 5_4})17_1 + (\overline{17_7 + 17_{13}})5_4 + 5_1 + 5_0 + 5_2 \\ L(13, 11) &= (\overline{11_2 + 11_4 + 11_7 + 11_9})13_0 + (\overline{13_5 + 13_{10}})11_9 \\ &\quad + 11_2 + 11_1 + 11_4 \\ L(17, 7) &= (\overline{7_1 + 7_3 + 7_5 + 7_6})17_1 + (\overline{17_6 + 17_{13}})7_6 + 7_1 + 7_0 + 7_3 \\ L(11, 5) &= (\overline{5_0 + 5_2 + 5_3 + 5_4})11_0 + (\overline{11_4 + 11_8})5_4 + 5_0 + 5_1 + 5_2 \\ L(1, 5) &= (\overline{5_1 + 5_2 + 5_3 + 5_4})1_1 + (\overline{1_1 + 1_1})5_4 + 5_1 + 5_2 + 5_3 \\ L(5, 7) &= (\overline{7_1 + 7_3 + 7_4 + 7_6})5_0 + (\overline{5_0 + 5_1})7_6 + 7_1 + 7_3 + 7_4 \\ L(17, 11) &= (\overline{11_2 + 11_5 + 11_6 + 11_9})11_1 + (\overline{17_1 + 17_3})11_9 \\ &\quad + 11_2 + 11_5 + 11_6 \\ L(7, 5) &= (\overline{5_1 + 5_2 + 5_3 + 5_4})7_0 + (\overline{7_0 + 7_1})5_4 + 5_1 + 5_2 + 5_3 \\ L(17, 13) &= (\overline{13_3 + 13_5 + 13_8 + 13_{10}})17_1 + (\overline{17_1 + 17_2})13_{10} \\ &\quad + 13_3 + 13_5 + 13_8 \\ L(5, 11) &= (\overline{11_3 + 11_4 + 11_7 + 11_8})5_0 + (\overline{5_0 + 5_1})11_8 + 11_3 + 11_4 + 11_7 \\ L(1, 11) &= (\overline{11_3 + 11_4 + 11_7 + 11_8})1_1 + (\overline{1_1 + 1_0})11_8 + 11_3 + 11_4 + 11_7 \end{aligned}$$

Rule c. Table

$\frac{n}{m}$	$\frac{5}{5} = 1$		
	$\frac{7}{5} = 1.4$	$\frac{7}{7} = 1$	
	$\frac{11}{5} = 2.2$	$\frac{11}{7} = 1.57$	$\frac{11}{11} = 1$
	$\frac{13}{5} = 2.6$	$\frac{13}{7} = 1.85$	$\frac{13}{11} = 1.2$
	$\frac{17}{5} = 3.4$	$\frac{17}{7} = 2.43$	$\frac{17}{11} = 1.54$
			$\frac{17}{13} = 1.3$

Group and sub-group of residual classes obtained by ordinary multiplication followed by reduction relative to the modulus 18

.	1	5	7	11	13	17
1	1	5	7	11	13	17
5	5	7	17	1	11	13
7	7	17	13	5	1	11
11	11	1	5	13	17	7
13	13	11	1	17	7	5
17	17	13	11	7	5	1

.	1	7	13
1	1	7	13
7	7	13	1
13	13	1	7

DETAILED ANALYSIS OF THE BEGINNING OF THE SCORE ($L(11, 13)$)¹⁸

Thanks to the metabola in 5. of the outside-time organization, the origins of the partial sieves ($13_3 \vee 13_5 \vee 13_7 \vee 13_9 \wedge 11_2 \vee (11_4 \vee 11_8) \wedge 13_9$ and $13_0 \vee 13_1 \vee 13_6$) correspond to $A_3\#$ and A_3 , respectively, for $A_3 = 440$ Hz. Hence the sieve $L(11, 13)$ will produce the following pitches: $\dots, C_2\sharp, C_2\#, D_2, D_2\sharp, F_2, F_2\#, G_2, G_2\#\#, A_2, B_2\sharp, C_3, C_3\#\#, D_3\#, D_3\#\#, F_3\sharp, F_3\#, G_3\#, A_3\sharp, A_3\#, B_3, C_4\sharp, D_4\sharp, E_4, E_4\sharp, G_4, A_4, A_4\#, A_4\#\#, \dots$

The order applied to the sonic complexes (S_n) and to the density, intensity, and duration combinations (K_n) are for transformation β :

$$S_1 = \text{dots} \quad K_1 = 1 \quad mf$$

$$S_2 = \text{dotted} \quad K_2 = 2.25 \quad fff$$

$$S_3 = \text{dotted} \quad K_3 = 22.5 \quad fff$$

$$S_4 = \text{wavy} \quad K_4 = 10 \quad mf$$

$S_5 =$		$K_5 = 2.83$	<i>f</i>
$S_6 =$		$K_6 = 3.72$	<i>ff</i>
$S_7 =$		$K_7 = 7.98$	<i>ff</i>
$S_8 =$		$K_8 = 6.08$	<i>f</i>

(In this text C_n is replaced by S_n .)

First sequence (see Fig. VIII-13):

1	2	3	4	5	6	7	8
↓	↓	↓	↓	↓	↓	↓	↓
$D(S_n) = S_2$	S_3	S_1	S_4	S_6	S_7	S_5	S_8
$D(K_n) = K_2$	K_3	K_1	K_4	K_6	K_7	K_5	K_8
2.25	22.5	1	10	3.72	7.98	2.83	6.08
fff	fff	mf	mf	ff	ff	f	f

This part begins with a pizzicato glide on the note C , *fff* (the sliding starts *ppp*). The slope of the glide is zero at first and then very weak (1/4 tone per 2.5 seconds).

S_3 consists of $C\sharp$ $C\#$ D struck col legno, *fff* (with *p* in the middle). In S_8 there is an introduction of beats obtained by raising $G\#$ towards A .

Second sequence, beginning at Q_{12}/Q_3 :

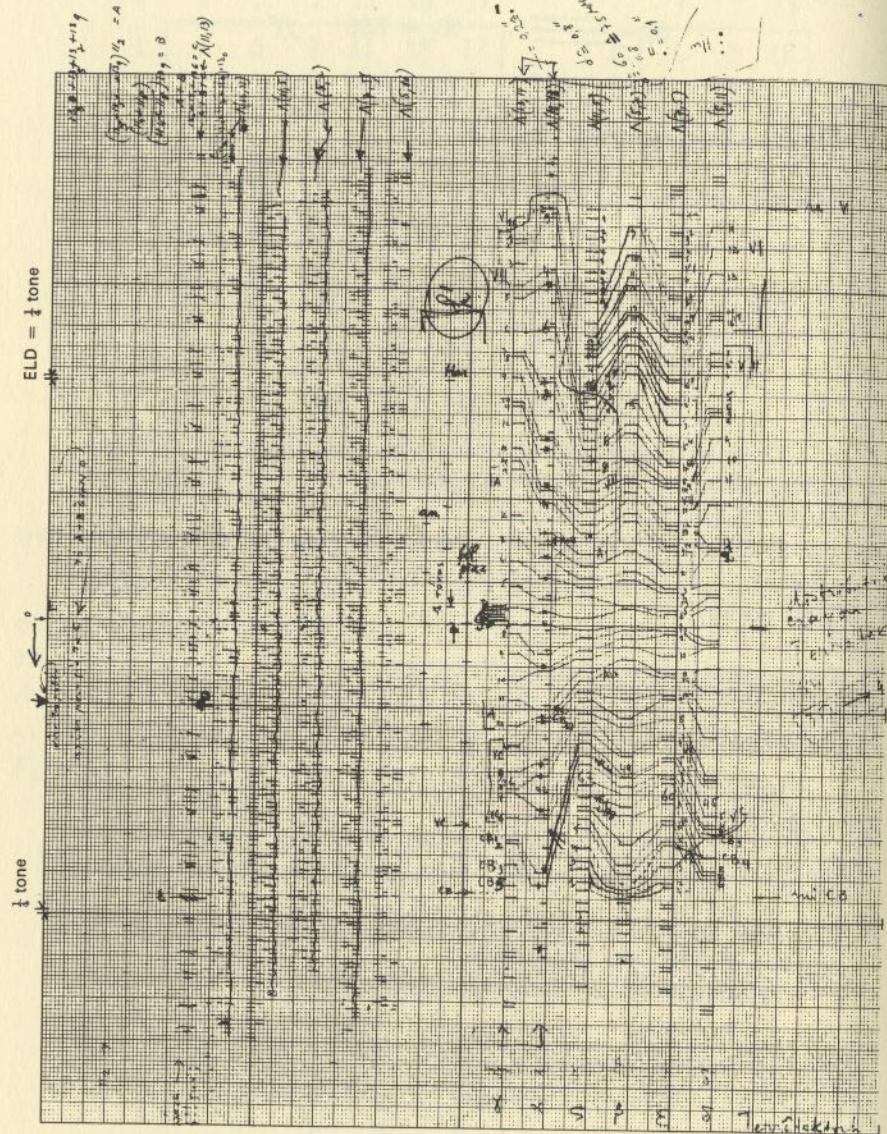
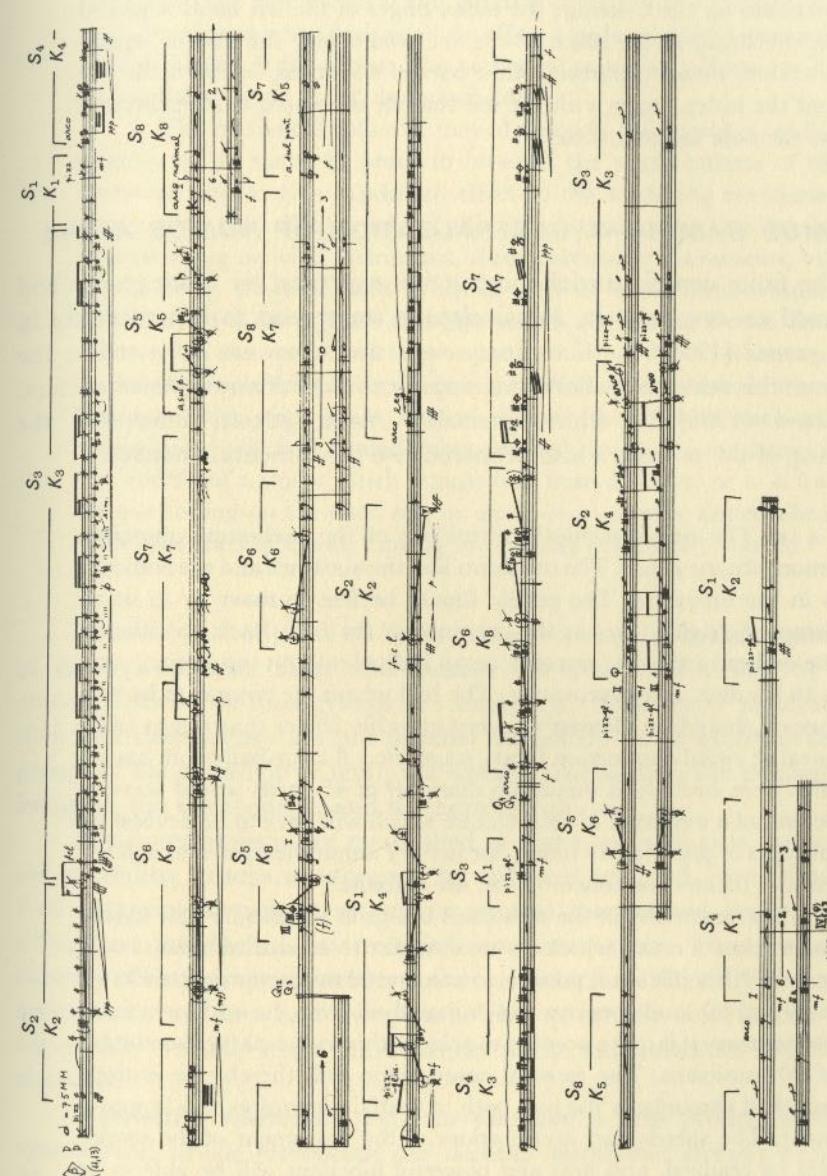
1	2	3	4	5	6	7	8
↓	↓	↓	↓	↓	↓	↓	↓
$Q_{12}(S_n) = S_5$	S_6	S_8	S_7	S_1	S_2	S_4	S_3
$Q_3(K_n) = K_8$	K_6	K_7	K_5	K_4	K_2	K_3	K_1
6.08	3.72	7.98	2.83	10	2.25	22.5	1.0
f	ff	ff	f	mf	fff	fff	mf

Note, as in the preceding part, the previously calculated contraction of the values of duration.

S_1 is atoxic, lasting more than a second.

Third sequence, beginning at Q_4/Q_7 :

1	2	3	4	5	6	7	8
↓	↓	↓	↓	↓	↓	↓	↓
$Q_4(S_n) = S_6$	S_7	S_8	S_5	S_2	S_3	S_4	S_1
$Q_7(K_n) = K_8$	K_7	K_5	K_6	K_4	K_3	K_1	K_2
6.08	7.98	2.83	3.72	10	2.25	1.0	2.25
f	ff	f	ff	mf	fff	mf	fff

Fig. VIII-12. *Nomos alpha* SievesFig. VIII-13. Opening Bars of *Nomos alpha* for Cello

In S_8 the slopes of the glissandi in opposite directions cancel each other. The enlargement in S_4 is produced by displacement of the lower line and the inducement of beats. The cloud is introduced by a pizzicato on the C string; the index finger of the left hand is placed on the string at the place where one would play the note in square brackets; then by plucking that part of the string between the nut and the index finger with the left thumb, the sound that results will be the note in parentheses.

NOMOS GAMMA—A GENERALIZATION OF NOMOS ALPHA

The finite combinatorial construction expressed by finite groups and performed on one cello in *Nomos alpha* is transposed to full orchestra in *Nomos gamma* (1967/68). The ninety-eight musicians are scattered in the audience; this scattering allows the amplification of *Nomos alpha*'s structure. *Terrétektorkh* (1965/66), which preceded *Nomos gamma*, innovated the scattering of the orchestra and proposed two fundamental changes:

a. The quasi-stochastic sprinkling of the orchestral musicians among the audience. The orchestra is in the audience and the audience is in the orchestra. The public should be free to move or to sit on camp-stools given out at the entrance to the hall. Each musician of the orchestra should be seated on an individual, but unresonant, daïs with his desk and instruments. The hall where the piece is to be performed should be cleared of every movable object that might cause aural or visual obstruction (seats, stage, etc.) A large ball-room having (if it were circular) a minimum diameter of 45 yards would serve in default of a new kind of architecture which will have to be devised for all types of present-day music, for neither amphitheatres, and still less normal theatres or concert-halls, are suitable.

The scattering of the musicians brings in a radically new kinetic conception of music which no modern electro-acoustical means could match.¹⁹ For if it is not possible to imagine 90 magnetic tape tracks re-laying to 90 loud speakers disseminated all over the auditorium, on the contrary it is quite possible to achieve this with a classical orchestra of 90 musicians. The musical composition will thereby be entirely enriched throughout the hall both in spatial dimension and in movement. The speeds and accelerations of the movement of the sounds will be realized, and new and powerful functions will be able to be made use of, such as logarithmic or Archimedean spirals, in-time and geometrically. Ordered or disordered sonorous masses, rolling one against the other like waves . . . etc., will be possible.

Terrétektorkh is thus a "Sonotron": an accelerator of sonorous particles, a disintegrator of sonorous masses, a synthesizer. It puts the sound and the music all around the listener and close up to him. It tears down the psychological and auditive curtain that separates him from the players when positioned far off on a pedestal, itself frequently enough placed inside a box. The orchestral musician rediscovers his responsibility as an artist, as an individual.

b. The orchestral colour is moved towards the spectrum of dry sounds, full of noise, in order to broaden the sound-palette of the orchestra and to give maximum effect to the scattering mentioned above. For this effect, each of the 90 musicians has, besides his normal string or wind instrument, three percussion instruments, viz. Wood-block, Maracas, and Whip as well as small Siren-whistles, which are of three registers and give sounds resembling flames. So if necessary, a shower of hail or even a murmuring of pine-forests can encompass each listener, or in fact any other atmosphere or linear concept either static or in motion. Finally the listener, each one individually, will find himself either perched on top of a mountain in the middle of a storm which attacks him from all sides, or in a frail barque tossing on the open sea, or again in a universe dotted about with little stars of sound, moving in compact nebulae or isolated.²⁰

Now the crux or thesis of *Nomos gamma* is a combinatorial organization of correspondences, finite and outside the time of the sets of sound characteristics. Various groups are exploited; their inner structure and their interdependency are put in relief musically: cyclic group of order 6, groups of the rectangle (Klein), the triangle, the square, the pentagon, the hexagon, the tetrahedron, and the hexahedron.

The isomorphisms are established in many ways, that is, each one of the preceding groups is expressed by different sets and correspondences, thus obtaining structures set up on several interrelated levels. Various groups are interlocked, intermingled, and interwoven. Thus a vast sonic tapestry of non-temporal essence is formed (which incidentally includes the organization of time and durations). The space also contributes, and is organically treated, in the same manner as the more abstract sets of sound elements.

A powerful *deterministic and finite machinery* is thus promulgated. Is it symmetrical to the *probabilistic and stochastic machineries* already proposed? The two poles, one of pure chance, the other of pure determinacy, are dialectically blended in man's mind (and perhaps in nature as well, as Epicurus or Heisenberg wished it). The mind of man should be able to

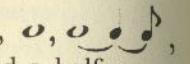
travel back and forth constantly, with ease and elegance, through the fantastic wall, of disarray caused by irrationality, that separates determinacy from indeterminacy.

We will now consider some examples. It goes without saying that *Nomos gamma* is not entirely defined by group transformations. Arbitrary ranges of decisions are disseminated into the piece, as in all my works except for those originated by the stochastic program in Chap. V. However *Nomos gamma* represents a stage in the method of mechanization by computers for this category of problem.

Measures 1–16 (three oboes, then three clarinets)

OUTSIDE-TIME STRUCTURE

Set of pitches: $H = \{H_1, H_2, H_3, H_4, H_5\}$. Origins: $D_3, G\sharp_3, D_4, G\sharp_4, D_5$, respectively, with range ± 3 semitones.

Set of durations: $U = \{U_1, U_2, U_3, U_4\}$. Origins: , respectively, with range \pm one sixteenth-note and a half note ~ 1 sec.

Set of intensities: $G = \{G_1, G_2, G_3, G_4\}$. $G_1 = \{ppp, pp\bar{p}, pp, \bar{p}p, \bar{p}\bar{p}, p\}$, $G_2 = \{\bar{p}, \bar{\bar{p}}, mp, \bar{m}p, \bar{m}\bar{p}, mf, \bar{m}f\}$, $G_3 = \{\bar{m}f, f, \bar{f}, \bar{f}, \bar{s}f, ff, \bar{f}f\}$, $G_4 = \{ff, \bar{s}ff, fff, \bar{f}ff, \bar{s}fff, \bar{s}\bar{f}ff\}$. Origins: $pp, \bar{m}p, f, \bar{f}ff$, respectively.

Product sets: $K = H \times U \times G$. Each one of the points of the product set is defined by a sieve modulo n considered as an element of an additive group (e.g., $n = 3, \dots, 3_0 \rightarrow 3_1 \rightarrow 3_1 \rightarrow 3_2 \rightarrow 3_0 \rightarrow 3_2 \rightarrow 3_2 \rightarrow 3_1 \rightarrow \dots$) and by its unit, that is, the elementary displacement ELD:

$K_1 = H_4 \times G_2 \times U_1$	$K_2 = H_4 \times G_3 \times U_2$	$K_3 = H_4 \times G_1 \times U_1$
Moduli: 2 2 2	2 2 2	2 2 2
ELD: $\frac{1}{4}$ tone	$\frac{1}{8}$ sec $\frac{1}{4}$ tone	$\frac{1}{5}$ sec $\frac{1}{4}$ tone
$K_4 = H_4 \times G_3 \times U_1$	$K_5 = H_4 \times G_2 \times U_2$	$K_6 = H_4 \times G_3 \times U_3$
Moduli: 3 2 3	3 2 3	3 2 3
ELD: $\frac{1}{4}$ tone	$\frac{1}{8}$ sec $\frac{1}{4}$ tone	$\frac{1}{5}$ sec $\frac{1}{4}$ tone
		$\frac{1}{6}$ sec

In addition, K_2 and K_3 are deformed by translations and homothetic transformations of the H values.

Let us now consider the three points K_1, K_2, K_3 of the product $H \times G \times U$, and map them one-to-one onto three successive moments of time. We thus define the triangle group with the following elements:

$$\{I, A, A^2, B, \vec{BA}, BA^2\} \leftrightarrow \{123, 312, 231, 132, 213, 321\}$$

IN-TIME STRUCTURE

For each transformation of the triangle the vertices are stated by K_1, K_2, K_3 , which are played successively by the oboes and the clarinets, according to the above permutation group and to the following circuit: $BA, BA^2, A, B, BA^2, A^2$.

Measures 16–22 (three oboes and three clarinets)

OUTSIDE-TIME STRUCTURE

Form the product $K_i \times C_i: K_1 \times C_1, K_2 \times C_2, K_3 \times C_3$, in which the C_i are the ways of playing. C_1 = smooth sound without vibrato, C_2 = flutter tongue, C_3 = quilisma (irregular oscillations of pitch).

Consider now two triangles whose respective vertices are the three oboes and the three clarinets. The $K_i \times C_i$ values are the names of the vertices. All the one-to-one mappings of the $K_i \times C_i$ names onto the three space positions of the three oboes or of the three clarinets form one triangle group.

IN-TIME STRUCTURE

To each group transformation the names $K_i \times C_i$ are stated simultaneously by the three oboes, which alternate with the three clarinets. The circuits are chosen to be $I, BA, BA, I, A^2, B, BA, A, BA^2$ and I, B, B .

Measures 404–42—A Sound Tapestry

The string orchestra (sixteen first violins, fourteen second violins, twelve violas, ten cellos, and eight double basses) is divided into two times three teams of eight instruments each: $\phi_1, \phi_2, \phi_3, \psi_1, \psi_2, \psi_3$. The remaining twelve strings duplicate the ones sitting nearest them. In the text that follows the ϕ_i and ψ_j are considered equivalent in pairs ($\phi_i \sim \psi_j$). Therefore we shall only deal with the ϕ_i .

LEVEL I—OUTSIDE-TIME STRUCTURE

The eight positions of the instruments of each ϕ_i are purposely taken into consideration. Onto these positions (instruments) we map one-to-one eight ways of playing drawn from set $X = \{\text{on the bridge tremolo, on the bridge tremolo and trill, sul ponticello smooth, sul ponticello tremolo, smooth natural harmonic notes, irregular dense strokes with the wood of the bow, normal arco with tremolo, pizzicato-glissando ascending or descending}\}$. We have thus formed a cube: KVBOS 1.

Onto these same eight positions (instruments) of ϕ_i we map one-to-one eight dynamic forms of intensity taken from the following sets: $g_\lambda =$

$\{ppp$ crescendo, ppp diminuendo, pp cresc, pp dim, p cresc, p dim, mp cresc, mp dim $\}$, $g_\mu = \{mf$ cresc, mf dim, f cresc, f dim, ff cresc, ff dim, fff cresc, fff dim $\}$, $g_\xi = \{p$ dim, p cresc, mp dim, mp cresc, mf dim, mf cresc, f dim, f cresc $\}$. We have thus defined a second cube: KVBOS 2.

LEVEL 1—IN-TIME STRUCTURE

Each one of these cubes is transformed into itself following the kinematic diagrams of the hexahedral group (cf. *Nomos alpha*, p. 225); for example, KVBOS 1 following $D^2Q_{12} \dots$ and KVBOS 2 following $Q_{11}Q_7 \dots$

LEVEL 2—OUTSIDE-TIME STRUCTURE

The three partitions ϕ_1, ϕ_2, ϕ_3 are now considered as a triplet of points in space. We map onto them, one-to-one, three distinct pitch ranges $H_\alpha, H_\beta, H_\gamma$ in which the instrumentalists of the preceding cubes will play. We have thus formed a triangle TRIA 1.

Onto these same three points we map one-to-one three elements drawn from the product (durations \times intensities), $U \times G = \{2.5 \text{ sec } g_\lambda, 0.5 \text{ sec } g_\mu, 1.5 \text{ sec } g_\xi\}$. We have thus defined a second triangle TRIA 2.

LEVEL 2—IN-TIME STRUCTURE

When the two cubes play a Level 1 transformation, the two triangles simultaneously perform a transformation of the triangle group. If I, A, A^2, B, BA, BA^2 are the group elements, then TRIA 1 proceeds according to the kinematic diagram $A, B, BA^2, A^2, BA, BA^2$, and TRIA 2 proceeds simultaneously according to A, BA^2, BA, A^2, B, AB .

LEVEL 3—OUTSIDE-TIME STRUCTURE

Form the product $C_i \times M_j$ with three macroscopic types: $C_1 =$ clouds of webs of pitch glissandi, $C_2 =$ clouds of sound-points, and $C_3 =$ clouds of sounds with quilisma. Three sieves with modulus $M = 3$ are taken: $3_0, 3_1, 3_2$. From this product we select five elements: $C_1 \times 3_0 = I, C_1 \times 3_1 = A, C_1 \times 3_2 = A^3, C_2 \times 3_0 = A^4, C_3 \times 3_1 = A^5$, which could belong to the cyclic group of order 6.

LEVEL 3—IN-TIME STRUCTURE

The nested transformation of Levels 1 and 2 are plunged into the product $C_i \times M_j$, which traverses successively $C_1 \times 3_2, C_2 \times 3_0, C_1 \times 3_1, C_3 \times 3_1, C_1 \times 3_0 \leftrightarrow A^3, A^4, A, A^5, I$, during the corresponding arbitrary durations of 20 sec, 7.5 sec, 12.5 sec, 12.5 sec, 7.5 sec.

LEVEL 4—OUTSIDE-TIME STRUCTURE

The partition of the string orchestra into teams ϕ_i, ψ_j is done in two modes: compact and dispersed. The compact mode is itself divided into two cases: Compact I and Compact II. For example,

in Compact I, $\phi_1 = \{VI_{13}, VII_1, VII_2, VII_{14}, A_7, VC_2, VC_6, CB_4\}$

in Compact II, $\phi_1 = \{VI_1, VI_7, VI_8, VI_9, VI_{10}, A_8, VC_3, CB_2\}$

in the dispersed mode, $\phi_1 = \{VI_2, VI_3, VI_6, VII_1, VII_6, VII_{11}, CB_3, CB_7\}$

($VI_i =$ i th first violin, $VII_i =$ i th second violin, $A_i =$ i th viola, $VC_i =$ i th cello, $CB_i =$ i th double bass.) These partitions cannot occur simultaneously.

LEVEL 4—IN-TIME STRUCTURE

All the mechanisms that sprang from Levels 1, 2, 3 are in turn plunged into the various above definitions of the ϕ_i and ψ_j teams, and successively into Compact I during the 27.5 sec duration, into the dispersed mode during the 17.5 sec duration, into Compact II during 5 sec, into the dispersed mode during 5 sec, and into Compact I during 5 sec.

DESTINY'S INDICATORS

Thus the *inquiry* applied to music leads us to the innermost parts of our mind. Modern axiomatics disentangle once more, in a more precise manner now, the significant grooves that the past has etched on the rock of our being. These mental premises confirm and justify the billions of years of accumulation and destruction of signs. But awareness of their limitation, their closure, forces us to destroy them.

All of a sudden it is unthinkable that the human mind forges its conception of time and space in childhood and *never* alters it.²¹ Thus the bottom of the cave would not reflect the beings who are behind us, but would be a filtering glass that would allow us to guess at what is at the very heart of the universe. It is this bottom that must be broken up. Consequences: 1. It would be necessary to change the ordered structures of time and space, those of logic, . . . 2. Art, and sciences annexed to it, should realize this mutation.

Let us resolve the duality *mortal-eternal*: the future is in the past and vice-versa; the evanescence of the present is abolished, it is everywhere at the same time; the *here* is also two billion light-years away. . . .

The space ships that ambitious technology have produced may not carry us as far as liberation from our mental shackles could. This is the fantastic perspective that *art-science* opens to us in the Pythagorean-Parmenidean field.

New Proposals in Microsound Structure

FOURIER SERIES—BASIC IMPORTANCE AND INADEQUACY

The physico-mathematical apparatus of acoustics [2, 23] is plunged into the theories of energy propagation in an elastic medium, in which harmonic analysis is the cornerstone.

The same apparatus finds in the units of electronic circuit design the practical medium where it is realized and checked.

The prodigious development of radio and TV transmissions has expanded the Fourier harmonic analysis to very broad and heterogeneous domains.

Other theories, quite far apart, e.g., servomechanisms and probability, find necessary backing in Fourier series.

In music ancient traditions of scales, as well as those of string and pipe resonances, also lead to circular functions and their linear combinations [24].

In consequence, any attempt to produce a sound artificially could not be conceived outside the framework of the above physico-mathematical and electronic apparatus, which relies on Fourier series.

Indeed the long route traversed by the *acoustics* of the Pythagoreans seemed to have found its natural bed. Musical theoreticians did base their theories on Fourier, more or less directly, in order to support the argument about the *natural harmony* of tonality. Moreover, in defining tonality, the 20th-century deprecators of the new musical languages based their arguments on the theory of vibration of elastic bodies and media, that is, in the end, on Fourier analysis. But they were thus creating a paradox, for al-

though they wanted to keep music in the intuitive and instinctive domain, in order to legitimize the tonal universe they made use of physico-mathematical arguments!

The Impasse of Harmonic Analysis and Some Reasons

Two major difficulties compel us to think in another way:

1. The defeat by the thrust of the new languages of the theory according to which harmony, counterpoint, etc., must stem, just from the *basis* formed by circular functions. E.g., how can we justify such harmonic configurations of recent instrumental or electro-acoustic music as a cloud of gliding sounds? Thus, harmonic analysis has been short-circuited in spite of touching attempts like Hindemith's explanation of Schönberg's system [25]. Life and sound adventures jostle the traditional theses, which are nevertheless still being taught in the conservatories (rudimentally, of course). It is therefore natural to think that the disruptions in music in the last 60 years tend to prove once again that music and its "rules" are socio-cultural and historical *conditionings*, and hence modifiable. These conditions seem to be based roughly on *a*. the absolute limits of our senses and their deforming power (e.g., Fletcher contours); *b*. our canvass of mental structures, some of which were treated in the preceding chapters (ordering, groups, etc.); *c*. the means of sound production (orchestral instruments, electro-acoustic sound synthesis, storage and transformation analogue systems, digital sound synthesis with computers and digital to analogue converters). If we modify any one of these three points, our socio-cultural conditioning will also tend to change in spite of an obvious inertia inherent in a sort of "entropy" of the social facts.

2. The obvious failure, since the birth of oscillating circuits in electronics, to reconstitute any sound, even the simple sounds of some orchestral instruments! *a*. The Trautoniums, Theremins, and Martenots, all pre-World War II attempts, prove it. *b*. Since the war, all "electronic" music has also failed, in spite of the big hopes of the fifties, to pull electro-acoustic music out of its cradle of the so-called electronic pure sounds produced by frequency generators. Any electronic music based on such sounds only, is marked by their simplistic sonority, which resembles radio atmospherics or heterodyning. The serial system, which has been used so much by electronic music composers, could not by any means improve the result, since it itself is much too elementary. Only when the "pure" electronic sounds were framed by other "concrete" sounds, which were much richer and much more interesting (thanks to E. Varèse, Pierre Schaeffer, and Pierre Henry),

could electronic music become really powerful. *c.* The most recent attempts to use the flower of modern technology, computers coupled to converters, have shown that in spite of some relative successes [26], the sonorous results are even less interesting than those made ten years ago in the classic electro-acoustic studios by means of frequency generators, filters, modulators, and reverberation units.

In line with these critiques, what are the causes of these failures? In my opinion the following are some of them:

1. Meyer-Eppler's studies [1] have shown that the spectral analysis of even the simplest orchestral sounds (they will form a reference system for a long time to come) presents variations of spectral lines in frequency as well as in amplitude. But these tiny (second order) variations are among those that make the difference between a lifeless sound made up of a sum of harmonics produced by a frequency generator and a sound of the same sum of harmonics played on an orchestral instrument. These tiny variations, which take place in the permanent, stationary part of a sound, would certainly require new theories of approach, using another functional basis and a harmonic analysis on a higher level, e.g., stochastic processes, Markov chains, correlated or autocorrelated relations, or theses of pattern and form recognition. Even so, analysis theories of orchestral sounds [27] would result in very long and complex calculations, so that if we had to simulate such an orchestral sound from a computer and from harmonic analysis on a first level, we would need a tremendous amount of computer time, which is impossible for the moment.

2. It seems that the transient part of the sound is far more important than the permanent part in timbre recognition and in music in general [28]. Now, the more the music moves toward complex sonorities close to "noise," the more numerous and complicated the transients become, and the more their synthesis from trigonometric functions becomes a mountain of difficulties, even more unacceptable to a computer than the permanent states. It is as though we wanted to express a sinuous mountain silhouette by using portions of circles. In fact, it is thousands of times more complicated. The intelligent ear is infinitely demanding, and its voracity for information is far from having been satisfied. This problem of a considerable amount of calculation is comparable to the 19th-century classical mechanics problem that led to the kinetic gas theory.

3. There is no pattern and form recognition theory, dependent on harmonic analysis or not, that would enable us to translate curves synthesized by means of trigonometric functions in the perception of forms or

configurations. For instance, it is impossible for us to define equivalence classes of very diversified oscilloscope curves, which the ear throws into the same bag. Furthermore, the ear makes no distinction between things that actual acoustic theories differentiate (e.g., phase differences, differential sensitivity ability), and vice versa.

The Wrong Concept of Juxtaposing Finite Elements

Perhaps the ultimate reason for such difficulties lies in the improvised entanglement of notions of finity and infinity. For example, in sinusoidal oscillation there is a unit element, the variation included in 2π . Then this *finite* variation is repeated endlessly. Seen as an economy of means, this procedure can be one of the possible optimizations. We labor during a limited span of time (one period), then repeat the product indefinitely with almost no additional labor. Basically, therefore, we have a mechanism (e.g., the sine function) engendering a finite temporal object, which is repeated for as long as we wish. This long object is now considered as a new element, to which we juxtapose similar ones. The odds are that one can draw any variation of one variable (e.g., atmospheric pressure) as a function of time by means of a finite superposition (sum) of the preceding elements. In doing this we expect to obtain an irregular curve, with increasing irregularity as we approach "noises." On the oscilloscope such a curve would look quite complex. If we ask the eye to recognize particular forms or symmetries on this curve it would almost certainly be unable to make any judgment from samples lasting say 10 microseconds because it would have to follow them too fast or too slowly: too fast for the everyday limits of visual attention, and too slow for the TV limits, which plunge the instantaneous judgment into the level of global perception of forms and colors. On the other hand, for the same sample duration, the ear is made to recognize forms and patterns, and therefore senses the correlations between fragments of the pressure curve at various levels of understanding. We ignore the laws and rules of this ability of the ear in the more complex and general cases that we are interested in. However, in the case in which we superpose sine curves, we know that below a certain degree of complexity the ear disentangles the constituents, and that above it the sensation is transformed into timbre, color, power, movement, roughness, and degree of disorder; and this brings us into a tunnel of ignorance. To summarize, we expect that by judiciously piling up simple elements (pure sounds, sine functions) we will create any desired sounds (pressure curve), even those that come close to very strong irregularities—almost stochastic ones. This same statement holds even when the unit element of the iteration is taken from a function

other than the sine. In general, and regardless of the specific function of the unit element, this procedure can be called *synthesis by finite juxtaposed elements*. In my opinion it is from here that the deep contradictions stem that should prevent us from using it.*

NEW PROPOSAL IN MICROCOMPOSITION BASED ON PROBABILITY DISTRIBUTIONS

We shall raise the contradiction, and by doing so we hope to open a new path in microsound synthesis research—one that without pretending to be able to simulate already known sounds, will nevertheless launch music, its psychophysiology, and acoustics in a direction that is quite interesting and unexpected.

Instead of starting from the unit element concept and its tireless iteration and from the increasing irregular superposition of such iterated unit elements, we can start from a disorder concept and then introduce means that would increase or reduce it. This is like saying that we take the inverse road: We do not wish to construct a complex sound edifice by using discontinuous unit elements (bricks = sine or other functions); we wish to construct sounds with continuous variations that are not made out of unit elements. This method would use stochastic variations of the sound pressure directly. We can imagine the pressure variations produced by a particle capriciously moving around equilibrium positions along the pressure ordinate in a non-deterministic way. Therefore we can imagine the use of any “random walk” or multiple combinations of them.

Method 1. Every probability function is a particular stochastic variation, which has its own personality (personal behavior of the particle). We shall then use *any* one of them. They can be discontinuous or continuous; e.g., Poisson, exponential (ce^{-cx}), normal, uniform, Cauchy ($t[\pi(t^2+x^2)]^{-1}$), $\text{arc sin}(\pi^{-1}[x(1-x)]^{-1/2})$, logistic $[(\alpha e^{-\alpha x - \beta})(1+e^{-\alpha x - \beta})^{-1}]$ distributions.

Method 2. Combinations of a random variable X with itself can be established. Example: If $f(x)$ is the probability function of X we can form $S_n = X_1 + X_2 + \dots + X_n$ (by means of the n -fold convolution of $f(x)$ with itself) or $P_K = X_1 \cdot X_2 \cdots X_K$, or any linear, polynomial, . . . , function of the variable X .

* In spite of this criticism I would like to draw attention to the magnificent manipulatory language Music V of Max V. Mathews, which achieves the final step in this procedure and automates it [29]. This language certainly represents the realization of the dream of an electronic music composer in the fifties.

Method 3. The random variables (pressure, time) can be functions of other variables (elastic forces), even of random variables. Example: The pressure variable x is under the influence of a centrifugal or centripetal force $\phi(x, t)$. For instance, if the particle (pressure) is influenced by a force wx (w being a constant) and also obeys a Wiener-Lévy process, then its density will be

$$q_t(x, y) = (w^{1/2}/[\pi(1 - e^{-2wt})]^{-1/2}) \exp [-w(y - xe^{-wt})^2/(1 - e^{-2wt})],$$

where x and y are the values of the variable at the instants 0 and t , respectively. (This is also known as the Ornstein-Uhlenbeck process.)

Method 4. The random variable moves between two reflecting (elastic) barriers. Example: If we again have a Wiener-Lévy process with two reflecting barriers at $a > 0$ and zero, then the density of this random walk will be

$$q_t(x, y) = (2\pi t)^{-1/2} \sum_{k=0}^{\pm\infty} (\exp [-(y - x + 2ka)^2/2t] + \exp [-(y + x + 2ka)^2/2t]),$$

where x and y are the values of the variables at the instants 0 and t , respectively, and $k = 0, \pm 1, \pm 2, \dots$

Method 5. The parameters of a probability function can be considered as variables of other probability functions (randomization, mixtures) [30]. Examples:

a. t is the parameter of a Poisson distribution $f(k) = (\alpha t)^k (k!)^{-1} e^{-\alpha t}$, and the random variable of the exponential density $g(t) = \beta e^{-\beta t}$. The combination is

$$f(k) * g(t) = w(k) = \int_{-\infty}^{\infty} (\alpha t)^k (k!)^{-1} e^{-\alpha t} \beta e^{-\beta t} dt = \beta (\alpha + \beta)^{-1} [\alpha(\alpha + \beta)^{-1}]^k,$$

which is a geometric distribution.

b. p and q are the probabilities of a random walk with jumps ± 1 (Bernoulli distribution). The time intervals between successive jumps are random variables with common density e^{-t} (Poisson distribution). Then the probability of the position n at instant t will be $f_n(t) = I_n(2t\sqrt{pq})e^{-t}(p/q)^{n/2}$, where

$$I_n(x) = \sum_{k=0}^{\infty} [k! \Gamma(k + n + 1)]^{-1} (x/2)^{2k+n}$$

is the modified Bessel function of the first kind of order n .

Method 6. Linear, polynomial, . . . , combinations of probability functions f_i are considered as well as composite functions (mixtures of a family of distributions, transformations in Banach space, subordination, etc.).

a. If A and B are any pair of intervals on the line, and $Q(A, B) = \text{prob } \{X \in A, Y \in B\}$ with $q(x, B) = \text{prob } \{X = x, Y \in B\}$ (q , under appropriate regularity conditions being a probability distribution in B for a given x and a continuous function in x for a fixed B ; that is, a conditional probability of the event $\{Y \in B\}$, given that $X = x$), and $\mu\{A\}$ is a probability distribution of $x \in A$, then $Q(A, B) = \int_A q(x, B) \mu\{dx\}$ represents a mixture of the family of distributions $q(X, B)$, which depends on the parameter x , with μ serving as the distribution of the randomized parameter [30].

b. Interlocking probability distributions (modulation). If f_1, f_2, \dots, f_n are the probability distributions of the random variables X^1, X^2, \dots, X^n , respectively, then we can form

$$S_{\sigma i}^i = X_1^i + X_2^i + \dots + X_{\sigma i}^i \quad \text{and} \quad S^n \left(\sum_{i=1}^n S_{\sigma i}^i \right) = S_{\sigma 1}^1 + S_{\sigma 2}^2 + \dots + S_{\sigma n}^n$$

or

$$P_{\gamma k}^k = X_1^k \cdot X_2^k \cdots X_{\gamma k}^k \quad \text{and} \quad P^n \left(\prod_{k=1}^n P_{\gamma k}^k \right) = P_{\gamma 1}^1 \cdot P_{\gamma 2}^2 \cdots P_{\gamma n}^n,$$

or any combination (functional or stochastic) of these sums and products. Furthermore, the σi and γk could be generated by either independent determined functions, independent stochastic processes, or interrelated determined or indetermined processes. In some of these cases we would have the theory of renewal processes, if, for instance, the σi were considered waiting times T_i . From another point of view, some of these cases would also correspond to the time series analysis of statistics. In reality, the ear seems to realize such an analysis when in a given sound it recognizes the fundamental tone pitch together with timbre, fluctuation, or casual irregularities of that sound! In fact, time series analysis should have been invented by composers, if they had—.

c. Subordination [30]. Suppose $\{X(t)\}$, a Markovian process with continuous transition probabilities

$$Q_t(x, \Gamma) = \text{prob } \{X(T(t+s)) \in \Gamma | X(T(s)) = x\}$$

(stochastic kernel independent of s), and $\{T(t)\}$, a process with non-negative independent increments. Then $\{X(T(t))\}$ is a Markov process

with transition probabilities

$$P_t(x, \Gamma) = \int_0^\infty Q_s(x, \Gamma) U_t\{ds\},$$

where U_t is the infinitely divisible distribution of $T(t)$. This P_t is said to be *subordinated* to $\{X(t)\}$, using the operational time $T(t)$ as the *directing* process.

Method 7. The probability functions can be filed into classes, that is, into parent curve configurations. These classes are then considered as elements of higher order sets. The classification is obtained through at least three kinds of criteria, which can be interrelated: a. analytical source of derived probability distribution; gamma, beta, . . . , and related densities, such as the density of χ^2 with n degrees of freedom (Pearson); Student's t density; Maxwell's density; b. other mathematical criteria, such as stability, infinite divisibility; and c. characteristic features of the curve designs: at level 0, where the values of the random variable are accepted as such; at level 1, where their values are accumulated, etc.

Macrocomposition

Method 8. Further manipulations with classes of distributions envisaged by Method 7 introduce us to the domain of macrocomposition. But we will not continue these speculations since many things that have been exposed in the preceding chapters could be used fruitfully in obvious ways. For example, sound molecules produced by the above methods could be injected into the ST(ochastic) program of Chap. V, the program forming the macrostructure. The same could be said about Chaps. II and III (Markovian processes at a macrolevel). As for Chaps. VI and VIII (symbolic music and group organization) establishing a complex microprogram is not as easy, but it is full of rich and unexpected possibilities.

All of the above new proposals are being investigated at the Centers for Mathematical and Automated Music (CMAM) at both the School of Music of Indiana University, Bloomington, Indiana, and the Nuclear Research Center of the Collège de France, in Paris. Digital to analogue converters with 16 bits resolution at a rate of $0.5 \cdot 10^5$ samples per second are available in both places.

Figs. IX, 1–8 were calculated and plotted at the Research Computing Center of Indiana University under the supervision of Cornelia Colyer. These graphs could correspond to a sound duration of 8 milliseconds, the ordinates being the sound pressures.

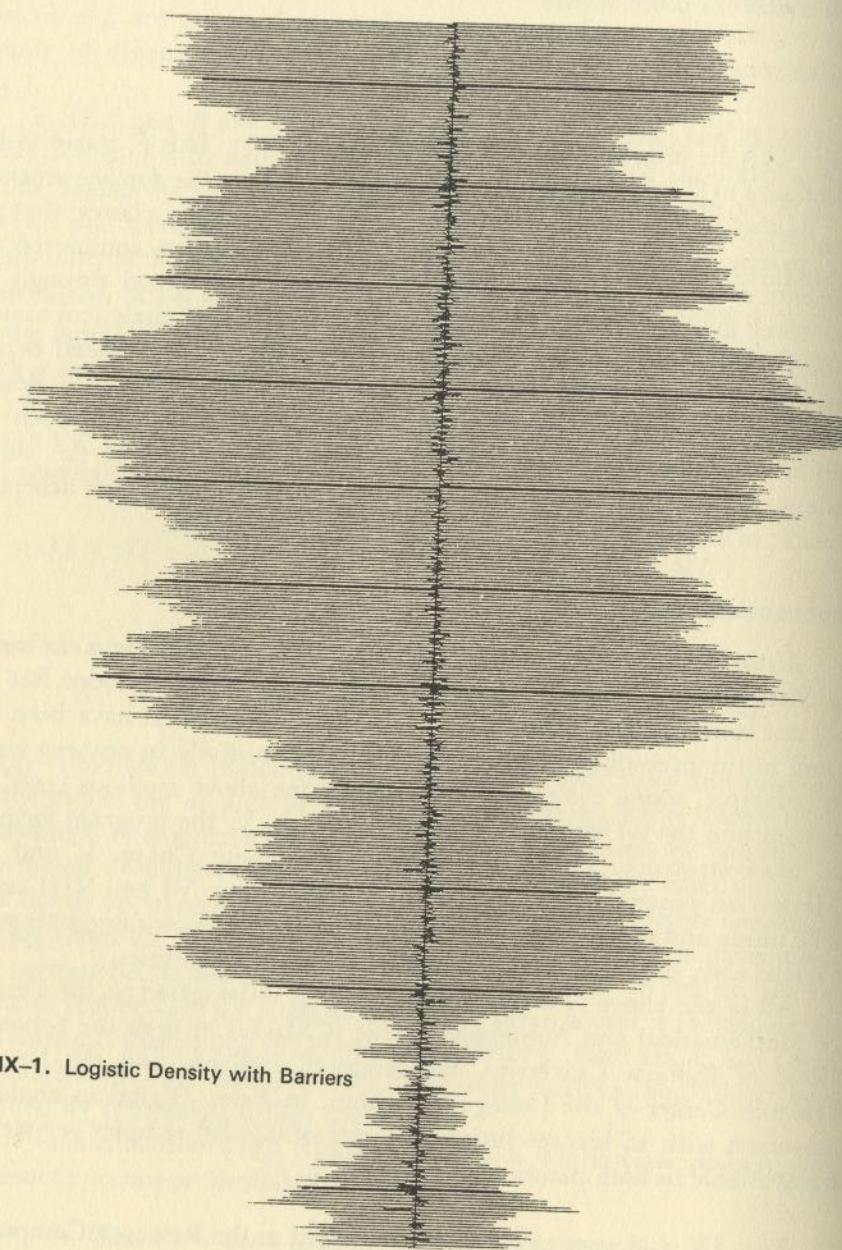


Fig. IX-1. Logistic Density with Barriers

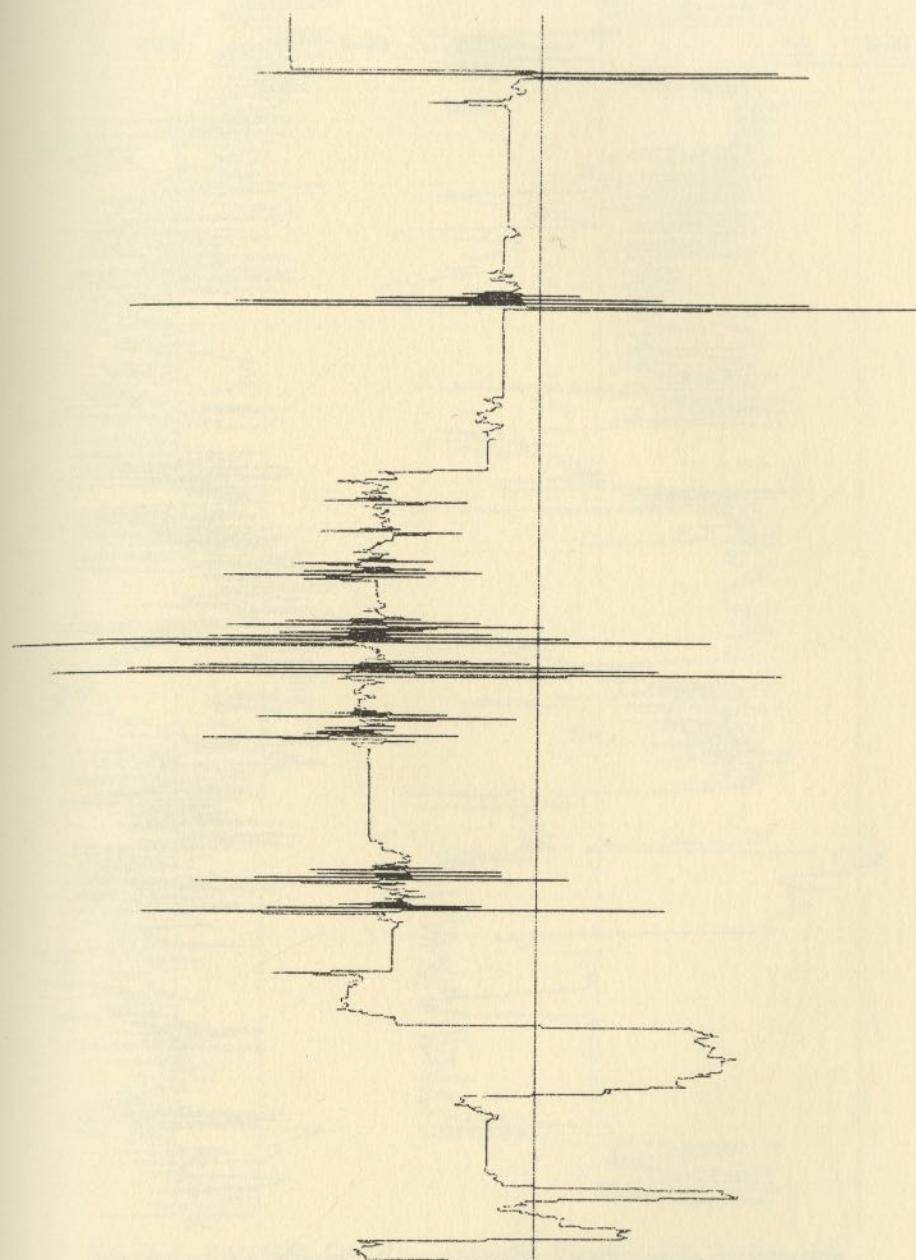


Fig. IX-2. Exponential \times Cauchy Densities with Barriers and Randomized Time

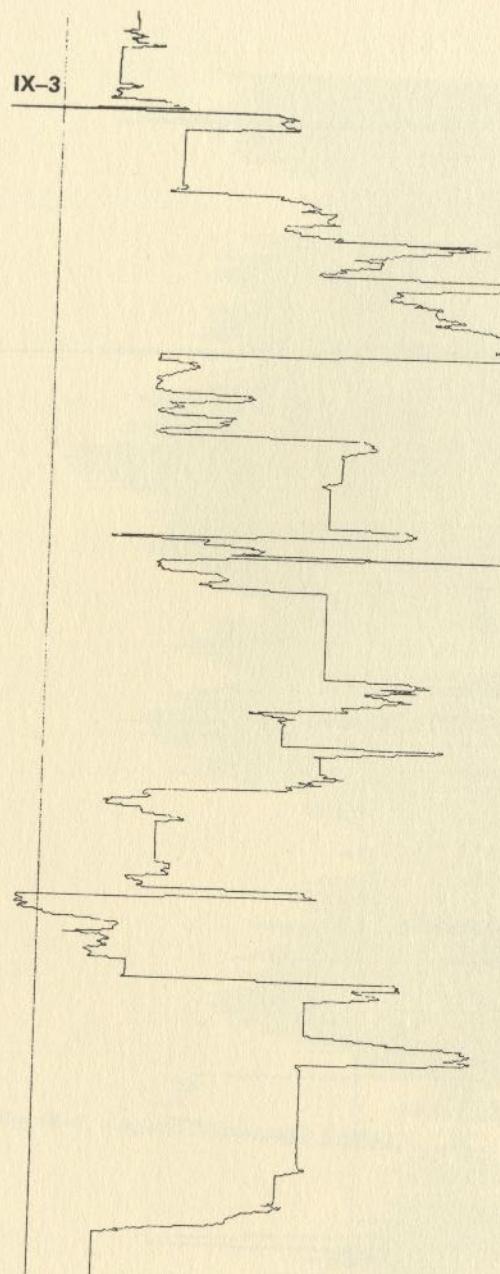


Fig. IX-3. Exponential \times Cauchy Densities with Barriers and Randomized Time



Fig. IX-4. Exponential \times Cauchy Densities with Barriers and Randomized Time

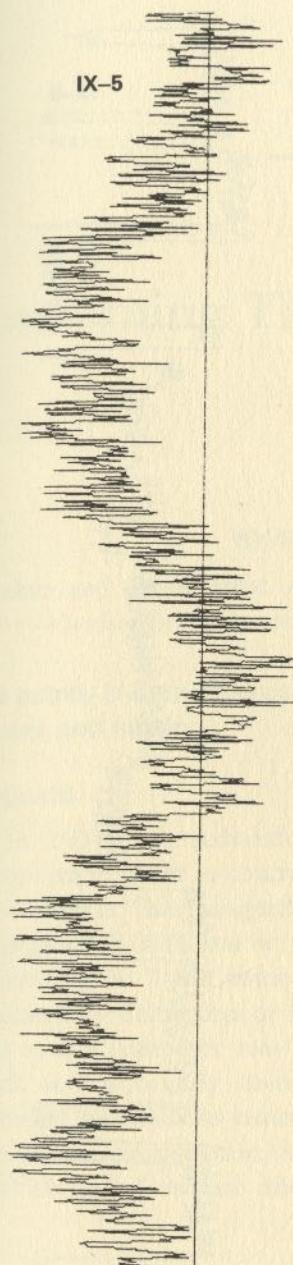


Fig. IX-5. Hyperbolic Cosine \times Exponential Densities with Barriers and Determined Time

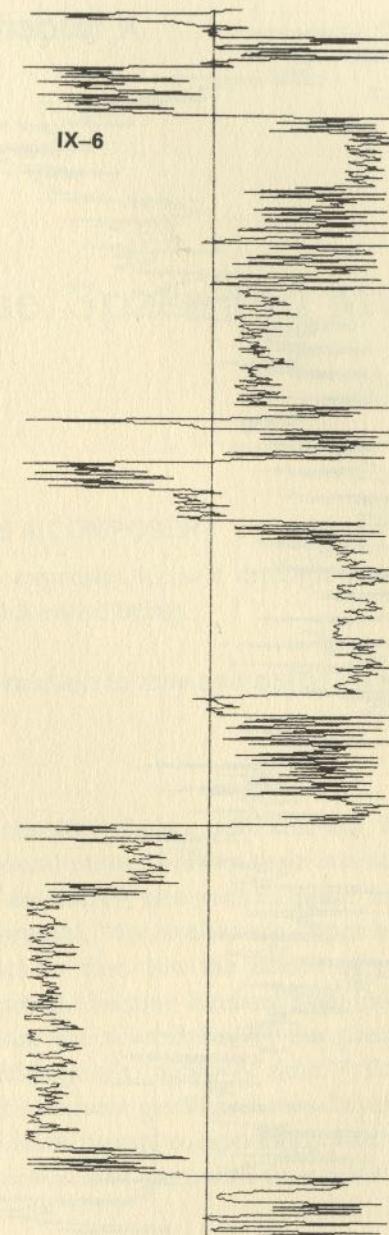


Fig. IX-6. Hyperbolic Cosine \times Exponential Densities with Barriers and Randomized Time

Concerning Time, Space and Music*

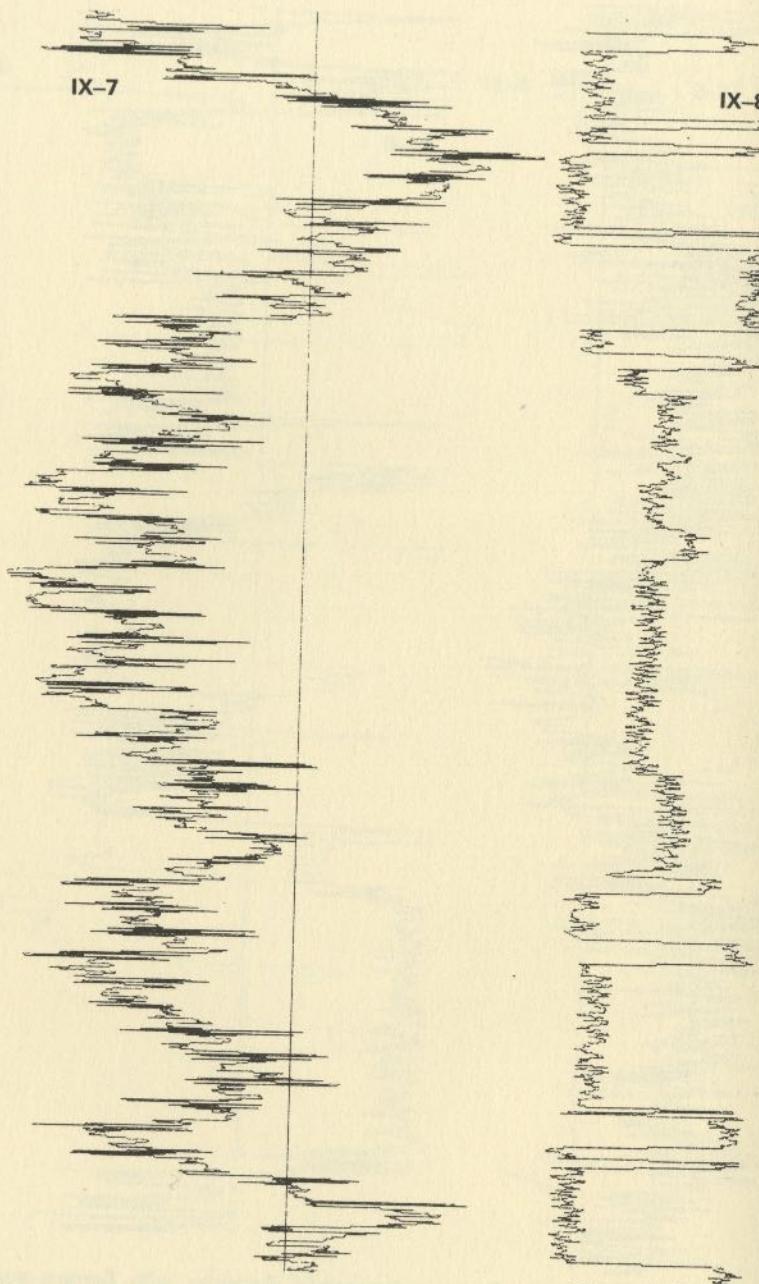


Fig. IX-7. Hyperbolic Cosine \times Exponential \times Cauchy Densities with Barriers and Determined Time
 Fig. IX-8. Logistic \times Exponential Densities with Barriers and Randomized Time

WHAT IS A COMPOSER?

A thinker and plastic artist who expresses himself through sound beings. These two realms probably cover his entire being.

A few points of convergence in relation to time and space between the sciences and music:

First point:

In 1954, I introduced probability theory and calculus in musical composition in order to control sound masses both in their invention and in their evolution. This inaugurated an entirely new path in music, more global than polyphony, serialism or, in general, "discrete" music. From hence came stochastic music. I will come back to that. But the notion of entropy, as formulated by Boltzmann or Shannon,¹ became fundamental. Indeed, much like a god, a composer may create the reversibility of the phenomena of masses, and apparently, invert Eddington's "arrow of time."² Today, I use probability distributions either in computer generated sound synthesis on a micro or macroscopic scale, or in instrumental compositions. But the laws of probability that I use are often nested and vary with time which creates a

*Excerpts of Chapter X originally appeared in English in *Perspectives of New Music*, Vol. 27, N° 1. Those excerpts appeared originally in French in *Redécouvrir le Temps*, Editions de l'Université de Bruxelles, 1988, Vol. 1-2.

stochastic dynamics which is aesthetically interesting. This procedure is akin to the mathematical analysis of Liouville's equation on non-unitary transformations proposed essentially by I. Prigogine,³ namely, if the microscopic entropy M exists, then $M = \Lambda^2$, where Λ acts on the distribution function or the density matrix. Λ is non-unitary which means that it does not maintain the size of probabilities of the states considered during the evolution of the dynamic system, although it does maintain the average values of those which can be observed. This implies the irreversibility of the system to the equilibrium state; that is, it implies the irreversibility of time.

Second point:

This point has no obvious relationship to music, except that we could make use of Lorentz-Fitzgerald and Einstein transformations in the macroscopic composition of music.⁴ I would nevertheless like to make some comments related to these transformations.

We all know of the special theory of relativity and the equations of Lorentz-Fitzgerald and Einstein, which link space and time because of the finite velocity of light. From this it follows that time is not absolute. Yet time is always there. It "takes time" to go from one point to another in space, even if that time depends on moving reference frames relative to the observer. There is no instantaneous jump from one point to another in space, much less "spatial ubiquity"—that is, simultaneous presence of an event or an object in two sites in space. On the contrary, one posits the notion of displacement. Within a local reference frame, what then does displacement signify? If the notion of displacement were more fundamental than that of time, one could undoubtedly reduce all macro and microcosmic transformations to extremely short chains of displacement. Consequently (and this is an hypothesis that I freely advance), if we were to adhere to quantum mechanics and its implications accepted now for decades, we would perhaps be forced to admit the notion of quantified space and its corollary, quantified time. But then, what could a quantified time and space signify, a time and space in which contiguity would be abolished? What would the pavement of the universe be if there were gaps between the paving stones, inaccessible and filled with nothing? Time has already been proposed as having a quantic structure by T. D. Lee of Columbia University.

Let us return to the notion of time considered as duration. Even after the experimental demonstration of Yang and Lee which has abolished the parity symmetry P ,⁵ it seems that the *CPT* theorem still holds for the symmetries of the electron (C) and of time (T), symmetries that have not yet

been completely annulled. This remains so even if the "arrow of time" appears to be nonreversible in certain weak interactions of particles. We might also consider the poetic interpretation of Feynman,⁶ who holds that when a positron (a positively charged particle created simultaneously with an electron) collides with an electron, there is, in reality, only one electron rather than three elementary particles, the positron being nothing but the temporal retrogression of the first electron. Let us also not forget the theory of retrograde time found in Plato's *Politicos*—or in the future contraction of the universe. Extraordinary visions!

Quantum physics will have difficulty discovering the reversibility of time, a theory not to be confused with the reversibility of Boltzmann's "arrow of entropy." This difficulty is reflected in the explanations that certain physicists are attempting to give even today for the phenomenon called the "delayed choice" of the two states—corpuscular or wave—of a photon. It has been proven on many occasions that the states depend entirely on observation, in compliance with the theses of quantum mechanics. These explanations hint at the idea of an "intervention of the present into the past," contrary to the fact that causality in quantum mechanics cannot be inverted. For, if the conditions of observation are established to detect the particle, then one obtains the corpuscular state and never the wave state, and vice versa. A similar discussion on non-temporality and the irreversibility of the notion of causality was undertaken some time ago by Hans Reichenbach.⁷

Another fundamental experiment has to do with the correlation of the movement of two photons emitted in opposite directions by a single atom. How can one explain that both either pass through two polarizing films, or that both are blocked? It is as if each photon "knew" what the other was doing and instantaneously so, which is contrary to the special theory of relativity.

Now, this experiment could be a starting point for the investigation of more deeply seated properties of space, freed from the tutelage of time. In this case, could the "nonlocality" of quantum mechanics perhaps be explained not by the hypothesis of "hidden variables" in which time still intervenes, but rather by the unsuspected and extravagant properties of nontemporal space, such as "spatial ubiquity," for example?

Let us take yet one more step. As space is perceptible only across the infinity of chains of energy transformations, it could very well be nothing but an appearance of these chains. In fact, let us consider the movement of a photon. Movement means displacement. Now, could this displacement be considered an autogenesis of the photon by itself at each step of its trajectory

(continuous or quantized)? This continuous auto-creation of the photon, could it not, in fact, be space?

Third point: Case of creating something from nothing

In musical composition, construction must stem from originality which can be defined in extreme (perhaps inhuman) cases as the creation of new rules or laws, as far as that is possible; as far as possible meaning original, not yet known or even foreseeable. Construct laws therefore from nothing, since without any causality.

But a construction from nothing, therefore totally engendered, totally original, would necessarily call upon an infinite mass of rules duly entangled. Such a mass would have to cover the laws of a universe different from our own. For example: rules for a tonal composition have been constructed. Such a composition therefore includes, *a priori*, the "tonal functions." It also includes a combinatory conception since it acts on entities, sounds, as defined by the instruments. In order to go beyond this slight degree of originality, other functions would have to be invented, or no functions should exist at all. One is therefore obliged to conceive of forms from thoughts bearing no relation to the preceding ones, thoughts without limits of shapes and without end. Here, we are obliged to progressively weave an unlimited web of entangled rules—and that alone in the combinatory realm which itself excludes, by definition, any possible continuums of sound. However, the insertion of continuity will consequently augment the spread of this web and its compacity. Furthermore, if one cared to engender the unengenderable in the realm of sound, then it would be necessary to provide rules other than those for sound machines such as pipes, strings, skins, etc. which is possible today thanks to computers and corresponding technologies. But technology is both but a semblance of thought and its materialisation. It is therefore but an epiphenomenon in this discussion. Actually, rules of sound synthesis such as those stemming from Fourier series should not be used any more as the basis of construction. Others, different ones, must be formulated.

Another perspective: We have seen how construction stems from an originality which is defined by the creation of rules and laws outside of an individual's or even the human species' memory. However, we have left aside the notion of rules or laws. Now the time has come to discuss this notion. A rule or law signifies a finite or infinite procedure, always the same, applied to continuous or "discrete" elements. This definition implies the notion of repetition, of recurrence in time, or symmetry in realms outside time (hors temps). Therefore, in order for a rule to exist, it must be applicable several

times in eternity's space and time. If a rule were to exist but once, it would be swallowed up in this immensity and reduced to a single point, therefore unobservable. In order for it to be observable, it must be repeatable an infinite number of times.

Subsidiary question: Can one repeat a phenomenon? (cf. Herakleitos: "It is impossible to step twice into the same river," and Kratyllos: "not even once.")

But the fact remains that the universe:

- a) seems, for the time being, to be made up of rules-procedures;
- b) that these rules-procedures are recurrent.

It is as though the Being (in disagreement with Parmenides), in order to continue existing, is obliged to die; and once dead, is obliged to start his cycle again. Existence, therefore, is a dotted line.

Can one, at last, imagine an infinitesimal microscopic rule that is engendered from nothing? Even if physics has yet to discover anything resembling this, despite "Lamb's shift" (which sees each point in space in our universe as seething in virtual pairs of particles and anti-particles), we can imagine such an eventuality which would, by the way, be of the same nature as the fact of pure chance, detached from any causality.

It is necessary to depend on such a conclusion of a Universe open to the unprecedented which relentlessly would be formed or would disappear in a truly creative whirlwind, beginning from nothingness and disappearing into nothing. The same goes for the basis of art as well as for man's destiny.

Here, below, is the thesis of a few astrophysicists such as Edward Tryon, Alexander Vilenkin, Alan Guth, Paul Steinhardt, adherents to the Big Bang theory:

If grand unified theories are correct in their prediction that baryon number is not conserved, there is no known conservation law that prevents the observed universe from evolving out of nothing. The inflationary model of the universe provides a possible mechanism by which the observed universe could have evolved from an infinitesimal region. It is then tempting to go one step further and speculate that the entire universe evolved from literally nothing. (cf. *Scientific American*, May, 1984)

The multiplicity of such universes according to Linde⁸ from Moscow is also quite intriguing.

Here, below, is an alternative to the Big Bang scenario. These studies have been followed by the physicists of the University of Bruxelles; namely R. Brout, E. Günzig, F. Englert and P. Spindel:

Rather than the Universe being born of an explosion, they propose that it appeared ex-nihilo following an *instability* of the minkowskian quantum void, meaning that space-time was devoid of any matter, therefore flat or yet-without any curvature." (cf. Coveney, Peter V., "L'irréversibilité du temps," *La Recherche*, Paris, February, 1989).⁹

What is extraordinary is that both propositions, Big Bang or not, admit a beginning, an origin from nothing, or nearly nothing with, however, cycles of re-creation! With a most extreme modesty, I would like to compare, especially the last hypothesis, with a scientific-musical vision I had made in 1958. At that time, I wanted to do away with all of the inherited rules of composition in order to create new ones. But the question that came to my mind at that time was whether a music could still have meaning even if it was not built on rules of occurrence. In other words, void of rules. Below are the steps in this thought process:

"For it is the same thing to think and to be"
(*The Poem, Parmenides*)

and my paraphrase

"For it is the same thing not to be and to be"

Ontology:

In a Universe of Void. A brief train of waves whose beginning and end coincide (nil Time), perpetually triggering off.

Nothingness resorbs, creates.

It is the generator of Being.

Time, Causality.

This text was first published in *Gravesaner Blätter*, N° 11/12, 1958, the revue published by the great conductor, Hermann Scherchen. At that time, I had temporarily resolved this problem in creating music *uniquely* through the help of probability distributions. I say "temporarily" since each probability function has its own finality and therefore is not a nothing.*

*Cf. also page 24 for a slightly different rendition of the same material (S.K.)

Another question

The actual state of knowledge seems to be the manifestation of the evolution of the universe since, let us say, some fifteen billion years.¹⁰ By that, I mean that knowledge is a secretion of the history of humanity, produced by this great lapse of time. Assuming this hypothesis, all that which our individual or collective brain hatches as ideas, theories or know-how, is but the output of its mental structures, formed by the history of the innumerable movements of its cultures, in its anthropomorphic transformations, in the evolution of the earth, in that of the solar system, in that of the universe. If this is so, then we face a frightening, fundamental doubt as to the "true objectivity" of our knowledge and know-how. For if, with bio-technologies already developing, one were to transform these mental structures (our own) and their heredity, therefore the rules for the functioning of the brain based on certain premises today, on logic or systems of logic, and so on ..., if one were to succeed in modifying them, one would gain, as if by sort of a miracle, another vision of our universe, a vision which would be built upon theories and knowledge which are beyond the realm of our present thought.

Let us pursue this thought. Humanity is, I believe, already on this path. Today, humanity, it seems to me, has already taken the first step in a new phase of its evolution, in which not only the mutations of the brain, but also the creation of a universe very different from that which presently surrounds us, has begun. Humanity, or generalizing, the species which may follow it, will accomplish this process.

Music is but a path among others for man, for his species, first to imagine and then, after many, many generations, to entail this existing universe into another one, one fully created by man. Indeed, if man, his species, is the image of his universe, then man, by virtue of the principle of creation from nothingness and disappearance into nothingness (which we are forced to set), could redefine his universe in harmony with his creative essence, such as an environment he could bestow upon himself.

IN MUSIC

In the following comments, the points of view on time are taken from music in gestation or under observation. This is not to say that my preceding comments do not concern the musician. On the contrary, if it is incumbent on music to serve as a medium for the confrontation of philosophic or scientific ideas on the being, its evolution, and their appearances, it is essential that the composer at least give some serious thought to these types of inquiry.

Furthermore, I have deliberately not approached the psychological apprehension of time from higher levels, for example, the effects of the temporal dynamic experience while listening to a symphony or to electronic music.

What is time for a musician? What is the flux of time which passes invisibly and impalpable? In truth, we seize it only with the help of perceptive reference-events, thus indirectly, and under the condition that these reference-events be inscribed somewhere and do not disappear without leaving a trace. It would suffice that they exist in our brain, our memory. It is fundamental that the phenomena-references leave a trace in my memory, for if not, they would not exist. Indeed, the underlying postulate is that time, in the sense of an impalpable, Heraclitian flux, has signification only in relation to the person who observes, to me. Otherwise, it would be meaningless. Even assuming the hypothesis of an objective flux of time, independent from me, its apprehension by a human subject, thus by me, must be subject to the phenomena-reference of the flux, first perceived, then inscribed in my memory. Moreover, this inscription must satisfy the condition that it be in a manner which is well circumscribed, well detached, individualized, without possible confusion. But that does not suffice to transform a phenomenon that has left traces in me into a referential phenomenon. In order that this trace-image of the phenomenon become a reference mark, the notion of anteriority is necessary. But this notion seems to be circular and as impenetrable as the immediate notion of flux. It is a synonym. Let us alter our point of view, if only slightly. When events or phenomena are synchronic, or rather, if all imaginable events were synchronic, universal time would be abolished, for anteriority would disappear. By the same token, if events were absolutely smooth, without beginning or end, and even without modifications or "perceptible" internal roughness, time would likewise find itself abolished. It seems that the notion of separation, of bypassing, of difference, of discontinuity, which are strongly interrelated, are prerequisite to the notion of anteriority. In order for anteriority to exist, it is necessary to be able to distinguish entities, which would then make it possible to "go" from one to the other. A smooth continuum abolishes time, or rather time, in a smooth continuum, is illegible, inapproachable. Continuum is thus a unique whole filling both space and time. We are once again coming back to Parmenides. Why is space included among those things that are illegible? Well, because of its non-roughness. Without separability, there is no extension, no distance. The space of the universe would find itself condensed into a mathematical point without dimensions. Indeed, Parmenides' Being,

which fills all space and eternity, would be nothing but an absolutely smooth "mathematical point."

Let us get back to the notion of separability, first in time. At the least, separability means non-synchronisation. We discover once again the notion of anteriority. It merges with the notion of temporal ordering. The ordering anteriority admits no holes, no empty spaces. It is necessary for one separable entity to be contiguous with the next, otherwise, one is subject to a confusion of time. Two chains of contiguous events without a common link can be indifferently synchronous or anterior in relation to each other; time is once again abolished in the temporal relation of each of the universes represented by the two chains. On the contrary, local clocks serve as chains without gaps, but only locally. Our biological beings have also developed local clocks but they are not always effective. And memory is a spatial translation of the temporal (causal) chains. We will come back to this.

I have spoken of chains without gaps. At the moment and to my knowledge, local gaps have not yet been discovered in sub-atomic physics or in astrophysics. And in his theory of the relativity of time, Einstein tacitly accepts this postulate of time without gaps in local chains, but his theory also constructs special chains without gaps between spatially separable localities. Here, we are definitely not concerned with the reversibility of time which was partially examined above in light of recent discoveries in sub-atomic physics, for reversibility would not abolish time.

Let us examine the notion of separability, of discontinuity in space. Our immediate consciousness (a mental category?) allows us to imagine separated entities which, in turn, necessitate contiguity. A void is a unity in this sense, contrarily to time, in which our inherited or acquired mental notions bar us from conceiving the absence of time, its abolition, as an entity sharing time, the primordial flux. Flux either is, or is not. We exist, therefore it is. For the moment, one cannot conceive of the halting of time. All this is not a paraphrase of Descartes or better yet, of Parmenides: it is a presently impassable frontier. (But certainly, by using Parmenides once more, passable: "TO ΓΑΡ ΑΥΤΟ ΝΟΕΙΝ ΕΣΤΙΝ ΤΕ ΚΑΙ ΕΙΝΑΙ").

To get back to space, the void can be imagined as a dwindling of the entity (phenomenon) down to an infinitesimal tenuousness, having no density whatsoever. On the other hand, to travel from one entity to another is a result of scale. If a person who voyaged were small, the person would not encompass the totality of entities, the universe at once. But if this person's scale were colossal, then yes. The universe would offer itself in one stroke, with hardly a scan, as when one examines the sun from afar.

The entities would appear, as in a snapshot, reunited in a dense network of nontemporal contiguities, uninterrupted, extending through the entire universe. I said, in a snapshot. This is to say that in the snapshot, the spatial relations of the entities, the forms that their contiguities assume, the structures, are essentially outside time (*hors-temps*). The flux of time does not intervene in any way. That is exactly what happens with the traces that the phenomenal entities have left in our memory. Their geographical map is outside time.

Music participates both in space outside time and in the temporal flux. Thus, the scales of pitch; the scales of the church modes; the morphologies of higher levels; structures, fugal architectures, mathematical formulae engendering sounds or pieces of music, these are outside time, whether on paper or in our memory. The necessity to cling against the current of the river of time is so strong that certain aspects of time are even hauled out of it, such as the durations which become commutable. One could say that every temporal schema, pre-conceived or post-conceived, is a representation outside time of the temporal flux in which the phenomena, the entities, are inscribed.

Due to the principle of anteriority, the flux of time is locally equipped with a structure of total order in a mathematical sense. That is to say that its image in our brain, an image constituted by the chain of successive events, can be placed in a one-to-one correspondance with the integers and even, with the aid of a useful generalization, with real numbers (rational and irrational). Thus, it can be counted. This is what the sciences in general do, and music as well, by using its own clock, the metronome. By virtue of this same structure of total order, time can be placed in a one-to-one correspondence with the points of a line. It can thus be drawn.

This is done in the sciences, but also in music. One can now design temporal architectures—rhythms—in a modern sense. Here is a tentative axiomatization of the temporal structures placed outside of time:

1. We perceive temporal events.
2. Thanks to separability, these events can be assimilated to *landmark points* in the flux of time, points which are instantaneously hauled up outside of time because of their trace in our memory.
3. The comparison of the *landmark points* allows us to assign to them distances, intervals, durations. A distance, translated spatially,

can be considered as the displacement, the step, the jump from one point to another, a nontemporal jump, a spatial distance.

4. It is possible to repeat, to link together these steps in a chain.
5. There are two possible orientations, one by an accumulation of steps, the other by a de-accumulation.

From here, we can construct an object which can be represented by points on a line, evenly spaced and symbolized by the numeral 1 with index zero: $1_0 = \{..., -3, -2, -1, 0, 1, 2, 3, \dots\}$. This is the regular rhythm, corresponding to the whole numbers. As the size of the step is not defined in the preceding propositions (recalling Bertrand Russell's observation concerning Peano's axiomatic of natural numbers¹¹), we can affix to the preceding object the following objects which I call "sieves," by using solely proposition 4:

$$\begin{aligned} 2_0 &= \{..., -4, -2, 0, 2, 4, 6, \dots\} \text{ or } 2_1 = \{..., -3, -1, 1, 3, 5, \dots\} \text{ or} \\ 3_0 &= \{..., -3, 0, 3, 6, 9, \dots\} \text{ or } 3_1 = \{..., -5, -2, 1, 4, 7, \dots\} \text{ or} \\ 3_2 &= \{..., -4, -1, 2, 5, 8, \dots\} \text{ etc...} \end{aligned}$$

From these objects and their modular nature, and with the help of these three logical operations:

$$\begin{aligned} \cup &\text{ union, disjunction ex. } 2_0 \cup 2_1 = 1_0 \\ \cap &\text{ intersection, conjunction ex. } 2_0 \cap 2_1 = 0 \\ - &\text{ complementarity, negation ex. } \overline{2_0} = 2_1 \end{aligned}$$

we can construct logical functions L—that is to say, very complex rhythmic architectures which can even go as far as a random-like distribution of points on a line—if the period is sufficiently long. The interplay between complexity and simplicity is, on a higher level, another way of defining the landmark points, which certainly plays a fundamental role in aesthetics, for this play is juxtaposed with the pair release/tension.

Example of a logical function L:

$$L = \overline{(M_k \cap N_j \cap P_l)} \cup (N_r \cup Q_s \cup \dots T_y) \cup \dots$$

The upper-case letters designate moduli and the subscripts designate shifts in relation to a zero point of reference.

Up to this point, we have examined time perceived by means of our faculties of attention and conscious thought—time on the level of forms and structures of an order ranging from tens of minutes to approximately one twenty-fifth of a second. A stroke of the bow is a referential event that can define durations of a fraction of a second. Now, there are some subliminal

events found on several even lower levels. Such an example is that of the temporal segmentation produced by a very choppy amplitude envelope on the sound of an unvarying sinusoidal wave form. If the duration of the note is long (about one minute), we perceive the rhythms of the beats as pleasant, moving vibratos. If the duration is relatively short (three seconds), the ear and the brain interpret it as a timbre. That is to say that the result of subliminal, unconscious counting is different in nature and is recognized as timbre.

Let us take a brief moment to consider the mechanism of the internal ear coupled with the brain which recognizes the wave form—that is to say, the timbre—and the frequency of a sound. On the one hand, it seems that the points of deformation of the basilar membrane play a fundamental role in the recognition; but, on the other hand, a sort of temporal Morse code of electrical discharges of neurons is taken statistically into account for the detection of tone. A remarkably complex subliminal counting of time is taking place. But knowledge of acoustics in this domain is still very limited.

On this subliminal level, here is another disconcerting phenomenon. It is the result of a new theory on the synthesis of computer sounds which circumvents the harmonic synthesis of Fourier, practiced everywhere today, a theory which I introduced now more than fifteen years ago.¹² It is a question of beginning with any form whatsoever of an elementary wave, and with each repetition, of having it undergo small deformations according to certain densities of probabilities (Gauss, Cauchy, logistic,...) appropriately chosen and implemented in the form of an abstract black box. The result of these deformations is perceptible on all levels, microstructure (= timbre), ministructure (= note), mesostructure (= polyrhythm, melodic scales of intensities), macrostructure (= global evolution on the order of some tens of minutes).

If the rate of sampling had been 1,000,000 or 2,000,000 samples per second instead of approximately 44,100 (commercial standard), one would have had an effect of sounding fractals, with a sonorous effect which is impossible to predict.

We see to what extent music is everywhere steeped in time: (a) time in the form of an impalpable flux or (b) time in its frozen form, outside time, made possible by memory. Time is the blackboard on which are inscribed phenomena and their relations outside the time of the universe in which we live. Relations imply architectural structures, rules. And, can one imagine a rule without repetition? Certainly not. I have already treated this subject. Besides, a single event in an absolute eternity of time and space would make

no sense. And yet, each event, like each individual on earth, is unique. But this uniqueness is the equivalent of death which lies in wait at every step, at every moment. Now, the repetition of an event, its reproduction as faithfully as possible, corresponds to this struggle against disappearance, against nothingness. As if the entire universe fought desperately to hang on to existence, to being, by its own tireless renewal at every instant, at every death. The union of Parmenides and of Heraclitus. Living species are an example of this struggle of life or death, in an inert Universe launched perhaps by the Big Bang (is it really inert, that is, without any changes in its laws?). This same principle of dialectical combat is present everywhere, verifiable everywhere. Change—for there is no rest—the couple death and birth lead the Universe, by duplication, the copy being more or less exact. The “more or less” makes the difference between a pendular, cyclic Universe, strictly determined (even a deterministic chaos), and a nondetermined Universe, absolutely unpredictable and chaotic. Unpredictability in thought obviously has no limits. On a first approach it would correspond to birth from nothingness, but also to disappearance, death into nothingness. At the moment, the Universe seems to be midway between these two chasms, something which could be the subject of another study. This study would deal with the profound necessity for musical composition to be perpetually original—philosophically, technically, aesthetically.¹³

In what follows and as a consequence of the preceding axioms, we will study in greater detail the practical questions of how to create a sieve (= series of points on a line), beginning from a logical function of moduli (periods), or inversely, from a series of points on a line, how to create a logical function of moduli which should be able to engender the given series. This time, we shall use series of “pitches” taken from musical space.

Sieves*

In music, the question of symmetries (spatial identities) or of periodicities (identities in time) plays a fundamental role at all levels: from a sample in sound synthesis by computers, to the architecture of a piece. It is thus necessary to formulate a theory permitting the construction of symmetries which are as complex as one might want, and inversely, to retrieve from a given series of events or objects in space or time the symmetries that constitute the series. We shall call these series "sieves."¹

Everything that will be said here could be applied to any set of characteristics of sound or of well-ordered sound structures, and especially, to any group which entails an additive operation and whose elements are multiples of a unity; that is to say that they belong to the set N of natural numbers. For example: pitches, time-points, loudnesses, densities, degrees of order, local timbres, etc. In the case of pitches, there must be a distinction between sieve (scale) and mode. Indeed, the white keys on a piano constitute a unique sieve (scale) upon which are formed the "modes" of C major, D, E, G, A (natural minor), etc. Just like Indian ragas or Olivier Messiaen's modes "of limited transpositions," modes are defined by cadential, harmonic, etc. formulas.

But every well-ordered set can be represented as points on a line, as long as a reference point is given for the origin and a length u for the unit distance, and this is a sieve. Historically, the invention of the well-tempered

*This chapter is scheduled to appear in a future issue of *Perspectives of New Music*. John Rahn's personal contribution to the following material is most appreciated (I.X.)

chromatic scale, attributed to the Renaissance, is of upmost importance since it provided a universal standardization of the realm of pitches, as fertile as that which already existed for rhythm. However, it should be remembered that the first theoretical attempt towards such an approach which opened the path to number theory in music dates back to Aristoxen of Tarent, during the IVth century B.C., in his "Harmonics."²

CONSTRUCTION OF A SIEVE

Starting from symmetries (repetitions), let us construct a sieve (scale). As a melodic example, we shall construct the diatonic scale formed by the white keys of the piano.

With u = one semitone = one millimeter and a zero reference point taken arbitrarily on a note, for example C3, we can notate the diatonic sieve (scale) on graph paper scaled to the millimeter, by means of points to the left and to the right of this zero reference point with successive intervals counted from left to right of 2, 2, 1, 2, 2, 1, 2, 2, 1, 2, 2, 1, ... millimeters, or we can write the sieve in a logical-arithmetic notation as $L = 12_0 \cup 12_2 \cup 12_4 \cup 12_5 \cup 12_7 \cup 12_9 \cup 12_{11}$ where 12 is the modulus of the symmetry (period) of the octave with u for the semitone. This notation gives all the Cs, all the Ds, ... all the Bs, considering that the moduli 12 repeat on both sides of the zero reference point. The indices, 0, 2, 4, 5, 7, 9, 11 of the modulus 12 signify shifts to the right of the zero of the modulus 12. They also represent the residue classes of congruence mod. 12.

With a different unit distance u , for example, a quartertone, one would have the same structure as the diatonic scale but the period of the series would no longer be an octave, but an augmented fourth.

In a similar fashion, a periodic rhythm, for example 3, 3, 2:



can be notated as $L = 7_0 \cup 7_3 \cup 7_5$. In both of these examples, the sign \cup is a logical union (and/or) of the points defined by the moduli and their shiftings.

The periodicity of the diatonic sieve (scale) is external to the sieve itself and is based on the existence of the modulus 12 (the octave). Its internal symmetry can be studied in the indices I (shiftings, residue classes) of the terms 12_i . But it would be interesting to give, when it exists, a more hidden

symmetry derived from the decomposition of the modulus 12 into simpler moduli (symmetries, periodicities), such as 3 and 4, a decomposition which would have the advantage of allowing a comparison among different sieves in order to study the degree of their difference and to be able to define a notion of distance in this way.

Let us take the elementary sieves 3_0 and 4_0 . In taking the points 3_0 and/or the points 4_0 , we obtain a series $H_1 = \{ \dots, 0, 3, 4, 6, 8, 9, 12, 15, 16, 18, 20, 21, 24, 27, 28, \dots \} = 3_0 \cup 4_0$, and if C is the zero and $u =$ one semitone, H_1 becomes $\{ \dots, C, D\#, E, F\#, G\#, A, C, D\#, E, \dots \}$. But if we take the points common to 3_0 and 4_0 , we obtain the series $H_2 = \{ \dots, 0, 12, 24, 36, \dots \} = 3_0 \cap 4_0$ where the sign \cap is the logical intersection (and) of the sets of points defined by these moduli and their respective shiftings.

Hence, we observe that the series H_2 can be defined by the modulus $12 = 3 * 4$ and by the logical expression $L = 12_0$ which gives the octaves. The number 12 is the smallest common multiple of 3 and 4, which are coprime, meaning their largest common denominator is 1.⁴

Let us imagine now the elementary sieves 2_0 and 6_0 . Then $G_1 = 2_0 \cup = \{ \dots, 0, 2, 4, 6, 8, 10, 12, \dots \}$ and the common points in $G_2 = 2_0 \cap 6_0 = \{ \dots, 0, 6, 12, 18, \dots \}$. But here, the series is no longer made into octaves as in the preceding case.

To understand this, let us take another example with elementary moduli $M1 = 6$ and $M2 = 15$ which have been adjusted to the original. We then form the pairs $6_0 = (M1, I1)$ and $15_0 = (M2, I2)$ with $I1 = 0$ and $I2 = 0$ as indices.

The series of the union $(M1, I1) \cup (M2, I2) = K1$ will be $K1 = \{ \dots, 6, 15, 18, 24, 30, 36, 42, 45, \dots \}$ and their common points (the intersection) will form the series $(M1, I1) \cap (M2, I2) = K2$ where $K2 = \{ \dots, 0, 30, 60, \dots \}$. The period is clearly equal to 30 and the largest common denominator D of 6 and 15 is 3 (which is, by multiplication, the part congruent to M1 and M2) and the smallest common multiple is M3, equals 30. Now, 6 divided by the largest common denominator D is C1, equals 2; and 15 divided by the largest common denominator D is C2, equals 5. Generalizing, the period of the points common to the two moduli M1 and M2 will be the smallest common multiple M3 of these two moduli. So, $(M1, I1) \cap (M2, I2) = (M3, I3)$ with $I3 = 0$, if $I1 = I2 = 0$ and $M3 = D * C1 * C2$, where $C1 = M1 / D$ and $C2 = M2 / D$.

It will also be noted that the operation of logical union, notated as \cup , of the two elementary moduli M1 and M2 is cumulative in that it takes into

account the periodic points of both moduli simultaneously. On the other hand, the logical operation of intersection, notated as \cap , is reductive since we take only the points common to both moduli.

When we mix the points of several moduli $M1, M2, M3, M4, \dots$:

a) by union, we obtain a sieve which is dense and complex depending on the elementary moduli;

$$P1 = (M1, I1) \cup (M2, I2) \cup (M3, I3) \cup \dots$$

b) by intersection, we obtain a sieve which is more rarified than that of the elementary moduli, and there would even be some cases in which the sieve would be empty of points when it lacks coincidences;

$$P2 = (M1, I1) \cap (M2, I2) \cap (M3, I3) \cap \dots$$

c) by simultaneous combinations of the two logical operations, we obtain sieves which can be very complex;

$$(0) \quad L = \{ (M11, I11) \cap (M12, I12) \cap \dots \} \cup \{ (M21, I21) \cap (M22, I22) \cap \dots \} \cup \{ \dots \}$$

$$= \sum_{i=1}^{k_0} \left(\prod_{j=1}^{k(i)} \right)$$

The intersection of each set of pairs between curly brackets should furnish a single final pair, if it exists. The final pairs will be combined by their union, which will provide the desired sieve.

Now let us examine the rigorous formulation of the calculation of the intersection of the two moduli $(M1, I1)$ and $(M2, I2)$ where the periods M1 and M2 start from some I1 and I2 respectively. First I1 and I2 are reduced by taking their moduli in relation to M1 and M2, $I1 = \text{MOD}(I1, M1)$ and $I2 = \text{MOD}(I2, M2)$.⁵

The first coincidence will eventually appear at a distance:

$$(1) \quad S = I1 + \lambda * M1 = I2 + \sigma * M2$$

where λ and σ are elements of N, and if $M1 = D * C1$ and $M2 = D * C2$ with D equal to the largest common denominator, C1 and C2 being coprime, then the period M3 of the coincidences will be: $M3 = D * C1 * C2$. From (1) there follows:

$$I1 - I2 = (\sigma * D * C2) - (\lambda * D * C1) \text{ and} \\ (I1 - I2) / D = (\sigma * C2) - (\lambda * C1).$$

Now, since the expression on the right of the equal sign is a whole number, the expression on the left of the equal sign should also be a whole number. But, if $I_1 - I_2$ is not divisible by D (for some I_1, I_2), then, there are no coincidences and the intersection $(M_1, I_1) \cap (M_2, I_2) = (M_3, I_3)$ will be empty. If not:

$$(2) \quad (I_1 - I_2) / D = \Psi \Sigma N \text{ and } \Psi = \sigma * C_2 - \lambda * C_1, \text{ as well as:}$$

$$\Psi + \lambda * C_1 = \sigma * C_2.$$

But following Bachet de Meziriac's theorem (1624), in order for x and y to be two coprimes, it is necessary and sufficient that there exist two relative whole numbers, ξ and ξ' , such that:

$$(3) \quad 1 + \xi * x = \xi * y \text{ or}$$

$$\xi' * x = \xi' * y + 1$$

where ξ and ξ' come from the recursive equations:

$$(4) \quad \text{MOD}(\xi * C_2, C_1) = 1 \text{ and}^6$$

$$(5) \quad \text{MOD}(\xi' * C_1, C_2) = 1$$

while letting ξ and ξ' run through the successive values $0, 1, 2, 3, \dots$ (except if $C_1 = 1$ and $C_2 = 1$).

But since C_1 and C_2 are coprime, there follows from (2) and (3):

$$\lambda / \sigma = \xi, \quad \sigma / \Psi = \xi, \quad \lambda / (-\Psi) = \xi' \text{ and}$$

$\sigma / (-\Psi) = \xi'$, and if $(M_1, I_1) \cap (M_2, I_2) = (M_3, I_3)$, then

$$(6) \quad I_3 = \text{MOD}(I_2 + \xi * (I_1 - I_2) * C_2, M_3) \text{ or}$$

$$I_3 = \text{MOD}(I_1 + \xi' * (I_2 - I_1) * C_1, M_3)$$

with $M_3 = D * C_1 * C_2$.

Example 1: $M_1 = 60, I_1 = 18, M_2 = 42, I_2 = 48, D = 6, C_1 = 10, C_2 = 7, M_3 = 6 * 10 * 7 = 420$, with C_1 and C_2 coprime.

From (3) and (4) we get: $\xi' = 5$.

From (6) we get: $I_3 = \text{MOD}(18 + 5 * (48 - 18) * 10, 420) = 258$.

Example 2: $M_1 = 6, I_1 = 3, M_2 = 8, I_2 = 3, D = 2, C_1 = 3, C_2 = 4, M_3 = 24$, with C_1 and C_2 coprime.

From (4) we get: $\xi = 1$.

And from (6) we get: $I_3 = \text{MOD}(3 + 1 * (3 - 3) * 4, 24) = 3$; that is, in the case that $I_1 = I_2$, then $I_3 = I_1 = I_2$, and here $M_3 = 24$ and $I_3 = 3$.

Take the preceding example but with $I_1 = I_3$ and $I_2 = 4$, so I_1 is not equal to I_2 . Since $I_1 / D = 1.5$, which is not an element of N , there are no coincidences and $M_3 = 0$ and $I_3 = 0$. But, if $I_1 = 2$ and $I_2 = 16$, and since $(I_1 - I_2) / D = 7 \Sigma N$, we obtain from (4) $\xi = 1$ and from (6) $I_3 = \text{MOD}(0 + 1 * (2 - 0) * 4, 24) = 8$ and $(M_3, I_3) = (24, 8)$.

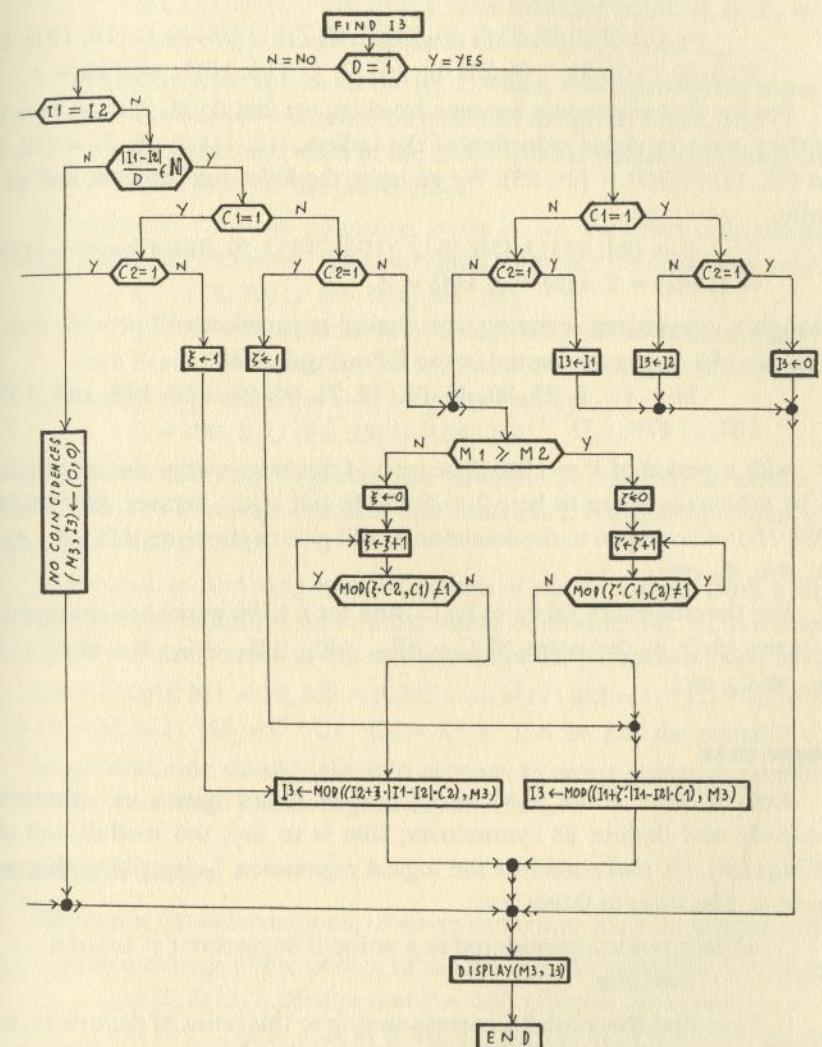
Computation of the Intersection $(M_1, I_1) \cap (M_2, I_2) = (M_3, I_3)$

Are given: M_1, M_2, I_1, I_2 , with $I_i = \text{MOD}(I_i, M_i) \geq 0$

$D = \text{the largest common denominator of } M_1 \text{ and } M_2$

$M_3 = \text{the smallest common multiple of } M_1 \text{ and } M_2$

$C_1 = M_1 / D, C_2 = M_2 / D, M_3 = D * C_1 * C_2$



To compute several simultaneous intersections (coincidences) from an expression between brackets in the equation (0) of L, it suffices to calculate the pairs in that expression two by two. For example:

$$\begin{aligned}
 k_0 &= 4 & k(i) \\
 L &= \sum_{i=1}^{k(i)} (\prod) \\
 &= \{(3, 2) \cap (4, 7) \cap (6, 11) \cap (8, 7)\} \cup \{(6, 9) \cap (15, 18)\} \cup \\
 &\quad \{(13, 5) \cap (8, 6) \cap (4, 2)\} \cup \{(6, 9) \cap (15, 19)\}, \text{ with } k_0 = 4.
 \end{aligned}$$

For the first expression between brackets, we first do $(3, 2) \cap (4, 7) = (12, 11)$, then, after modular reduction of the indices, $(12, 11) \cap (6, 5) = (12, 11)$, then $(12, 11) \cap (8, 7) = (24, 23)$. We go on to the following brackets, and so on. Finally,

$$\begin{aligned}
 L &= (24, 23) \cup (30, 3) \cup (104, 70) \cup (0, 0) \text{ for } k_0 = 4, k(1) \\
 &= 4, k(2) = 2, k(3) = 3, k(4) = 2.
 \end{aligned}$$

Through a convenient scanning, this logical expression will provide us with the points of a sieve constructed in the following fashion:

$$H = \{ \dots 3, 23, 33, 47, 63, 70, 71, 93, 95, 119, 123, 143, 153, 167, \dots 479, \dots \}$$

with a period of $P = 1560$. The zero of this sieve within the set of pitches can be arbitrarily taken to be $C_2 = 8.25$ Hz and at ten octaves, $(2^{10} * 8.25 = 16384$ Hz) with u equal to the semitone. It will give us the notes $\#D_2, B_1, A_0, B_1, \#D_3, \#A_3, B_3$, etc.

For the same zero taken to be C_2 and for u to be equal to a quartertone, the series gives us the notes $\#C_2, +B_2, +E_1, +B_1, +G_0, B_0, \#A_1, +B_1, +B_2, \#C_3$, etc.

Inverse case

Let us start from a series of points either given or constructed intuitively and deduce its symmetries; that is to say, the moduli and their shiftings (M_j, I_j) , and construct the logical expression L describing this series of points. The steps to follow are:

- each point is considered as a point of departure ($= I_n$) of a modulus.
- to find the modulus corresponding to this point of departure, we begin by applying a modulus of value $Q = 2$ unities. If each one of its multiples meets a point which has not already been encountered and which belongs to the given sieve, then we

keep the modulus and it forms the pair (M_n, I_n) . But if any one of its multiples happens not to correspond to one of the points of the series, we abandon it and pass on to $Q + 1$. We proceed so until each one of the points in the given series has been taken into account.

- if for a given Q , we garner all its points (Q, I_k) under another pair (M, I) ; that is, if the set (Q, I_k) is included in (M, I) , then, we ignore (Q, I_k) and pass on to the following point I_{k+1} .
- similarly, we ignore all the (Q, I) which, while producing some of the not-yet-encountered points of the given series, also produce, upstream of the index I , some parasitical points other than those of the given series.

An example: from the preceding series H, we will select only the points between 3 and 167 inclusive. Then, we could construct the following union:

$$\begin{aligned}
 L &= (73, 70) \cup (30, 3) \cup (24, 23), \\
 &\text{with } P = 8760 \text{ as its period. However, if the same series } H \\
 &\text{were limited between the points 3 and 479 inclusive, (this time} \\
 &\text{having 40 points), it would be generated by:} \\
 L &= (30, 3) \cup (24, 23) \cup (104, 70),
 \end{aligned}$$

the modulus 30 covering 16 points, the modulus 24 covering 20 points, and the modulus 104 covering 4 points. The function L is identical to that given earlier. Its period is 1560.

In general, to find the period of a series of points derived from a logical expression whose definitive form is the union of moduli (M_j, I_j) , it is enough to compose the intersection of the moduli within the parentheses two by two.

For example: $M_1 = 12, M_2 = 6, M_3 = 8; M_1 \cap M_2 = D * C_1 * C_2 = 6 * 2 * 1 = 12 = M; M \cap M_3 = D * C_1 * C_2 = 4 * 3 * 2 = 24$. And the period $P = 24$.

In general, one should take into account as many points as possible in order to secure a more precise logical expression L.

Metabolae of Sieves

Metabolae (transformations) of sieves can come about in various ways:

- by a change of the indices of the moduli. For example: $L = (5, 4) \cup (3, 2) \cup (7, 3)$ of period $P = 105$ will give the series: $H = \{ \dots, 2, 3, 4, 5, 8, 9, 10, 11, 14, 17, 19, 20, 23, 24, 26, 29, 31, \dots \}$. But if a whole number n is added to the indices, the expression L becomes for $n = 7$:

$L' = (5, 11) \cup (3, 9) \cup (7, 10)$ and after modular reduction of the indices:

$L' = (5, 1) \cup (3, 0) \cup (7, 3)$, of the same period $P = 105$.

The series $H' = \{ \dots, 0, 1, 3, 6, 9, 10, 11, 12, 15, 16, 17, 18, 21, 24, 26, 27, 30, \dots \}$ derived from this last expression L' , having the same intervallic structure as the H series and differing from it only by its initial point, which is given by the smallest index of the expression L' and by a shifting n of the intervallic structure of H . Indeed, if in the series H , the intervals start from 2, which is the index of the smallest modulus of M , then the same intervals are to be found starting from $2 + 7 = 9$ within the series H' . This case is what musicians call "transposition" upwards and is part of the technique of "variations." If, on the other hand, we add to each index any whole number n , then the intervallic structure of the sieve changes while its period is maintained. For example: add 3, 1, and -6 respectively to the three indices of L , which becomes after their modular reductions:

$L = (5, 2) \cup (3, 0) \cup (7, 4)$ of period $P = 105$, and which gives:

$H = \{ \dots, 0, 2, 3, 4, 6, 7, 9, 11, 12, 15, 17, 18, 21, 22, 24, 25, 27, 30, 32, \dots \}$.

- b) by transformations of the logical operations in some manner, using the laws of logic and mathematics, or arbitrarily.
- c) by the modification of its unity u . For example, sing the national anthem, which is based on the diatonic scale (white keys), while transforming the semitones into quartertones or into eighthtones, etc. If this metabola is used rarely melodically or harmonically, it does however occur in other characteristics of sound such as time by changes in tempo, and this, as far as history can remember.

Conclusion

In provisional conclusion, it will be said that sieve theory is the study of the internal symmetries of a series of points either constructed intuitively, given by observation, or invented completely from moduli of repetition.

In what has been demonstrated above, the examples have been taken from instrumental music. But it is quite conceivable to apply this theory to computer generated sound synthesis, imagining that the amplitude and/or the time of the sound signal can be ruled by sieves. The subtle symmetries thus created should open a new field for exploration.

Chapter XII

Sieves: a User's Guide

I would like to give credit and express my thanks to Gérard Marino, a programmer who works with me at CEMAMu. He has adapted my own program which I originally wrote in "Basic" into "C."

The program is divided into two parts:

A. Generation of points on a straight line from the logical formula of the sieve.

B. Generation of the logical formula of the sieve from a series of points on a straight line.

A. GENERATION OF POINTS ON A STRAIGHT LINE FROM THE LOGICAL FORMULA OF THE SIEVE

Example:

DEFINITION OF A SIEVE:

$$\begin{aligned} L = & [0 * 0 * \dots * 0] \\ & + [0 * 0 * \dots * 0] \\ & + \dots \\ & + [0 * 0 * \dots * 0] \end{aligned}$$

In each parenthesis are given in order: modulus, starting point
(taken from the set of integers)

$[] + []$ is a union

$(*)()$ is an intersection

Given the formula of a sieve made out of unions and intersections of moduli, the program reduces the number of intersections to one and keeps only the given unions. The abscissa of the final points of the sieve are computed from these unions and displayed.

NUMBER OF UNIONS ? = 2

union 1: number of modules ? = 2	modulus 1 ? = 3
start ?	= 2
modulus 2 ?	= 4
start ?	= 7

FORMULA OF THE SIEVE:

$$L = [[3, 2] * (4, 7)] \\ + [[6, 9] * (15, 18)]$$

REDUCTION OF THE INTERSECTIONS:

union 1

$$[(3,2) * (4,7)] = (12,11)$$

decompression into prime modules ?
(press 'y' for yes, any other key for no)

$$(12,11) = 4,3) * (3,2)$$

union 2

$$[(6,9) * (15,18)] = (30,3)$$

decompression into prime modules ?
(press 'y' for yes, any other key for no): y

$$(30,2) = 2,1) * (3,0) * (5,3)$$

SIMPLIFIED FORMULA OF THE SIEVE:

$$L = L(12, 11) + (30, 3)$$

POINTS OF THE SIEVE CALCULATED WITH THIS FORMULA:

rank of first displayed point? = 0

press <enter> to get a series of 10 points

Rank

Rank	0	3	11	23	33	35	47	59	63	71	83
10	93	95	107	119	123	131	143	153	155	155	167
20	179	183	191	203	213	215	227	239	243	251	
30	263	273	275	287	299	303	311	323	333	335	
40	347	359	363	371	383	393	395	407	419	423	
50	431	443	453	455	467	479	483	491	503	513	
60	515	527	539	543	551	563	573	575	587	599	
70	603	611	623	633	635	647	659	663	671	683	
80	693	695	707	719	723	731	743	753	755	767	
90	779	783	791	803	813	815	827	839	843	851	
100	863	873	875	887	899	903	911	923	933	935	
110	947	959	963	971	983	993	995	1007	1019	1023	
120	1031	1043	1053	1055	1067	1079	1083	1091	1103	1113	
130	1115	1127	1139	1143	1151	1163	1173	1175	1187	1199	

Sieves: A User's Guide

Line# Source Line

```

52      "[] + [] is a union\n"
53      "() * () is an intersection\n\n";
54      printf("-----\n"
55      "Given the formula of a sieve made out of unions and\n"
56      "intersections of moduli, the program reduces the number of\n"
57      "intersections to one and keeps only the given unions.\n"
58      "Then, the abscissa of the final points of the sieve are\n"
59      "computed from these unions and displayed.\n\n";
60      /* ----- get the formula of the sieve ----- */
61      while (unb == 0)
62      {
63          printf("NUMBER OF UNIONS ? = ");
64          scanf("%d",&unb);
65      }
66      fCrib = (inter *)(malloc (sizeof(inter) * unb));
67      if (fCrib == NULL)
68      {
69          printf("not enough memory\n");
70          exit(1);
71      }
72      printf("-----\n");
73      for (u = 0; u < unb; u++)
74      {
75          printf("union %d: number of modules ? = ", u + 1);
76          scanf("%d",&fCrib[u].clnb);
77          printf("\n");
78          fCrib[u].cl = (periode *)(malloc (sizeof(periode) * fCrib[u].clnb));
79          if (fCrib[u].cl == NULL)
80          {
81              printf("not enough memory\n");
82              exit(1);
83          }
84          for (i = 0; i < fCrib[u].clnb; i++)
85          {
86              printf("\n      modulus %d ? = ", i + 1);
87              scanf("%d",&fCrib[u].cl[i].mod);
88              printf("      start ? = ");
89              scanf("%d",&fCrib[u].cl[i].ini);
90          }
91          printf("-----\n");
92      }
93      /* ----- reduction of the formula ----- */
94      printf("FORMULA OF THE SIEVE:\n\n"
95      "      L = [ ];\n"
96      for (u = 0; u < unb; u++)
97      {
98          if (u != 0)
99          printf("      + [ ];\n"
100         for (i = 0; i < fCrib[u].clnb; i++)
101         {
102             if (i != 0)
103             {

```

Line# Source Line

```

104      if (i % 4 == 0)
105          printf("\n      ");
106          printf("* ");
107      }
108      printf("(%5d,%5d) ", fCrib[u].cl[i].mod, fCrib[u].cl[i].ini);
109      }
110      printf("]\n");
111      }
112      printf("-----\n");
113      printf("REDUCTION OF THE INTERSECTIONS:\n\n");
114      for (u = 0; u < unb; u++)
115      {
116          printf("union %d\n      [ ",u + 1);
117          for (i = 0; i < fCrib[u].clnb; i++)
118          {
119              printf("(%d,%d) ", fCrib[u].cl[i].mod, fCrib[u].cl[i].ini);
120              if (i != fCrib[u].clnb - 1)
121                  printf("* ");
122          }
123          fCrib[u].clr = ReducInter(u); /* reduction of an intersection */
124          printf("] = (%d,%d)\n\n", fCrib[u].clr.mod, fCrib[u].clr.ini);
125          printf("      decomposition into prime modules ?\n"
126          "      (press 'y' for yes, any other key for no): ");
127          if (getche() == 'y')
128          {
129              printf("\n\n      (%d,%d)", fCrib[u].clr.mod, fCrib[u].clr.ini);
130              Decompos(fCrib[u].clr);
131          }
132          else
133              printf("\n\n");
134      }
135      printf("-----\n");
136      /* ----- display the simplified formula ----- */
137      printf("SIMPLIFIED FORMULA OF THE SIEVE:\n\n");
138      printf("      L = ");
139      for (u = 0; u < unb; u++)
140      {
141          if (u != 0)
142          {
143              if (u % 4 == 0)
144                  printf("\n      ");
145                  printf("+ ");
146          }
147          printf("(%5d,%5d) ", fCrib[u].clr.mod, fCrib[u].clr.ini);
148      }
149      printf("\n-----\n");
150      /* ----- points of the sieve ----- */
151      printf("POINTS OF THE SIEVE CALCULATED WITH THIS\n"
152      "FORMULA:\n");
153      printf("rank of first displayed point ? = ");
154      scanf("%lu",&n0);

```

```

Line# Source Line
155 printf("\npress <enter> to get a series of 10 points\n\n"
156     "Rank |");
157 for (u = 0; u < unb; u++)
158 {
159     if (fCrib[u].clr.mod != 0 || fCrib[u].clr.ini != 0)
160     {
161         fCrib[u].ptval = fCrib[u].clr.ini;
162         flag = NONEMPTY;
163     }
164     else
165         fCrib[u].ptval = 0xFFFFFFFF;
166 }
167 if (flag != NONEMPTY)
168     return;
169 u0 = u1 = 0;
170 lastval = 0xFFFFFFFF;
171 while (1)
172 {
173     for (u = (u0 + 1) % unb; u != u0; u = (u + 1) % unb)
174     {
175         if (fCrib[u].ptval == fCrib[u1].ptval)
176             u1 = u;
177     }
178     if (fCrib[u1].ptval != lastval) /* new point */
179     {
180         lastval = fCrib[u1].ptval;
181         if (ptnb == n0)
182         {
183             if (ptnb % 10 == 0)
184             {
185                 getch(); /* get a character from the keyboard */
186                 printf("\n%7lu ", ptnb);
187             }
188             printf("%6lu ", fCrib[u1].ptval);
189         }
190         ptnb++;
191     }
192     fCrib[u1].ptval += fCrib[u1].clr.mod;
193     u0 = u1;
194 }
195
196 /* ===== reduction of an intersection ===== */
197 periode ReducInter(short u)
198 {
199     periode cl,cl1,cl2,cl3;
200     short pgcd,T,n;
201     long c1,c2;
202
203     cl3 = fCrib[u].cl[0];
204     for (n = 1; n < fCrib[u].clnb; n++)
205     {
206         cl1 = cl3;

```

```

Line# Source Line
207     cl2 = fCrib[u].cl[n];
208     if (cl1.mod == cl2.mod)
209     {
210         cl1 = cl1;
211         cl1 = cl2;
212         cl2 = cl1;
213     }
214     if (cl1.mod != 0 && cl2.mod != 0)
215     {
216         cl1.ini %= cl1.mod;
217         cl2.ini %= cl2.mod;
218     }
219     else
220         return CL_EMPTY;
221     /* module resulting from the intersection of 2 modules */
222     pgcd = Euclide(cl1.mod, cl2.mod);
223     cl1 = cl1.mod / pgcd;
224     cl2 = cl2.mod / pgcd;
225     if (pgcd != 1
226         && ((cl1.ini - cl2.ini) % pgcd != 0))
227         return CL_EMPTY;
228     if (pgcd != 1
229         && ((cl1.ini - cl2.ini) % pgcd == 0)
230         && (cl1.ini != cl2.ini) && (cl1 == cl2) )
231     {
232         cl3.mod = pgcd;
233         cl3.ini = cl1.ini;
234         continue;
235     }
236     T = Meziriac((short) cl1, (short) cl2);
237     cl3.mod = (short) (cl1 * cl2 * pgcd);
238     cl3.ini = (short) ((cl1.ini
239                     + T * (cl2.ini - cl1.ini) * cl1) % cl3.mod);
240     while (cl3.ini == cl1.ini || cl3.ini == cl2.ini)
241         cl3.ini += cl3.mod;
242     }
243     return cl3;
244 }
245 /* ===== decomposition into an intersection ===== */
246 /* of prime modules */
247 void Decompos (periode pr)
248 {
249     periode pf;
250     short fct;
251
252     if (pr.mod == 0)
253     {
254         printf(" = (%d,%d)\n", pr.mod, pr.ini);
255         return;
256     }
257     printf(" =");
258     for (i = 0, fct = 2; pr.mod != 1; fct++)

```

```

Line#  Source Line
259  {
260  pf.mod = 1;
261  while (pr.mod % fct == 0 && pr.mod != 1)
262  {
263  pf.mod *= fct;
264  pr.mod /= fct;
265  }
266  if (pf.mod != 1)
267  {
268  pf.ini = pr.ini % pf.mod;
269  pr.ini %= pr.mod;
270  if (i != 0)
271  printf(" *");
272  printf("(%d,%d)", pf.mod, pf.ini);
273  i++;
274  }
275  }
276  printf("\n");
277  }
278 /* ====== Euclide's algorithm ===== */
279 short Euclide (a1, a2) /* a1 = a2 0 */
280 short a1;
281 short a2;
282 {
283 short tmp;
284
285 while ((tmp = a1 % a2) != 0)
286 {
287 a1 = a2;
288 a2 = tmp;
289 }
290 return a2;
291 }

292 /* ===== De Meziriac's theorem ===== */
293 short Meziriac (c1, c2) /* c1 = c2 0 */
294 short c1;
295 short c2;
296 {
297 short T = 0;
298
299 if (c2 == 1)
300 T = 1;
301 else
302 while (((+T * c1) % c2) != 1)
303 ;
304 return T;
305 }

```

B. GENERATION OF THE LOGICAL FORMULA OF THE SIEVE FROM A SERIES OF POINTS ON A STRAIGHT LINE

Example:

Given a series of points, find the starting points with their moduli (periods).

NUMBER OF POINTS ? = 12

abscissa of the points

point 1 = 59	point 2 = 93	point 3 = 47	point 4 = 3
point 5 = 63	point 6 = 11	point 7 = 23	point 8 = 33
point 9 = 95	point 10 = 71	point 11 = 35	point 12 = 83

POINTS OF THE SIEVE (ordered by their increasing abscissa):

Rank	0	3	11	23	33	35	47	59	63	71	83
	10	93	95								

FORMULA OF THE SIEVE:

In each parenthesis are given in order:

(modulus, starting point, number of covered points)

$L = (30, 3, 4) + (12, 11, 8)$

period of the sieve: $P = 60$

```

Line#  Source Line
1  #include <stdio.h>
2  #include <stdlib.h>
3  #include <string.h>
4  #include <string.h>
5
6  /* ----- types definitions ----- */
7  typedef struct /* period (congruence class) */
8  {
9      short mod; /* modulus of the period */
10     short ini; /* starting point */
11     short couv; /* number of covered points */
12 } periode;
13 /* ----- function prototypes ----- */
14 unsigned long Euclide(unsigned long m1, /* computation of the LCD */
15                         unsigned long m2); /* */
16 /* ----- variables and constants ----- */
17 periode*perCrib; /* periods of the sieve */
18 short perTotNb = 0; /* number of periods in the formula */
19 long *ptCrib; /* points of the crible */
20 long *ptReste; /* points outside the periods */
21 short ptTotNb = 0; /* number of points in the sieve */
22 short p,ptnb;
23 long ptval;
24 unsigned long percrib;
25

```

Line #	Source Line
26	periodeper;
27	
28	#define NON_REDUNDANT 0
29	#define REDUNDANT 1
30	#define COVERED -1L
31	short flag;
32	
33	/* ======
34	void main(void)
35	{
36	printf("B. GENERATION OF THE LOGICAL FORMULA OF THE SIEVE FROM\n")
37	" A SERIES OF POINTS ON A STRAIGHT LINE\n\n"
38	"Example:\n"
39	"-----\n"
40	"Given a series of points, find the starting points\n"
41	"with their moduli (periods).\n\n";
42	/* ----- entry of the points of the sieve and their sorting -----*/
43	while (ptTotNb == 0)
44	{
45	printf("NUMBER OF POINTS ? = ");
46	scanf("%d",&ptTotNb);
47	}
48	ptCrib = (long *)malloc (ptTotNb * sizeof(long)));
49	ptReste = (long *)malloc (ptTotNb * sizeof(long)));
50	perCrib = (periode *)malloc (ptTotNb * sizeof(periode)));
51	if (ptCrib == NULL ptReste == NULL perCrib == NULL)
52	{
53	printf("not enough memory\n");
54	exit(1);
55	}
56	printf("-----\n")
57	"abscissa of the points:\n";
58	for (p = 0; p < ptTotNb; p++)
59	{
60	if (p % 4 == 0)
61	printf("\n ");
62	printf("point %2d = ", p + 1);
63	scanf("%ld", &ptval);
64	for (ptnb = 0;
65	ptnb p && ptval ptCrib[ptnb];
66	ptnb++)
67	;
68	if (ptnb p)
69	{
70	if (ptval ptCrib[ptnb]) /* new point
71	memmove(&ptCrib[ptnb + 1], &ptCrib[ptnb],
72	sizeof(long) * (p - ptnb));
73	else /* point already exist */
74	{
75	p--;

Line #	Source Line
76	ptTotNb--;
77	}
78	}
79	ptCrib[ptnb] = ptval;
80	}
81	printf("\n-----\n");
82	/* ----- points of the sieve ----- */
83	printf("POINTS OF THE SIEVE (ordered by their increasing abscissa):\n\n"
84	"Rank ");
85	for (p = 0; p < ptTotNb; p++)
86	{
87	if (p % 10 == 0)
88	printf("\n%7d ", p);
89	printf("%6ld ", ptCrib[p]);
90	}
91	printf("\n\n-----\n");
92	/* ----- compute the periods of the sieve ----- */
93	memcpy(ptReste, ptCrib, ptTotNb * sizeof(long));
94	
95	for (p = 0; p < ptTotNb; p++)
96	{
97	if (ptReste[p] == COVERED)
98	continue;
99	/* ----- compute a period starting at current point ----- */
100	per.mod = 0;
101	do
102	{
103	per.mod++;
104	per.ini = (short) (ptCrib[p] % (long)per.mod);
105	per.couv = 0;
106	for (ptnb = 0, ptval = per.ini;
107	ptnb < ptTotNb && ptval = ptCrib[ptnb];
108	ptnb++)
109	{
110	if (ptval == ptCrib[ptnb])
111	{
112	per.couv++;
113	ptval += per.mod;
114	}
115	}
116	}
117	while (ptnb < ptTotNb);
118	/* ----- check the redundancy of the period ----- */
119	for (ptnb = 0, ptval = per.ini, flag = REDUNDANT;
120	ptnb < ptTotNb;
121	ptnb++)
122	{
123	if (ptval == ptCrib[ptnb])
124	{
125	if (ptval == ptReste[ptnb])
126	{

Line #	Source Line
127	ptReste[ptnb] = COVERED;
128	flag = NON_REDUNDANT;
129	}
130	ptval += per.mod;
131	}
132	}
133	if (flag == NON_REDUNDANT)
134	perCrib[perTotNb++] = per;
135	}
136	/* ----- compute the period of the sieve ----- */
137	percrib = perCrib[0].mod;
138	for (p = 1; p < perTotNb; p++)
139	{
140	if ((long) perCrib[p].mod == percrib)
141	percrib *= (long) perCrib[p].mod / Euclide((long)perCrib[p].mod,
142	percrib);
143	else
144	percrib *= (long) perCrib[p].mod / Euclide(percrib,
145	(long)perCrib[p].mod);
146	/* ----- display the formula of the sieve ----- */
147	printf("FORMULA OF THE SIEVE:\n" -
148	"In each parenthesis are given in order:\n"
149	"(modulus, starting point, number of covered points)\n\n");
150	printf(" L = ");
151	for (p = 0; p < perTotNb; p++)
152	{
153	if (p != 0)
154	{
155	if (p % 3 == 0)
156	printf("\n ");
157	}
158	printf("(%5d,%5d,%5d) ", perCrib[p].mod, perCrib[p].ini,
159	perCrib[p].couv);
160	printf("\n\n period of the sieve: P = %lu\n", percrib);
161	}
162	/* ===== Euclide's algorithm ===== */
163	unsigned long Euclide (a1, a2) /* a1 = a2 0 */
164	unsigned long a1;
165	unsigned long a2;
166	{
167	unsigned long tmp;
168	
169	while ((tmp = a1 % a2) != 0)
170	{
171	a1 = a2;
172	a2 = tmp;
173	}
174	return a2;
175	}

Chapter XIII

Dynamic Stochastic Synthesis

What is the most economical way to create a plane wave in an amplitude-time space (atmospheric pressure-time), encompassing all possible forms from a square wave to white noise? From an informatics point of view, a square wave is quite simple with only two amplitudes, $\pm a$ over n of fixed samplings. White noise is also quite simple and generated by a compound of stochastic functions whose samplings are dovetailed, nested, or not.

But what about waves representing melodies, symphonies, natural sounds ...?

The foundation of their nature and therefore of their human intelligibility is temporal periodicity and the symmetry of the curves. The brain can marvelously detect, with a fantastic precision, melodies, timbres, dynamics, polyphonies, as well as their complex transformations in the form of a curve, unlike the eye which has difficulty perceiving a curve with such a fast mobility.

An attempt at musical synthesis according to this orientation is to begin from a probabilistic wave form (random walk or Brownian movement) constructed from varied distributions in the two dimensions, amplitude and time (a, t), all while injecting periodicities in t and symmetries in a . If the symmetries and periodicities are weak or infrequent, we will obtain something close to white noise. On the other hand, the more numerous and complex (rich) the symmetries and periodicities are, the closer the resulting music will resemble a simple held note. Following these principles, the whole gamut of music past and to come can be approached. Furthermore, the relationship between the macroscopic or microscopic levels of these injections plays a fundamental role.

Below, is a first approach to constructing such a wave.

Procedure

A1. Following the absciss of t , we begin with a length (period) T where $T = 1/f$ seconds and f is a freely chosen frequency. At the start, this period T is subdivided into n equal segments; for example, $n = 12$ (this is one

macroscopic level). Every time T is repeated, each segment $t_i - t_{i-1}$ of ($i = 0, 1, 2, 3, \dots, n-1$) undergoes a stochastic alteration which increases or reduces it within certain limits imposed, for example, by elastic barriers.

B1. Following the amplitude axis, a value is given to each extremity of the 12 preceding segments. These values form a polygon inscribed or enveloping a sine wave, or a rectangular form, or a form born of a stochastic function such as that of Cauchy, or even a polygon flattened at the zero level. The E_i ordinates of these n summits undergo a stochastic alteration at each repetition which is sufficiently weak and even more, compressed between two adequate elastic barriers.

C1. The E ordinates of the samplings found between the two extremities of a segment T will be calculated by a linear interpolation of the ordinates E_{i-1} and E_i of these extremities.

A2. Abscissa of the polygon's summits

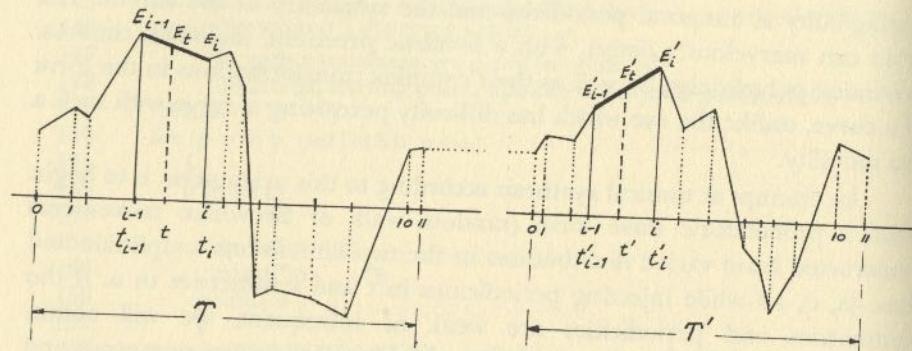


Figure 1.

$$\theta \text{ prec(eding)} = t_i - t_{i-1}; \theta \text{ pres(ent)} = t'_i - t'_{i-1}$$

Procedures

Construction of θ present from "logistic" distribution:

$$(1) \quad U(\xi) = \frac{ae^{-\alpha\xi - \beta}}{(1 + e^{-\alpha\xi - \beta})^2}$$

and its distribution function,

$$F(\xi) = \int_{-\infty}^{\xi} U(\zeta) d\zeta = \frac{1}{1 + \text{EXP}(-a\xi - \beta)}$$

we obtain $\xi = -(\beta + \ln(\frac{1-y}{y})) / a$ with y coming from the

uniform distribution:

$$0 \leq y \leq 1.$$

(2)

take: $\xi \text{ pres} = \xi \text{ prec} + \xi$

(3)

Pass this ξ pres into local elastic barriers

± 100 taken from $\beta/2$, to obtain ξ' :

(4)

Then do:

$$\theta \text{ pres} = \theta \text{ prec} + \xi' * \text{Rdct}$$

where Rdct is a reduction factor.

(5)

Finally pass θ pres into general elastic barriers θ_{\min} and θ_{\max} obtained as follows:

a) the minimum frequency is, say 3 HZ. Then the maximum period is $T = \frac{1}{3}$ sec and each of the 12 segments will have a mean length of $\theta_{\max} = \frac{1}{3 * 12}$ sec.

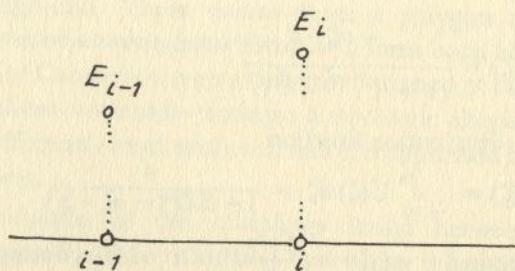
b) The maximum frequency could be $\frac{\text{SAMP}}{12}$ HZ where

SAMP is the sampling rate, say 44100 HZ. Therefore each of the 12 segments could have a minimum length of the period

$$\frac{T}{12} = \frac{1}{\text{SAMP}} = \theta_{\min}.$$

(6)

Repeat the above procedures for each of the $n = 12$ segments.

B2. Ordinates of the Polygon's summits**Figure 2.**

The i^{th} present ordinate is obtained from the i^{th} preceding ordinate in the following manner:

Construction of the E_i pres:

(1) Take a probability distribution $W(\sigma)$; then its distribution

$$\text{function } Q(W) = \int_{-\infty}^{\sigma} W(\sigma) d\sigma. \text{ We obtain } \sigma = V(Q, y) \text{ with}$$

$0 \leq y \leq 1$ (the uniform distribution) and $W(\sigma)$ any distribution.

(2) Pass σ through local barriers (± 0.2)

(3) Add this σ to the E_i prec.

$$E_i \text{ pres} = E_i \text{ prec} + \sigma$$

(4) Pass E_i pres through limitative barriers ± 8 bits (± 32768) and this is the final E_i pres.

(5) Do this for each of the 10 summits within the two boundary summits of the polygon.

(6) The last boundary summit will be taken as the first boundary summit of the next period.

C2. Construct the E_i ordinate of the sampling point t which can be found on the segment $t_i - t_{i-1}$ between the ordinates E_{i-1} and E_i through a linear interpolation.

$$E_t = \frac{(E_i - E_{i-1})(t - t_{i-1})}{t_i - t_{i-1}}$$

Therefore, a *microscopic* construction.

General comment: the distribution functions $U(\zeta)$ and $W(\sigma)$ can be either simple, for example, sine, Cauchy, logistic, ...; or more complex, through nesting, etc.

The data given above is naturally an arbitrary starting point which I used in *La Légende d'Eer*.

This approach to sound synthesis represents a non-linear dynamic stochastic evolution which bypasses the habitual analyses and harmonic syntheses of Fourier since it is applied to the $f(t)$ part on the left of the equal sign of Fourier's transformation. This approach can be compared to current research on dynamic systems, deterministic chaoses or fractals. Therefore, we can say that it bears the seed of future exploration.

More Thorough Stochastic Music

Introduction

This chapter deals with a generalisation of sound synthesis by using not periodic functions, but quite the opposite, non-recurring, non-linear functions. The sound space in question is one which will produce a likeness of live sounds or music, unpredictable in the short or long run, but, for example, being able to vary their timbre from pure "sine - wave" sound to noise.

Indeed, the challenge is to create music, starting, in so far as it is possible, from a minimum number of premises but which would be "interesting" from a contemporary aesthetical sensitivity, without borrowing or getting trapped in known paths.

The ontological ideas behind this subject have already been exposed in the chapters treating ACHORRIPSIS (cf. chapters I and V) some 33 years ago, and still form the background canvas to this new, somewhat more thorough scope, which should result in more radical experimental solutions.

If, at that time, the "waves" in the "black universe" were still produced by musical instruments and human beings, today, these "waves" would be produced mainly by probability distributions (adorned with some restrictions) and by computers.

Therefore, we find ourselves in front of an attempt, as objective as possible, of creating an automated art, without any human interference except at the start, only in order to give the initial impulse and a few premises, like in the case of the Demiourgos in Plato's *Politicos*, or of Yahweh in the Old Testament, or even of Nothingness in the Big Bang Theory.

Microstructure

The fundamental ingredients used are (almost like in the case of *La Légende d'Eer*) four in number :

- a) A temporal fictitious length divided into a given number of segments, at whose ends we draw amplitudes in order to form a stochastic polygonal wave-form (PWF);
- b) As a matter of fact, this polygon is built continuously and endlessly through the help of probability distributions by cumulatively varying temporal lengths as well as the amplitudes of the vertices;
- c) In order to avoid excessive cumulated values, elastic barriers are imposed;
- d) A linear interpolation joins the vertices.

Under certain conditions, this procedure, although chaotic and undeterministic, produces a relatively stable sound.

The computation of the stochastic polygonal waveforms uses one stochastic law that governs the amplitudes and another one that governs the durations of the time-segments. The user chooses among several distinct stochastic laws (Bernouilli, Cauchy, Poisson, Exponential. . .). The sizes of the elastic-mirrors that are applied to the amplitudes and the durations can be chosen too.

Macrostructure

- A) The preceding procedure therefore produces a sound of a certain duration;
- B) A sequence (PARAG(psi%)) results from a simultaneous and temporal multiplicity of such sounds. This sequence is equally constructed through decisions governed by probability distributions;
- C) An arbitrary chain of such sequences could produce an interesting musical composition.

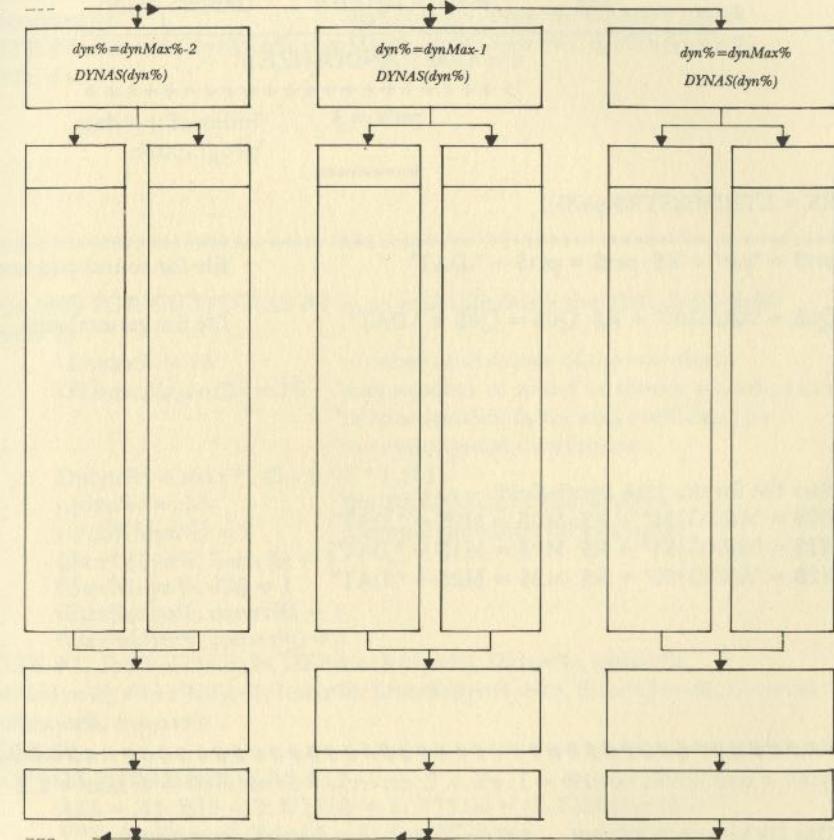
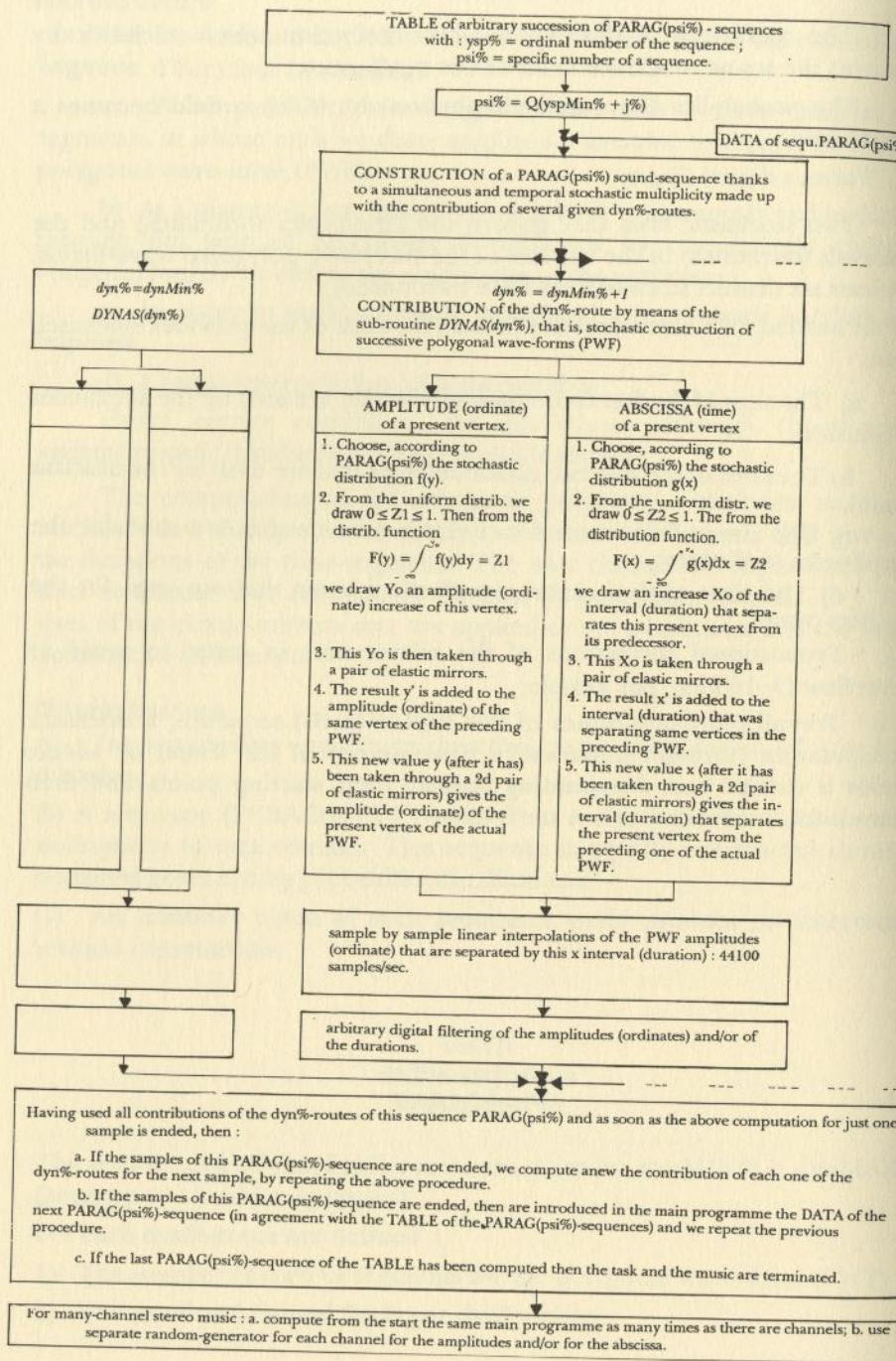
DATA of the sequence PARAG(psi%)

The end-figures of the dyn%-routes are given by $\text{dynMIN\%} \leq \text{dynMAX\%}$ (here, up to 16 arbitrary routes)

For each dyn%-route are defined :

- 1) The number $I_{\text{max}\%}$ of segments for the polygonal wave-form (PWF),
- 2) The number of sound fields per dyn%-route,

- 3) The coefficient of the exponential distribution which stochastically governs the sound or silence fields of this dyn%-route,
- 4) The probability (Bernoulli distribution) by which a field becomes a sound field,
- 5) Various digital filters,
- 6) Two stochastic laws that govern the amplitudes (ordinates) and the intervals (durations) of the vertices of the successive polygonal wave-forms, (at least six distinct stochastic laws are introduced),
- 7) If needed, two numerical coefficients for each of the previous stochastic laws,
- 8) a) The sizes of the first two elastic-mirrors that are used for the amplitudes (ordinates),
b) The sizes of the first two elastic-mirrors that are used for the abscissa (time),
c) The sizes of the second two elastic-mirrors that are used for the amplitudes (ordinates),
d) The sizes of the second two elastic-mirrors that are used for the abscissa (time),
- 9) Proportional corrections of the mirror-sizes in order to avoid an overflow (> 16 bits) per sample,
- 10) For all the dyn%-routes of this PARAG(psi%) sequence, a stochastic computation (through exponential distribution) of the sound or silence fields is carried out, determining namely their starting points and their durations.



*This programme has been technically realized with the help of Marie-Hélène Serra (I.X.).

```

DIM V2&(0 TO 20)           'size of the lower second-elastic-mirror
DIM filter%(0 TO 20, 0 TO 10) 'there are ten possible filters per dyn%-field

OPEN Q0$ FOR OUTPUT AS #1  'general data for the sequence
vertec% = 1: vertcon% = 2  'indexes of the ampl.-ordinate for the screen
                           'and the converter.

Nmax& = 10000000
dynMin% = 1
dynMax% = 16
flrt%(vertec%) = 0          'vert.screen-filter for GENDY1.BAS
flrt%(vertcon%) = 1          'vert.convert.-filter for GENDY1.BAS
WRITE #1, Nmax&, dynMin%, dynMax%, flrt%(vertec%), flrt%(vertcon%)
CLOSE #1

'
'
'
'
'

*****



OPEN M0$ FOR OUTPUT AS #1 'as an example, this is the 13th dyn%-field
dyn% = 13
I13max% = 13              'number of divisions of the waveform
DEBmax&(dyn%) = 25         'max.number of sound or silence sound-patches.
                           'proportionality factor and coefficient for
                           'the exponential distribution:
D(dyn%) = mkr * .45 / (1.75 * 1.25)
p(dyn%) = .35              'the BERNOULLI distribution.
ralon%(dyn%) = 9            'minimal time interval extention
filter%(dyn%, horiz%) = 1
filter%(dyn%, e%) = 1
filter%(dyn%, ecrvrt%) = 1
filter%(dyn%, convrt%) = 1
WRITE #1, dyn%, I13max%, DEBmax&(dyn%), D(dyn%), p(dyn%),
ralon%(dyn%), filter%(dyn%, horiz%), filter%(dyn%, e%), filter%(dyn%, ecrvrt%),
filter%(dyn%, convrt%)
CLOSE #1

OPEN M1$ FOR OUTPUT AS #1
A13 = .01: B13 = 5: U131& = 1: V131& = -1: U2&(dyn%) = 7:
V2&(dyn%) = -7: Rdct13 = 1: distrPC13 = 1
WRITE #1, A13, B13, U131&, V131&, U2&(dyn%), V2&(dyn%), Rdct13, distrPC13
CLOSE #1

OPEN M2$ FOR OUTPUT AS #1
Ad13 = 1: Bd13 = 6: Ud131& = 2: Vd131& = -2: Ud132& = 20:
Vd132& = 0: Rdcd13 = 1: distrPD13 = 2
WRITE #1, Ad13, Bd13, Ud131&, Vd131&, Ud132&, Vd132&, Rdcd13, distrPD13
CLOSE #1
*****
```

```

',

',

'##### DIM TH&(0 TO 20, 0 TO 100)      'starting point (sample) of a sound/silence patch
DIM DUR&(0 TO 20, 0 TO 100)      'duration of that patch
DIM THpr&(0 TO 20, 0 TO 100)     'present starting point
DIM BED&(0 TO 20, 0 TO 100)      'variable for the computation of the patches
DIM sTHend&(0 TO 20)           'last sample
'@@@@@@@@@@@ OPEN prt$ FOR OUTPUT AS #1 'COMPUTING the sound or silence patches .
FOR dyn% = dynMin% TO dynMax%
  'n = 4000 + 100 * psi% + 10 * dyn%: RANDOMIZE n
  DEB& = 0 : IF p(dyn%) <= 0 THEN
    'ignore this dyn%field
    GOTO Gp2
  END IF
Gp1:
  DEB& = DEB& + 1: y1 = RND: y2 = RND
  DR = -(LOG(1 - y2)) / D(dyn%)          'patch-duration=EXPON.
  'distrib/sec.
  DUR&(dyn%, DEB&) = DR * 44100          'same in samples.
  THpr&(dyn%, DEB&) = THpr&(dyn%, DEB& - 1) + DUR&(dyn%, DEB&)
  IF y1 <= p(dyn%) THEN
    'the sound is in this patch!
    TH&(dyn%, DEB&) = THpr&(dyn%, DEB& - 1)
    THDUR = THDUR + DR
    BED&(dyn%, DEB&) = BED&(dyn%, DEB&) + 1
    DBE& = DBE& + 1
  END IF
  IF DEB& < DEBmax&(dyn%) THEN
    GOTO Gp1
  ELSE
    FOR xi% = 1 TO DEBmax&(dyn%)
      THend& = TH&(dyn%, xi%) + DUR&(dyn%, xi%): TELOS& =
      TELOS& + DUR&(dyn%, xi%)
      WRITE #1, BED&(dyn%, xi%), TH&(dyn%, xi%), DUR&(dyn%, xi%),
      THend&, TH&(dyn%, xi%) / 44100, DUR&(dyn%, xi%) / 44100,
      THend& / 44100
      'last sample of this dyn%-field
      ',

      IF THend& >= sTHend&(dyn%) THEN
        sTHend&(dyn%) = THend&
      END IF
    NEXT xi%
    WRITE #1, THDUR, THDUR / (TELOS& / 44100), sTHend&(dyn%)
    DURsec = (sTHend&(dyn%)) / 44100
  END IF

```

END IF

'against the overflow
',

```

sU2& = sU2& + U2&(dyn%)
sV2& = sV2& + V2&(dyn%)
THDUR = 0: TELOS& = 0: DBE& = 0
sTHend&(dyn%) = 0

```

Gp2:

```

  NEXT dyn%
  'Proportionality for less than 16 bits amplitudes (upper mirrors)
  ++++++ FOR dyn% = dynMin% TO dynMax%
    IF p(dyn%) > 0 THEN
      IF dyn% = 1 THEN

```

```

OPEN A1$ FOR OUTPUT AS #1: U2&(dyn%) = (98 / sU2&) * U2&(dyn%)
WRITE #1, A1, B1, U11&, V11&, U2&(dyn%), V2&(dyn%), Rdct1, distrPC1
CLOSE #1
  ELSEIF dyn% = 2 THEN
  ,
  ,
  ,
  ,

```

```

  ELSEIF dyn% = 13 THEN
OPEN M1$ FOR OUTPUT AS #1: U2&(dyn%) = (98 / sU2&) * U2&(dyn%)
WRITE #1, A13, B13, U131&, V131&, U2&(dyn%), V2&(dyn%), Rdct13, distrPC13
CLOSE #1
  ELSEIF dyn% = 14 THEN
  ,
  ,
  ,
  ,

```

END IF

END IF

NEXT dyn%

```

  'Proportionality for less than 16 bits amplitudes(lower mirrors)
  ++++++ FOR dyn% = dynMin% TO dynMax%
    IF p(dyn%) > 0 THEN
      IF dyn% = 1 THEN

```

```

OPEN A1$ FOR OUTPUT AS #1: V2&(dyn%) = (-98 / sV2&) * V2&(dyn%)
WRITE #1, A1, B1, U11&, V11&, U2&(dyn%), V2&(dyn%), Rdct1, distrPC1
CLOSE #1
  ELSEIF dyn% = 2 THEN
  ,
  ,
  ,
  ,

```

'GENDY1.BAS

'This programme controls several stochastic-dynamic sound-fields.
 'A stochastic-dynamic sound-field is made out of a wave-length T_1
 'divided in $I_{max\%}$ segments (durations). Each one of these segments
 'is stochastically varied by a cumulated probability-distribution.
 'At the ends of each one of these segments are computed the amplitudes
 '(ordinates) that will form the waveform polygone. Are defined:
 'for the duration abscissa a probability distribution and 2 times 2
 'elastic mirrors; for the amplitude ordinates a probability distri-
 'bution and 2 times 2 elastic mirrors. In between the vertices a linear
 'interpolation of points completes the waveform polygone.
 '@@@@@@@@@@@@@@@@
 '1st field:
 'compute one sound-sample:
 DECLARE SUB DYNAS1 (I1max%, SMP&, C11&, C12&, t11&, t12&, I1%, N1&,
 fh&, hf&, hh&)
 'compute the amplitude-ordinate:
 DECLARE SUB PG1 (T1, I1%) T1, I1%, I1%,

```
DECLARE SUB PCI (Tab11(), Tab12(), I1%, N1&c)
'compute the time-abscissa:
DECLARE SUB PD1 (Tad11(), Tad12(), I1%, N1&c)
'2d field:
,
,
,
```

```

'are to be found in PARAG(psi%).
'===== COMPUTATION'S BEGINING =====
'ysp% = yspMin%
*****1bg1:
          CALL ARCHSEQ1(ysp%, yspMax%)
'$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
'Free dimensioning of the tables
'-----
'Tables for the ordinate values of the I1% segment for cumulation.
DIM Tab11(1 TO 2, 0 TO 90): DIM Tab12(1 TO 2, 0 TO 90)'K=1 or 2:IjMax%
      = 90
'Tables for the abscissa values of the I1% segment for cumulation.
DIM Tad11(1 TO 2, 0 TO 90): DIM Tad12(1 TO 2, 0 TO 90)

'Tables for the ordinate values of the I2% segment for cumulation.
DIM Tab21(1 TO 2, 0 TO 90): DIM Tab22(1 TO 2, 0 TO 90)
,
,
,
,
'Tables for the ordinate values of the I13% segment for cumulation.
DIM Tab131(1 TO 2, 0 TO 90): DIM Tab132(1 TO 2, 0 TO 90)
'Tables for the abscissa values of the I13% segment for cumulation.
DIM Tad131(1 TO 2, 0 TO 90): DIM Tad132(1 TO 2, 0 TO 90)
'Tables for the ordinate values of the I14% segment for cumulation.
DIM Tab141(1 TO 2, 0 TO 90): DIM Tab142(1 TO 2, 0 TO 90)
,
,
,
,
*****dyn% = index of the stochastic subroutine DYNAS(dyn%);
DEB&(dyn%) = ordinal index of the sound-patches of this routine;
DEBmax&(dyn%) = last sound-patch of this routine;
DUR&(dyn%,DEB&(dyn%)) = sound-duration whose ordinal number is
DEB&(dyn%);
TH&(dyn%,DEB&(dyn%)) = the SMP& sample at which each sound-patch
commences;
SMP& = number of the running sample;
Ijmax% = number of subdivisions of a waveform time-length.
DIM DEBmax&(0 TO 20)

```

DIM DEB&(0 TO 90)	'max.patch numb.:dynMin%=0 TO 'dynMax%=20
DIM D(0 TO 20)	'current patch numb.: 0 TO 'DEBmax&(dyn%)=90
DIM pp(0 TO 20)	"in expon.dens.;dynMin%=0 TO 'dynMax%=20
DIM TH&(0 TO 20, 0 TO 90)	"in Bernoulli dens.;dynMin%=0 TO 'dynMax%=20
DIM DUR&(0 TO 20, 0 TO 90)	'patch start: dyn%=0 TO '20,DEB&(dyn%)=0 TO 90
DIM BED&(0 TO 20, 0 TO 90)	'patch dur.:dyn%=0 TO '20,DEB&(dyn%)=0 TO 90
DIM U2&(0 TO 20)	'patch param.:dyn%=0 TO '20,DEB&(dyn%)=0 TO 90
DIM V2&(0 TO 20)	'upper mirror size: dynMin%=0 TO 'dynMax%=20
DIM sTHend&(0 TO 20)	'lower mirror size: dynMin%=0 TO 'dynMax%=20
DIM flrt%(0 TO 2)	'last sample of the considered dyn%.
DIM filter%(0 TO 20, 0 TO 10)	'final screen or converter filter.
DIM ralon%(1 TO 20)	'ten available filters per field (dyn%).
	'extention of abscissa.

'readings of sequences' data from files written by PARAG(psi%).	
'	
R\$ = LTRIM\$(STR\$(psi%))	
prt\$ = "prt" + R\$: prt\$ = prt\$ + ".DAT"	'sound-patches data-files.
Q0\$ = "ARAG00" + R\$: Q0\$ = Q0\$ + ".DAT"	'general data-file for all 'sequences.
A0\$ = "ARAG10" + R\$: A0\$ = A0\$ + ".DAT"	'specific data for 1st 'dyn%-field.
A1\$ = "ARAG11" + R\$: A1\$ = A1\$ + ".DAT"	
A2\$ = "ARAG12" + R\$: A2\$ = A2\$ + ".DAT"	
B0\$"ARAG20" + R\$: B0\$ = B0\$ + ".DAT"	
,	
,	
,	
,	
M0\$ = "ARAG130" + R\$: M0\$ = M0\$ + ".DAT"	'specific data for 13th 'dyn%-field.
M1\$ = "ARAG131" + R\$: M1\$ = M1\$ + ".DAT"	
M2\$ = "ARAG132" + R\$: M2\$ = M2\$ + ".DAT"	
N0\$ = "ARAG140" + R\$: N0\$ = N0\$ + ".DAT"	


```

lbg10:
  NEXT dyn%
  CLOSE #1
  '++++++++
  chD&(psi%) = megDUR
  sDURech = sDURech + chD&(psi%)
  'cumulation of the longest
  'sequence -durations.
  DURlept = INT(sDURech / (44100 * 60))
  DURsec = (sDURech / 44100) MOD 60
  'duration in minutes.
  'duration in seconds.
  =====
  megDUR = 0
  '$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
  FOR dyn% = dynMin% TO dynMax%
    DEB&(dyn%) = 1
    'starting number of the
    'sound or
    'silence patch for each
    'dyn%-field.
  NEXT dyn%
  *****
  CALL WINDO
  SECN = SMP& / 44100
  'screen window.
  SPM& = SMP& MOD 44100
  'running sample.
  PRINT SMP&: PRINT SECN: PRINT sampl&: PRINT sec
  'running seconds.
  dyn% = dynMin%
  *****
  SMP& = 0
  ff% = 0
  'sample number.
  hf& = 0
  'screen amplitude of
  'a current sample.
  'converter amplitude of
  'a current sample.
  Kdyn% = dynMax% + 1
  'check of the dyn%-fields
  TELEN% = 0
  'amount still available.
  'for testing the music-piece
  *****
  ***** MAIN PROGRAMME *****

```

lbg2:

'This part concerns the computation of the amplitude (ordinate) at
 'a given sample SMP& by adding up the sound contributions of all
 'dyn%-fields in a row from dynMin% to dynMax% with their
 'patches DEB&(dyn%), their starting samples TH&(dyn%,DEB&(dyn%)) and
 'their durations DUR&(dyn%,DEB&(dyn%)). This computation defines
 'concurrently the amplitude and time elements of the waveform polygons.

IF DEB&c(dyn%) > DEBmax&c(dyn%) THEN

IF Kdyn% = dynMin% THEN
TELEN% = 1: GOTO lbg5


```

***** hf& = Q *****

IF SMP& < 400000000 THEN
  '(400000000 is an arbitrary number.)
  'for the screen, if we wish to show the resultant:
  LINE (abs1%, ord1%)-(abs2%, ord2%)
  abs1% = abs2%
  ord1% = ord2%
  SMP& = SMP& + 1
  sampl& = sampl& + 1
  END IF
  'global sampling
  'point

'change of sequence.
***** IF SMP& <= chD&(psi%) THEN
  GOTO lbg9
ELSE
  ysp% = ysp% + 1
  IF ysp% < yspMax% THEN
    GOTO lbg1
  ELSE
    GOTO lbg8
  END IF
END IF
lbg9:
  abs2% = sampl& MOD 639
  IF abs2% = 0 THEN
    abs1% = 0
  END IF
  ++++++
  'every point is now written in the converter file.
  -----
  sample% = hf&
  hf& = 0
  PUT #3, , sample%
  ++++++
  'in the converter
  ++++++
  'chronologies and beep signals.
  -----
  SECN = SMP& / 44100
  secd = sampl& / 44100
  SPM& = SMP& MOD 44100
  IF SPM& > = 0 AND SPM& 2 THEN
    SOUND 1000, 1000 / 500
  END IF
  IF abs2% = 0 THEN
    SOUND 500, 500 / 200
  END IF
END IF

```

```

ELSEIF ClePenetr% = 0 THEN
    GOTO lbl139
ELSEIF DEB&(dyn%) 1 THEN
    GOTO lbl137
END IF
'-----'
lbl137:
    N13& = 2: PSET (0, 0): C131& = SMP&
lbl131:
    IF N13& MOD 2 = 0 THEN
        'K% = alternating switch for cumulating in tables:preced. or present period.
        K% = 2: GOTO lbl132
    ELSE
        K% = 1: GOTO lbl135
    END IF
lbl132:
    'first ordinate of the new period = last ordinate of the preceding one.
    Tab131(K%, 0) = Tab131(K% - 1, I13max%)
    Tab132(K%, 0) = Tab132(K% - 1, I13max%)
    Tad131(K%, 1) = Tad131(K% - 1, I13max%)
    Tad132(K%, 1) = Tad132(K% - 1, I13max%)
    GOTO lbl136
lbl135:
    Tab131(K%, 0) = Tab131(K% + 1, I13max%)
    Tab132(K%, 0) = Tab132(K% + 1, I13max%)
    Tad131(K%, 1) = Tad131(K% + 1, I13max%)
    Tad132(K%, 1) = Tad132(K% + 1, I13max%)
lbl136:
    I13% = 1
lbl133:
    p13& = 0
    'computing the Imax ordinates.
    '-----'
    CALL PC13(Tab131(), Tab132(), I13%, N13&)
    'computing the Imax abscissa-intervals.
    '-----'
    CALL PD13(Tad131(), Tad132(), I13%, N13&)
    e13& = Qd13
    'horizontal abscissa filter
    '-----'
    IF filter%(dyn%, horiz%) = 0
    THEN
        GOTO f13lr1
    END IF
    e13& = (PDprc131& + PDprc132& + Qd13) / 3/filter
    PDprc132& = PDprc131&
    PDprc131& = Qd13
    'filter
    'filter
f13lr1:
    IF N13& MOD 2 = 0 THEN
        K% = 2: GOTO lbl134

```

```

ELSE
    K% = 1: GOTO lbl134
END IF
'-----'
lbl134:
    'Drawing the polygone of period T113
    '-----'
    C132& = C131& + e13&
    sc% = 639
    t131& = Tab132(K%, I13% - 1): t132& = Tab132(K%, I13%)
    t13& = t132& - t131&
    '-----'
    'LINEAR INTERPOLATION OF ORDINATES
    'in-between the abscissa C131& and C132&
    '-----'
lbl139:
    'extension of abscissa (= ralon%)
    '-----'
    IF filter%(dyn%, e%) = 1 THEN
        e13& = ralon%(dyn%)
    END IF
    IF p13& > e13& THEN
        GOTO lbl1310
    ELSEIF p13& = e13& AND e13& <> 0 THEN
        GOTO lbl1310
    ELSEIF e13& = 0 AND filter%(dyn%, e%) = 0 THEN
        e13& = 1
    END IF
    p13& = p13& + 1
    fh& = p13& * t13& / e13& + t131&
    'Attack and decay of a sound-patch.
    '-----'
    DIAFA& = SMP& - TH&(dyn%, DEB&(dyn%))
    IF DIAFA& = 0 AND DIAFA <= 500 THEN
        fh& = fh& * DIAFA& / 500
    END IF
    DIAFDIM& = TH&(dyn%, DEB&(dyn%)) + DUR&(dyn%, DEB&(dyn%)) -
    SMP&
    IF DIAFDIM& <= 1000 THEN
        fh& = fh& * DIAFDIM& / 1000
    END IF
    'Acoustic Normalisation
    '-----'
    hh& = fh& * 32767 / 100
    'screen's vertical filter
    '-----'
    IF filter%(dyn%, ecrvrt%) = 0
    THEN

```

IF ClePenetr% = 1 THEN

'Input of the stochastic-distribution coefficients, of the elastic-mirror sizes, of a reduction factor and of the specific stochastic-distribution used for computing the amplitude-ordinates of the waveform polygone.

```
OPEN M1$ FOR INPUT AS #1
INPUT #1, A13, B13, U131&c, V131&c, U2&(dyn%), V2&(dyn%), Rdct13,
distrPC13
CLOSE #1
IF
```

END IF

IF N13& MOD 2 =

ELSE

$$K\% = 2$$

```

END IF
R'S random-number generator:
  xampl = ((xampl * aampl + campl) / mampl - INT((xampl * aampl +
  campl) / mampl)) * mampl
  z = xampl / mampl

```

'Built-in random-number generator:

```
z = RND
pi = 3.14159265359#: vang = 2 * pi / 44100
DO WHILE z = 0
    z = RND
LOOP
```

"CAUCHY

Cauchy = A13 * TAN((z -.5) * pi): Q13 = Tab13(K%, I13%) + Cauchy
 ELSEIF distrPC13 = 2 THEN

"LOGIST.:
L = -(LOG((1 - z) / z) + B13) / A13: Q13 = Tab131(K%, I13%) + L
ELSEIF distrPC13 = 3 THEN

```

"HYPERBCOS.:
hypc = A13 * LOG(TAN(z * pi / 2)): Q13 = Tab131(K%, I13%) + hypc
      ELSEIF distrPC13 = 4 THEN

```

"ARCSINE:

arcsin = A13 * (.5 - .5 * SIN((.5 - z) * pi)): Q13 = Tab131(K%, I13%) + arcsin
 ELSEIF distrPC13 = 5 THEN

"EXPON.:

expon = -(LOG(1 - z)) / A13: Q13 = Tab131(K%, I13%) + expon
ELSEIF distrPC13 = 6 THEN

"SINUS:

sinu = A13 * SIN(SMP& * vang * B13): Q13 = sinu 'validate coresp.expression
END IF

U&c = U131&c: V&c = V131&c: Q = Q13

CALL MIR0(U&, V&)

$$Q_{13} = Q$$

```

IF K% = 1 THEN
  Tab131(2, I13%) = Q13
ELSE
  Tab131(1, I13%) = Q13
END IF
  Q13 = Q13 * Rdct13
  'Q13 = Q13
  Q13 = Tab132(K%, I13%) + Q13
U& = U2&(dyn%): V& = V2&(dyn%): Q = Q13
CALL MIR0(U&, V&)
  'valeur filtree
  'filter
  Q = (prc131 + Q) / 2
  'Q = (prc131 + prc132 + Q) / 3
  'prc132 = prc131
  prc131 = Q
  Q13 = Q
  IF K% = 1 THEN
    Tab132(2, I13%) = Q13
  ELSE
    Tab132(1, I13%) = Q13
  END IF
END SUB

SUB PD13 (Tad131(), Tad132(), I13%, N13&)
  'Subroutine of the 13th dyn%-field that computes the time-interval
  'between two vertices of the waveform polygon.

SHARED Q, Qd13, I13max%, SMP&, fh&, hf&, ClePenetr%, M2$%
SHARED aabsc, cabsc, mabsc, xabsc
STATIC Ad13, Bd13, Ud131&, Vd131&, Ud132&, Vd132&, Rdcd13, distrPD13

IF ClePenetr% = 1 THEN
  'Input of the stochastic-distribution coefficients, of the elastic-
  'mirror sizes, of a reduction factor and of the specific stochastic-
  'distribution used for computing the time-interval in-between
  'two verices of the waveform polygon.

  OPEN M2$ FOR INPUT AS #1
  INPUT #1, Ad13, Bd13, Ud131&, Vd131&, Ud132&, Vd132&,
  Rdcd13, distrPD13
  CLOSE #1
END IF

IF N13& MOD 2 = 0 THEN
  K% = 1
ELSE
  K% = 2
END IF

```

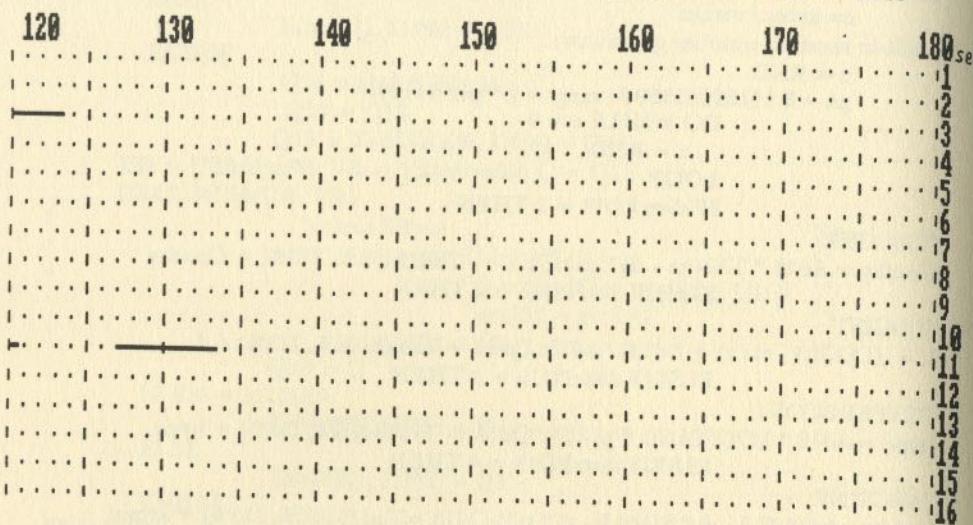
```

'LEHMER'S random-number generator:
'xabsc=((xabsc*aabsc+cabsc)/mabsc-INT((xabsc*aabsc+cabsc)/mabsc))*mabsc
 z= xabsc / mabsc
'Built-in random-number generator:
 z = RND
 pi = 3.14159265359#: vang = 2 * pi / 44100
 DO WHILE z = 0
   z = RND
   LOOP
   IF distrPD13 = 1 THEN
     "CAUCHY:
     Cauchy = Ad13 * TAN((z - .5) * pi): Qd13 = Tad131(K%, I13%) + Cauchy
     ELSEIF distrPD13 = 2 THEN
     "LOGIST.:
     L = -(LOG((1 - z) / z) + Bd13) / Ad13: Qd13 = Tad131(K%, I13%) + L
     ELSEIF distrPD13 = 3 THEN
     "HYPERBCOS.:
     hypc = Ad13 * LOG(TAN(z * pi / 2)): Qd13 = Tad131(K%, I13%) + hypc
     ELSEIF distrPD13 = 4 THEN
     "ARCSINE:
     'arcsin = Ad13 *(.5 - .5 * SIN((.5 - z) * pi)): Qd13 = Tad131(K%, I13%) + arcsin
     ELSEIF distrPD13 = 5 THEN
     "EXPON.:
     expon = -(LOG(1 - z)) / Ad13: Qd13 = Tad131(K%, I13%) + expon
     ELSEIF distrPD13 = 6 THEN
     "SINUS:
     sinu = Ad13 * SIN(SMP& * vang * Bd13): Qd13 = sinu 'validate corespx.expression
   END IF
   U& = Ud131&: V& = Vd131&: Q = Qd13
   CALL MIR0(U&, V&)
   Qd13 = Q
   IF K% = 1 THEN
     Tad131(2, I13%) = Qd13
   ELSE
     Tad131(1, I13%) = Qd13
   END IF
   Qd13 = Qd13 * Rdcd13
   'Qd13 = Qd13

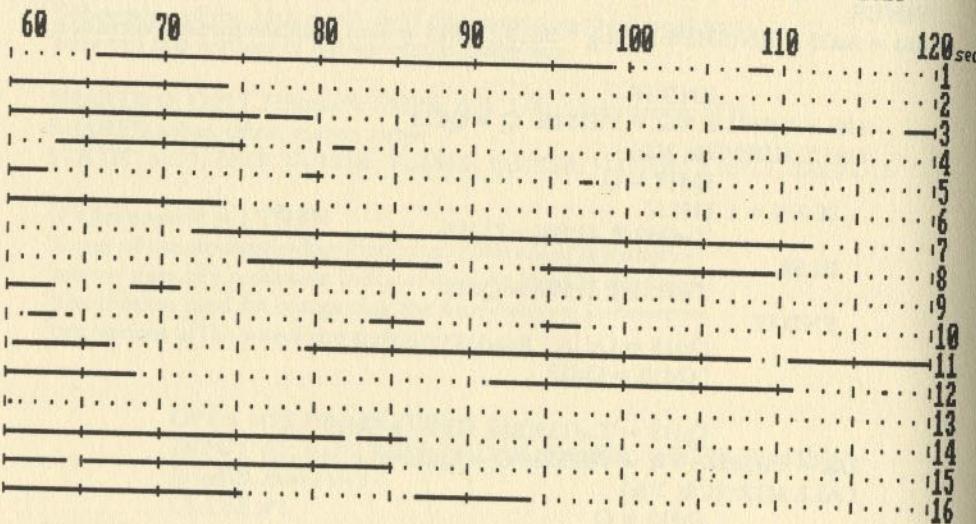
   Qd13 = Tad132(K%, I13%) + Qd13
   U& = Ud132&: V& = Vd132&: Q = Qd13
   CALL MIR0(U&, V&)
   Qd13 = Q
   IF K% = 1 THEN
     Tad132(2, I13%) = Qd13
   ELSE
     Tad132(1, I13%) = Qd13
   END IF
END SUB

```

SON=S352; ysp% = 7; psix% = 8; Pge = 3; dur. Min:Sec = 2:13; tot. Min : Sec = 2:13



SON=S352; ysp% = 7; psix% = 8; Pge = 2; dur. Min:Sec = 2:13; tot. Min : Sec = 2:13



Two pages of the "score" resulting from the programme reproduced here.

Appendix I

[7] [20]

TWO LAWS OF CONTINUOUS PROBABILITY

First Law

$$P_x = ce^{-cx} dx.$$

Let OA be a segment of a straight line of length l on which we place n points. Their linear density is $c = n/l$. Suppose that l and n increase indefinitely while c remains constant. Suppose also that these points are numbered A_1, A_p, A_q, \dots and are distributed from left to right beginning at the origin 0. Let

$$x_1 = A_1 A_p, x_2 = A_p A_q, x_3 = A_q A_r, \dots, x_i = A_s A_t.$$

The probability that the i th segment will have a length x_i between x and $x + dx$ is

$$P_x = e^{-cx} c dx.$$

Now the probability p_n , that there will be n points on a segment x , is given by the recurrence formula

$$\frac{p_{n+1}}{p_n} = \frac{cx}{n+1}.$$

Therefore $p_1 = (cx/1)p_0$. But $p_0 = e^{-cx}$ and

$$e^{-cx} = 1 - \frac{cx}{1!} + \frac{(cx)^2}{2!} - \frac{(cx)^3}{3!} + \dots$$

If x is very small and if we denote it by dx , we have

$$p_0 = 1 - c dx + \frac{c^2(dx)^2}{2!} \dots$$

Since the powers of dx are infinitely small for high values, $p_0 = 1 - c dx$ and $p_1 = c dx p_0 = c dx$. Hence, the probability P_x is composed of the probability $p_0 = e^{-cv}$, that there will be no point on the segment x , and the probability $p_1 = c dx$, that there will be a point in dx .

APPROXIMATE CALCULATION OF THE SAME PROBABILITY
(FOR CALCULATION BY HAND)

Let there be d points to be placed on a straight line of length l . The linear density is $c = d/l$ points on length l . If the lengths are expressed in units v then $l = av$ ($a > 0$) and $cv = d/a$ points in the unit of length v .

Then $x_i = iv$ ($i = 0, 1, 2, 3, \dots$), and the probability, the asymptotic limit of the relative frequency of the segment x_i , will be

$$P_{x_i} = e^{-cv} c \Delta x_i. \quad (1)$$

We shall now define the quantity Δx_i . The probability (1) is composed of the probability $p_0 = e^{-cv}$, that there will be no point on x_i , and the probability $p_1 = c \Delta x_i$, that there will be a point in Δx_i if $(c \Delta x_i)^2$ is small enough to be ignored. Set

$$0 < (c \Delta x_i)^2 < 10^{-n}.$$

where n is a sufficiently large natural number; this expression becomes

$$0 < \Delta x_i < c^{-1} \cdot 10^{-n/2}.$$

Substitute a constant z for Δx_i such that for every x_i

$$z \leq \Delta x_i < c^{-1} \cdot 10^{-n/2}. \quad (2)$$

Then equation (1) is written

$$P_{x_i} = e^{-cv} \cdot cz \quad (3)$$

and must satisfy the condition

$$\sum_{i=0}^{i=\infty} e^{-cv} \cdot cz = 1,$$

or

$$z = 1/c \cdot \sum_{i=0}^{i=\infty} e^{-cv}.$$

But since $cv > 0$, $e^{-cv} < 1$, so that

$$\sum_{i=0}^{i=\infty} (e^{-cv})^i = 1/(1 - e^{-cv})$$

and finally

$$z = \frac{1 - e^{-cv}}{c}.$$

Now from (2)

$$\frac{1 - e^{-cv}}{c} < \frac{10^{-n/2}}{c}.$$

Therefore

$$0 < (1 - e^{-cv}) < 10^{-n/2},$$

then

$$(1 - 10^{-n/2}) < e^{-cv} < 1.$$

Thus, for $cv > 0$ we have $e^{-cv} < 1$, and for $cv < -\log(1 - 10^{-n/2})$ we have $e^{-cv} > (1 - 10^{-n/2})$. And since $0 < 10^{-n/2} < 1$ we have

$$-\log(1 - 10^{-n/2}) = 10^{-n/2} + \frac{10^{-(n/2)2}}{2} + \frac{10^{-(n/2)3}}{3} + \frac{10^{-(n/2)4}}{4} + \dots$$

and

$$10^{-n/2} < -\log(1 - 10^{-n/2}).$$

In order that $e^{-cv} > 1 - 10^{-n/2}$ it is therefore sufficient that

$$cv \leq 10^{-n/2}. \quad (4)$$

Then we may take

$$\Delta x_i = z = \frac{1 - e^{-cv}}{c} \quad (5)$$

and substitute this value in formula (1), from which we can now set up probability tables. Here is an example:

Let $d = 10$ points as mean value to be spread on a straight line segment of length $l = 100$ cm. We have to define x_i and P_{x_i} as a function of i , given that $(c \Delta x_i)^2 = 10^{-4}$ is considered to be negligible.

From (4), $cv = 10^{-4/2} = 0.01$ points in v . Now $c = d/l = 10/100$ points/cm, therefore $c = 0.1$ points/cm, $v = 0.01/0.1 = 0.1$ cm, and $x_i = 0.1i$ cm $\hat{=} i$ mm.

From (5),

$$\Delta x_i = \frac{1 - e^{-0.01}}{0.1} = (1 - 0.9905)10 = 0.0995 \hat{=} 0.1 \text{ cm}.$$

From (1),

$$P_{x_i} = e^{-0.01i} \cdot 0.1 \cdot 0.1 = 0.01 \cdot (0.099005)^i.$$

For calculation by machine see Chapter V.

Second Law

$$f(j) dj = \frac{2}{a} \left(1 - \frac{j}{a}\right) dj.$$

Each variable (pitch, intensity, density, etc.) forms an interval (distance) with its predecessor. Each interval is identified with a segment x taken on the axis of the variable. Let there be two points A and B on this axis corresponding to the lower and upper limits of the variable. It is then a matter of drawing at random a segment within AB whose length is included between j and $j + dj$ for $0 \leq j \leq AB$. Then the probability of this event is:

$$P_j = f(j) dj = \frac{2}{a} \left(1 - \frac{j}{a}\right) dj. \quad (1)$$

for $a = AB$.

APPROXIMATE DEFINITION OF THIS PROBABILITY FOR CALCULATION BY HAND

By taking dj as a constant and j as discontinuous we set $dj = c, j = iv$ with $v = a/m$ for $i = 0, 1, 2, 3, \dots, m$. Equation (1) becomes

$$P_j = \frac{2}{a} \left(1 - \frac{iv}{a}\right) c. \quad (2)$$

But

$$\sum_{i=0}^{i=m} P_j = \frac{2c}{a} (m+1) - \frac{2cv}{a^2} \sum_{i=0}^{i=m} i = \frac{2c(m+1)}{a} - \frac{2cm(m+1)}{2a^2} = 1,$$

whence

$$dj = c = \frac{a}{m+1}.$$

On the other hand P_j must be taken as a function of the decimal approximation required:

$$P_j = \frac{2}{m+1} \left(1 - \frac{i}{m}\right) \leq 10^{-n} \quad (n = 0, 1, 2, 3, \dots).$$

P_j is at a maximum when $i = 0$, whence $m \geq 2 \cdot 10^n - 1$; so for $m = 2 \cdot 10^n - 1$ we have $v = a/(2 \cdot 10^n - 1)$ and $dj = a/(2 \cdot 10^n)$, and (1) becomes

$$P_j = P_i = \frac{1}{10^n} \left(1 - \frac{i}{2 \cdot 10^n - 1}\right).$$

DEFINITION OF THE SAME PROBABILITY FOR COMPUTER CALCULATION

We know that the computer can only draw numbers y_0 at random (of equal probability) $0 \leq y_0 \leq 1$. Using the probability law of density $P_j = f(j) dj$, we have for some interval x_0

$$\text{prob. } (0 \leq j \leq x_0) = \int_0^{x_0} f(j) dj = \frac{2x_0}{a} - \frac{x_0^2}{a^2} = F(x_0),$$

where $F(x_0)$ is the distribution function of j . But $F(x_0) = \text{prob. } (0 \leq y \leq y_0) = y_0$. Therefore

$$\frac{2x_0}{a} - \frac{x_0^2}{a^2} = y_0 \quad \text{and} \quad x_0 = a[1 \pm \sqrt{(1 - y_0)}],$$

and by rejecting the positive root, since x_0 must remain smaller than a , we obtain

$$x_0 = a[1 - \sqrt{(1 - y_0)}]$$

for all $0 \leq x_0 \leq a$.

Appendix II
[14]

Let there be states $E_1, E_2, E_3, \dots, E_r$ with $r < \infty$; and let one of these events necessarily occur at each trial. The probability that event E_k will take place when E_h has occurred at the previous trial is p_{hk} ;

$$\sum_k p_{hk} = 1, \quad \text{with } k = 1, 2, \dots, r.$$

$P_{hk}^{(n)}$ is the probability that in n trials we will pass from state E_h to state E_k ;

$$\sum_k P_{hk}^{(n)} = 1, \quad \text{with } k = 1, 2, \dots, r.$$

If for $n \rightarrow \infty$ one of the $P_{hk}^{(n)}$ tends towards a limit P_{hk} , this limit is expressed by the sum of all the products $P_{hj} p_{jk}$, j being the index of one of the intermediate states E_j ($1 \leq j \leq r$):

$$P_{hk} = P_{h1} p_{1k} + P_{h2} p_{2k} + \dots + P_{hr} p_{rk}.$$

The sum of all the limits P_{hk} is equal to 1:

$$P_{h1} + P_{h2} + P_{h3} + \dots + P_{hr} = 1.$$

We can form tables or matrices $D^{(n)}$ as follows:

$$D^{(n)} = \begin{bmatrix} P_{11}^{(n)}, & P_{21}^{(n)}, & \dots, & P_{r1}^{(n)} \\ \vdots & & & \\ P_{1m}^{(n)}, & P_{2m}^{(n)}, & \dots, & P_{rm}^{(n)} \\ \vdots & & & \\ P_{1r}^{(n)}, & P_{2r}^{(n)}, & \dots, & P_{rr}^{(n)} \end{bmatrix}$$

Regular case. If at least one of the tables $D^{(n)}$ contains at least one line m of which all the elements are positive, then the $P_{hk}^{(n)}$ have limits P_{hk} , and among the P_{hk} there exists at least one, P_m , which has a non-zero limit independent of n and of h . This is the *regular case*.

Positive regular case. If at least one of the tables $D^{(n)}$ has all positive elements, then all the P_{hk} have non-zero limits P_k independent of the initial index h . This is the *positive regular case*.

The probabilities $P_k = X_k$ constitute the system of solutions of the $r + 1$ equations with r unknowns:

$$X_1 = X_1 p_{11} + X_2 p_{21} + \dots + X_r p_{r1}$$

$$X_2 = X_1 p_{12} + X_2 p_{22} + \dots + X_r p_{r2}$$

$$X_3 = X_1 p_{13} + X_2 p_{23} + \dots + X_r p_{r3}$$

 \vdots

$$X_m = X_1 p_{1m} + X_2 p_{2m} + \dots + X_r p_{rm}$$

 \vdots

$$X_r = X_1 p_{1r} + X_2 p_{2r} + \dots + X_r p_{rr}$$

$$1 = X_1 + X_2 + \dots + X_r$$

But these equations are not independent, for the sum of the first r equations yields an identity. After the substitution of the last equation for one of the first r equations, there remains a system of r equations with r unknowns. Now there is a demonstration showing that in the regular case the system has only one solution, also that $D^{(n)} = D^n$ (n th power of D).

Appendix III

THE NEW UPIC SYSTEM *

Introduction

UPIC (Unité Polyagogique Informatique du CEMAMu)¹ is a machine dedicated to the interactive composition of musical scores. The new and final version of this system runs on an AT 386 microcomputer connected to a real-time synthesis unit. The new software offers a mouse-controlled, "user-friendly" window style graphical interface and allows real-time drawing, editing and playing of a musical page as well as the recording of a "performance."

Description

The UPIC is a music composing system which combines a graphic score editor, a voice editor and a powerful "performance" (or play-back) system, all sharing the same data. Therefore, all drawing and editing operations are available while the music plays. All the commands are mouse driven. A menu command allows one to switch the drawing input device from the mouse to the digitizer and vice versa.

A UPIC score is a collection of notes that are called "arcs." An arc is a pitch (frequency) versus time curve. The frequency variations are continuous and can cover the whole ambitus. The durations can range from 6 ms to the total duration of the musical page (1 hour maximum).

¹CEMAMu (Centre for studies in mathematics and automation of music), founded by Iannis Xenakis in 1965 with grants from the French Cultural Ministry.

*This appendix is freely inspired by a similar paper published by ICMC in Glasgow, 1990 in "Proceedings," written by Gérard Marino, Jean-Michel Raczinski, and Marie-Hélène Serra of CEMAMu. My gratitude for their faithful dedication is herewith expressed. (I.X.)

Tools are provided for obtaining quantified values of frequency and duration. In this way, the notion of an arc is an extension of the classical notion of a note. In addition, each arc has a set of sound attributes that can be changed real-time, during playback.

Voice editing on the UPIC includes redrawing and redefinition of waveforms, envelopes, frequency and amplitude tables, modulating arc assignment, and modification of audio channel parameters (dynamic and envelope). All these operations are feasible during playback and immediately heard.

Different sound interpretations of the same graphic score may be tested with the help of arc groups. Groups contain from one arc to the whole page and allow instantaneous and global modifications of sound parameters (waveform change, transposition, etc.).

During performance, the musician can switch from one page to another and may control the tempo and play position by moving the mouse across the page. The resulting live interpretation may be recorded in an editable object called a "sequence." The tempo and the position in the sequence is controllable while the sequence is being played.

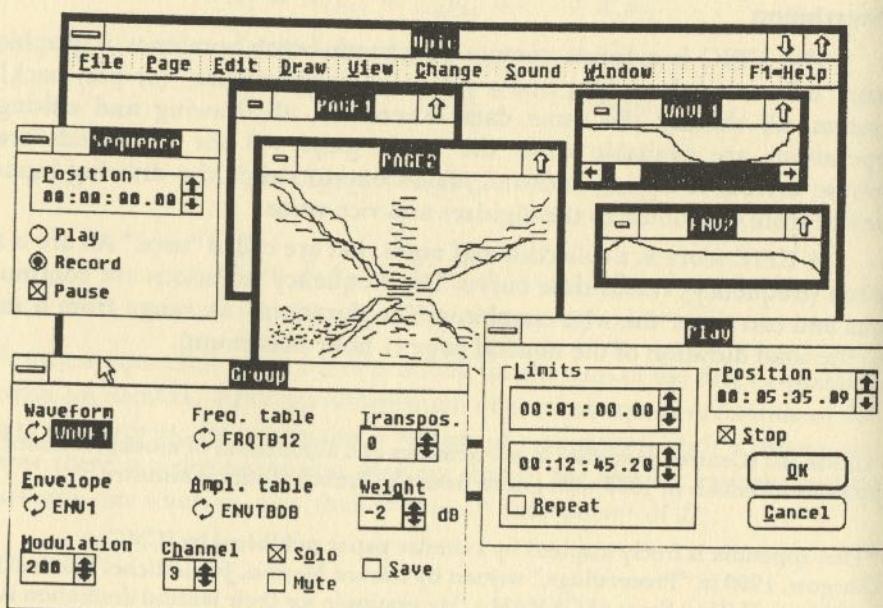


Figure 1. Sample screen from UPIC

Page Drawing and Editing

A maximum of four pages of music can be opened and displayed in moveable and resizable windows. Opening a page stored on the disk loads it into the memory of the real-time unit. Therefore, all the subsequent operations can be carried out while the page is being played.

Arcs can be drawn by using one of the drawing modes (free hand, broken line, etc.) If accepted, an arc is inserted in the page as soon as its drawing is over; if the limit of 64 oscillators is reached, the arc will be refused. At any time, it is possible to modify the set of the default attributes (waveform, envelope, frequency table, amplitude table, weight, modulating arc, audio channel). One page holds a maximum of 4000 arcs.

Usual editing commands (cut, copy, paste) are available. For each page, four groups of any number of arcs can be created by using different types of selection (block, list, criteria) Geometric operations like symmetry, rotation and vertical alignment can be applied to a group. Instantaneous modifications of the attributes (waveforms, envelope, frequency table, amplitude table, weight, modulating arc, audio channel) of the arcs belonging to a group can be temporarily applied and saved, if necessary. Furthermore, groups can be instantaneously muted, "solo-ed," and/or transposed.

Voice Edition

Each arc is associated with an oscillator whose configuration is given by the following arc attributes: waveform, envelope, modulating arc, audio channel. Before being transmitted to the oscillator, the graphic data of the arc and of the envelope are converted respectively by a frequency table and an amplitude table.

Waveforms and envelopes can be drawn or extracted from sampled sound, and normalized.

The contents of the conversion tables are defined either by a drawing or by a menu command and are redrawable.

The frequency table definition menu command enables the user to set the boundaries of the ambitus (in hertz or half-tones) and the musical scale parameters (tuning note and number of equal divisions in the octave). The frequency table can be inverted and can be made continuous or discrete. In the latter case, the steps are the octave divisions. When played with a discrete frequency table, the pitch variations within the arcs follow the frequency steps of the table.

Performance

Only one page can be played at a time. The four pages maximum in the window may be chained or not. The user chooses which page to play simply by clicking on it, stops or restarts the progression of the performance, defines the time limits of the performance with optional looping.

The tempo and play position can be defined by mouse motions on the page or by entering their values. All types of motions (forward, backward, jumps, acceleration, slowing down) within the page are permitted. When not user-controlled, the page is played at a constant tempo.

A set of channel parameters (dynamic and envelope) is assigned to each page. The dynamic and envelope of the 16 output audio channels are real-time controllable during performance. As the channel envelope spreads over the whole page, it is therefore possible to locally weight arcs assigned to a given channel.

In the UPIC, a sequence is the recording, during the performance (controlled or not) of all the successive positions in the page, with a 6 ms accuracy. It holds a maximum of 12 minutes of performance. It is displayable as a position versus time curve. Any piece of the sequence can be overwritten by a new recording or redrawn. The performance of a sequence is carried out inside its window with mouse motion controls (like the page itself). When four pages are loaded, the user has two sequences with which to work.

Storage

Pages, waveforms, envelopes, conversion tables and sequences are stored in separate banks (DOS files) on disk. Banks are user-protected. Copying, renaming, and deleting objects and banks is possible.

The user can load objects that come from different banks. Saving an object can be done in any bank.

Conclusion

This summarizes the principal characteristics of the UPIC system today. Additional commands are going to be integrated to the application, especially sampling utilities (record, play, simple edition fuctions) The synchronization of the performance with an external device as well as the communication between UPIC and MIDI devices is presently being studied. Tools will be provided to allow another application access the data of UPIC banks.

The system is being industrialized and will be commercialized in the course of 1991.

TECHNICAL DESCRIPTION

A) Hardware Specifications

Host computer

PC-AT 386 with 3 Megabytes memory minimum, hard disk, mouse, MIDI board, optional digitizer tablet. All Summagraphics compatible digitizers are supported (size A0 to A4).

Real-time synthesis unit

64 oscillators at 44.1 kHz with FM
(future extension to 128)

converter board:

4 audio output channels
2 audio input channels
AES/EBU interface
(extension to 4 converter boards)

capacity :

4 pages of 4000 arcs
64 waveforms (4K entries)
4 frequency tables (16K entries)
128 envelopes (4K entries)
4 amplitude tables (16K entries)
2 sequences (12 minutes each, 6 ms accuracy)

B) Software Main Features

Environment : DOS with Microsoft WINDOWS 3.x (graphical multi-application environment with pull-down menus and pop-up windows)

Storage : pages, waveforms, envelopes, frequency tables, amplitude tables and sequences are stored in separate banks on disk. Banks are user-protected.

Drawing : every object is initialized either by a command or by a drawing, and is redrawable. Objects are displayed in overlapped, resizable and zoomable windows.

Edition : several types of selection (block, list, criteria) allow the creation of up to four groups of arcs per page. Each group can be muted, solo-ed, graphically transformed and real-time controlled.

Sound : See C (Real-time controls)

C) Real-Time Controls

Page controls

- Tempo
- Play time interval (with or without looping)
- Page switching
- Position in the page
- For each audio channel : dynamic, envelope

Sequence controls

- Tempo
- Position
- Sequence switching

Group controls

- Solo
- Mute
- Transposition
- Intensity
- Frequency modulation
- Output channel
- Waveform (among 64)
- Frequency table (among 4)
- Envelope (among 128)
- Amplitude table (among 4)

Drawing while playing

While a page is being played, the user can modify its waveforms, envelopes and conversion tables.

A new arc can be heard as soon as its drawing is finished.

An existing arc can be redrawn within its endpoints and heard at the same time.

A Selected Bibliography of Iannis Xenakis*

compiled by Henning Lohner

The bibliography of the works of Iannis Xenakis is arranged as follows:

I. Works by Xenakis:

1. Books, and
2. Articles in periodicals, booklets, and encyclopedias. In general, the source of the first printing is given. The writings of Xenakis have been translated and reprinted many times.

II. Writings about Xenakis:

1. Monographs and collections, the majority of which are dedicated to Xenakis, and
2. Articles in periodicals, encyclopedias, and anthologies.

Works in daily and weekly newspapers, with some minor exceptions, are not included here. That applies, in particular, to the numerous festival catalogs and concert programs, which frequently have original material. Likewise, record notes by or about Xenakis, record reviews and introductions to works by Xenakis are excluded.

It should be mentioned here that Xenakis, as an independent architect and long-time collaborator of Le Corbusier is responsible for an extensive, architectural body of work, which is noted in the literature. We can, at present, include only the most important, useful, interdisciplinary works.

The writings are organized chronologically, and, within each year, arranged alphabetically. Articles without a specific author are listed by the initial letter of the title.

This bibliography is based on the private collection of the author, and also utilizes other relevant bibliographies on the subject at hand.

The list of abbreviations can be found at the end, p. 364.

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Abbreviations and symbols

Diss.	Dissertation
ff.	following
MQ	<i>Musical Quarterly</i>
No.	number
NMZ	<i>Neue Musikzeitung</i>
NZ	<i>Neue Zeitschrift für Musik</i>
ÖMZ	<i>Österreichischen Musikzeitschrift</i>
PNM	<i>Perspectives of New Music</i>
p.	page(s)
RILM	<i>Repertoire International de Litterature Musicale</i>
SMZ	<i>Schweizerische Musikzeitschrift</i>
Univ.	University
Vol.	Volume(s)
WOM	<i>World of Music</i>

DISCOGRAPHY

The following list is arranged in alphabetical order by recording company (and record number). Barring the various format exceptions (CD = compact disk; MC = cassette) the recordings are presented on long-playing records. Each entry also indicates the presence of works by another composer (with an asterisk [*]); the title of the compositions by Iannis Xenakis; and the performers.

1. Adda 81042 *
Mikka, Mikka S
P. Zukovsky, vl.
2. Adda 581224 CD *
Oophaa
E. Chojnacka, clav; S. Gualda, perc.
3. Adès 14.122-2 CD *
Metastasis, Nuits
Orchestra del Südwestfunk; H. Rosbaud, dir.; Soloists from the ORTF chorus; M. Couraud, dir.
4. Adès 16005 *
Herma
J. Mefano, pf.
5. Angel S-36560 *
Atrées
Ensemble Instrumental de musique contemporaine de Paris; K. Simonovich, dir.
6. Angel S-36655 *
Herma
G. Pludemacher, pf.
7. Angel S-36656
Achorripsis, Akrata, Polla ta dhina, ST/10
8. Arion ARN 38775 *
Nuits
Groupe Vocal de France; M. Tranchant, dir.
9. Babel 9054-1 CD
Echange, Palimpsest, Waarg, Eonta
A. Takahashi, pno; H. Sarnaay, clB; Asko Ensemble, D. Porcelijn, dir.
10. Barclay 920217 *
Anaktoria, Morisima-Amorsima
Octuor de Paris
11. Bis 256 *
Psappha
G. Mortensen, perc.
12. Bis 338 *
Keren
Christian Lindberg, Tb.
13. Bis CD 482 CD
Pleiades
The Kroumata Percussion Ensemble

Formalized Music

14. Boîte à Musique 070 *
Diamorphoses
 (electronic composition)

15. CBS 3461226
Akrata
 The Festival Chamber Ensemble; R. Dufallo, dir.

16. CBS Sony 32 DC 673 CD *
Psappha
 S. Joolihara, perc.

17. CBS Sony 32 DC 691 CD *
Pléades
 M. Aruga Ensemble

18. CBS Sony 32DC691 CD
Pleiades
 Makoto Aruga Percussion Ensemble

19. Candide 31049
Medea, Polytope de Montréal, Syrmos
 Ensemble Ars Nova; ORTF Chorus; M. Constant, dir.

20. Candide 31000
Nomos Gamma, Terretekthorh
 ORTF Philharmonic Orchestra; Ch. Bruck, dir.

21. Chant du Monde LDX 78308
 (also LDX-A 8368, CD LDC 278368)
Eonta, Metastasis, Pithoprakta
 Ensemble Instrumental de musique contemporaine de Paris; K. Simonovich, dir.; Orchestre National de l'ORTF; Le Roux, dir.; Y. Takahashi, pf.; L. Longo, P. Thibaud, tr.; M. Chapellier, G. Moisan, J. Toulon, trb.

22. Chant du Monde LDC 278 368
 CD *
Eonta
 Y. Takahashi, pno; Paris Contemporary Music Ensemble, K. Simonovitch, dir.
Metastasis Pithoprakta
 National Orch ORTF; M. Le Roux, dir.

23. Colosseum 3447253 CD *
Stratégie
 Yomiuri Nippon Symphony Orchestra; Seiji Ozawa and H. Wakasugi, dirs.

24. Colombia MS-7281 *
Akrata
 The Festival Chamber Ensemble; R. Dufallo dir.

25. Connaisseur Musik CP 6 *
Mikka, Mikka S
 P. Zukovsky, vl.

26. Cybernetics Serendipity Music ICA 01.02 *
Stratégie
 Yomiuri Nippon Symphony Orchestra; S. Ozawa and E. Wakasugi, dirs.

27. Decca 411610-1
Jonchaises

28. Decca HEAD 13 (also 591171)
Antikhton, Aroura, Synaphai
 G. Madge, pf.; New Philharmonia Orchestra; E. Howarth, dir.

29. Denon CO 73 768 CD
Pleiades
 Les Percussions de Strasbourg

30. Denon OX-7063 ND (also CD CO-1052) *
Evryali, Herma
 Y. Takahashi, pf.

Discography

31. Deutsche Grammophon 2530562
 *
Nomos Alpha
 S. Palm, vcl.

32. Disques Montaigne 782002 CD
Okho
 Le cercle Trio (perc.).

33. EMI Angel EAA-85013-5 *
Herma
 A. Takahashi, pf.

34. EMI CO631001
Achorripsis, Akrata, Polla ta dhina, ST/10
 Children's chorus of Notre-Dame de Paris; Ensemble Instrumental de musique contemporaine de Paris; K. Simonovich, dir.

35. EMI CVB 2190 *
Herma
 G. Pludemacher, pf.

36. EMI CVC-2086 (also MC MCV-2086c)
Atréés, Morsima-Amorsima, Nomos Alpha, ST/4
 P. Penassou, vcl.; Quatuor Bernède; Ensemble Instrumental de musique contemporaine de Paris; K. Simonovich, dir.

37. EMI Columbia SCXG-55 *
 ST/10
 Th. Antoniou, dir.

38. EMI HMV CSDG-63 *
Anaktoria
 Th. Antoniou, dir.

39. Elektrola (Hör Zu)
Concret PH, Medea, Orient-Occident
 (electronic composition)

40. Erato 2292-45030-2 CD
A l'île de Gorée, Naama

41. Erato Musicfrance 2292 45019-2
 CD *
Ikhoor
 Paris String Trio

42. Erato Musicfrance 2292 45030-2
 CD *
Khoaï, Komboï
 E. Chojnacka, clar.; S. Gualda, perc.

43. Erato NUM 75104 *
Komboï
 E. Chojnacka, clav.; S. Gualda, perc.

44. Erato STU 70457 *
Nuits
 Soloists and Chorus of ORTF; M. Couraud, dir.

45. Erato STU 70526 (also 9088)
Medea, Polytope de Montréal, Syrmos
 Ensemble Ars Nova; Chorus of ORTF; M. Constant, dir.

46. Erato STU 70527/28 *
Kraanerg
 M. Constant, dir.

47. Erato STU 70529 (also 91 19)
Nomos Gamma, Terretekthorh
 ORTF Philharmonic Orchestra; Ch. Bruck, dir.

48. Erato STU 70530
Bohor, Concret PH, Diamorphoses, Orient-Occident
 (electronic composition)

49. Erato STU 70656 (also 9137)
Oresteia
 S. Caillat Chorale ; Maîtrise de Notre-Dame de Paris; Ensemble Ars Nova; M. Constant, dir.

50. Erato STU 71106 *
Psappha
 S. Gualda, perc.

51. Erato STU 71266 *
Khoäi
 E. Chojnacka, clav.

52. Erato STU 71513
Cendrées, Jonchaises, Nomos Gamma
 Gulbenkian Foundation Chorus
 of Lisbon; Orchestre National
 de France; M. Tabachnik, dir.;
 ORTF Philharmonic Or-
 chestra; Ch. Bruck, dir.

53. Erato Interfaces (for
 Hewlett-Packard)
Aës.
 Spyros Sakkas, baritone; Sylvio
 Gualda, percussion; Sym-
 phonieorchester des Bay-
 erischen Rundfunk, dir.
 Michel Tabachnik;
Kekuia.
 Kölner Rundfunk-Symphonieor-
 chester; Kölner Rundfunchor,
 dir. Michel Tabachnik.
N'shima.
 Anne Bartolloni, Geneviève
 Renon, mezzo-sopranos; En-
 semble Instrumentale, dir. Mi-
 chel Tabachnik.

54. Ernst Klett 92422 *
Diamorphoses
 (electronic composition)

55. Etcetera KTC 1075 *
Kraanberg
 Alpha Centauri Ens., R. Wood-
 ward, dir.

56. Eterna Stereo 827906 *
Dmaathen

57. Finlandia 120 366-2 CD *
Anaktoria
 Members of the Avanti Chamber
 Orchestra

58. Finlandia FACD 357 *
Khoäi
 J. Tiensu, clav.

59. Gaudeamus Foundation - Radio
 Netherland (3 CD, 1988)
Gmeevoorh
 K. Hoek, Organ

60. Gramavision R2 79440 CD *
Tetras
 Arditti String Quartet

61. HMV S-ASD 2441
Morsima-Amorsima, Nomos Alpha,
ST/4
 P. Penassou, vcl.; Quatuor
 Bernède; Ensemble Instru-
 mental de musique contem-
 poraine de Paris; K.
 Simonovich, dir.

62. Harmonia Mundi HMC 5172 *
Mists
 C. Helffer, pf.

63. Harmonia Mundi HMC 905185
 CD
Pléiades
 Les Percussions de Strasbourg

64. Hungaroton 12569 (also CD
 HCD 12569) *
Mists
 K. Körmendi, pf.

65. Jeugden Muziek BVHAAST 007
Eonta, Eryiali, Herma
 G. Madge, pf. P. Eötvös, dir.

66. Limelight 86047 *
Orient-Occident
 (electronic composition)

67. Lyra 251
Eonta, Metastasis, Pithoprakta
 Y. Takahashi, pf.; Ensemble In-
 strumental de musique con-
 temporaine de Paris; K.
 Simonovich, dir.

68. Mainstream 5000 *
Herma
 Y. Takahashi, pf.

69. Mainstream MS-5008 *
Achorripsis
 Hamburger Kammersolisten; F.
 Travis, dir.

70. Musical Society MHS 1187 *
Medea, Nuits
 Ensemble Ars Nova; ORTF
 Chorus; M. Couraud, dir.

71. Musical observations CP 2/6 *
Mikka, Mikka S
 P. Zukovsky, vl.

72. Musidisc RC-16013
Anaktoria, Morsima-Amorsima
 Octuor de Paris

73. Neuma 450-71 *
Theraps
 R. Black, Cb.

74. Neuma 450-74 CD *
Mycène A
 (electronic composition)

75. Nieuwe Muziek 004
Dmaathen, Epeï, Palimpsest,
Phlegra
 Xenakis Ensemble; H. Kerstens,
 dir.

76. Nippon SFX-8683 *
Persépolis
 (electronic composition)

77. Nonesuch 32818 (also H-71201) *
Akrata, Pithoprakta
 Buffalo Philharmonic Orchestra;
 Lukas Foss, dir.

78. Nonesuch H-71245
Bohor, Concert PH, Diamorphoses,
Orient-Occident
 (electronic composition)

79. Owl 26 *
Charisma
 Jungerman, cl.; Banks, vcl.

80. Performance PER 84061 *
Jonchaises
 Leicestershire Schools Sym-
 phony Orchestra; P. Fletcher,
 dir.

81. Philips 6521020 (also 6718040) *
Persephassa
 Les Percussions de Strasbourg

82. Philips 835485/86 (also A
 00565/66 L, 836897 DSY) *
Orient-Occident
 (electronic composition)

83. Philips 835487
Analogique A et B, Concret PH
 Ensemble Instrumental de
 musique contemporaine de
 Paris; K. Simonovich, dir.

84. Philips T 6521045
Persépolis
 (electronic composition)

85. PNM (Perspectives of New
 Music) 28 CD
Voyage absolu des Unaris vers
Andromède
 (electronic composition com-
 posed on UPIC at CEMAMu)

86. RCA RS 9009 (also RE 25444)
Dikhthas, Embellie, Ikhoor, Kottos,
Mikka, Mikka S, ST/4
 Arditti Quartet

87. RCA Victor JRZ-2501 *
Hibiki Hana Ma
 (electronic composition)

88. RCA Victor SJV-1513 *
Stratégie
 Yomiuri Nippon Symphony Orchestra; S. Ozawa and H. Wakasugi, dirs.

89. Salabert Actuels SCD 8906 CD
 (dist. Harmonium Mundi)
Oresteia, Kassandra
 U. of Strasbourg Chorus;
 Maîtrise de Colmar; Anjou Vocal Ensemble; Ensemble de Basse-Normandie; D. Debart, dir; R. Weddle, Vocal dir; S. Sakkos, bar; S. Gualda, perc.
 Sony CBS SONC-10163 *
Akra
 The Festival Chamber Ensemble; R. Dufallo, dir.

90. Teldec 6.42339 AG (also CD 8.42339 ZK) *
Retours-Windungen
 The 12 cellists from the Berlin Philharmonic.

91. Telec/Warner Classics 2292 46442-2 CD *
Eonta
 R. Hind, pno; London Brass

92. Toshiba TA-72034 *
Evryali
 A. Takahashi, pf.

93. Vanguard Cardinal 10030
Eonta, Metastasis, Pithoprakta
 Y. Takahashi, pf.; Ensemble Instrumental de musique contemporaine de Paris, K. Simonovich, dir.

94. Varèse Sarabande 81060 *
Stratégie
 Yomiuri Nippon Symphony Orchestra; S. Ozawa and H. Wakasugi, dirs.

95. Wergo WER 6178-2 CD
Akanthos, Dikhthas, Palimpsest, Epeï
 I. Arditti, vln; C. Helffer, pno; P. Walmsey-Clark, sop; Spectrum Ensemble; G. Protheroe, dir.

In Preparation:

96. Disques Montaigne 782xxx 3 CD
Evryali, Mists, Herma, Dikhthas, Akea, Tetras, ST/4, Mikka, Mikka "S", Kottos, Nomos Alpha, Ikhoo, Embellie
 C. Helffer, pno; Arditti String Quartet

97. MFA (collection Musique Français d'Aujourd'hui)
Charisma
 A. Damiens, cl; P. Strauch, vlc.

98. Salabert Actuels SCD 9102 CD
 (dist. Harmonium Mundi)
Bohor, La Légende D'Eer
 (electronic compositions)

Iannis Xenakis Biographical Information

1957: Geneva, European Cultural Foundation Award

1963: Athens, Manos Hadjidakis Award

1963-64: Berlin, Ford Foundation Grant plus Grant from the West-Berlin Senate

1964: Paris, *Musiques Formelles* chosen by the Permanent Committee of the French Book and Graphic Arts Exhibits, to be one of the 50 "Books of the Year."

1965: Paris, Grand Prize awarded by the French Recording Academy Competition.

1968: Edinburgh, First Prize at the Computer-assisted Music Competition, IFIP Congress

Paris, Grand Prize awarded by the French Recording Academy

London, Bax Society Prize (Harriet Cohen International Music Awards)

1970 Paris Grand Prize awarded by the French Recording Academy

1971: Tokyo Modern Music Award from the Nippon Academy Awards

1972: London, Honorary Member of the British Computer Arts Society

1974: Paris, Gold Medal Maurice Ravel Award from the SACEM

1975: Honorary Member of the American Academy of Arts and Letters

1976: Paris, Sorbonne, Doctorat ès Letters and Humanities

Paris, National Grand Prize in Music from the French Cultural Secretary of State

1977: Paris, Grand Prize, Charles Cros Academy for Recordings (Grand Prix du Président de la République in honorem)
 :Bonn, Beethoven Prize
 :Amsterdam, Edison Award for the best recording of contemporary music

1981: Paris, Officier de l'Ordre des Arts et des Lettres

1982: Paris, Chevalier de la Légion d'Honneur

1983: Paris, Member of the Institut de France (Académie des Beaux Arts)
 :Berlin and Munich, Member of the Akademie der Kunste

1985: Paris, Officier de l'Ordre National du Mérite
 :Athens, Medal of Honor of the City

1986: Paris, Ordre National du Mérite

1987: Honorary Member of the Scottish Society of Composers
 :Grand Prize from the City of Paris

1988: Paris, Nominated to the Victoires de la Musique

1989: Foreign member of the Swedish Royal Academy of Music

1990: Professor Emeritus of the Université de Paris I, Panthéon-Sorbonne
 :Honorary Doctor of the University of Edinburgh
 :Honorary Doctor of the University of Glasgow

Notes

I. Free Stochastic Music

1. Jean Piaget, *Le développement de la notion de temps chez l'enfant* (Paris: Presses Universitaires de France, 1946).
2. I. Xenakis, *Gravesaner Blätter*, no. 1 (1955).
3. I. Xenakis, *Revue technique Philips*, vol. 20, no. 1 (1958), and Le Corbusier, *Modulor 2* (Boulogne-Seine: Architecture d'Aujourd'hui, 1955).
4. I. Xenakis, "Wahrscheinlichkeitstheorie und Musik," *Gravesaner Blätter*, no. 6 (1956).
5. Ibid.
6. Ibid.

II. Markovian Stochastic Music—Theory

1. The description of the elementary structure of the sonic symbols that is given here serves as a point of departure for the musical realization, and is consequently only a hypothesis, rather than an established scientific fact. It can, nevertheless, be considered as a first approximation to the considerations introduced in information theory by Gabor [1]. In the so-called Gabor matrix a sonic event is resolved into elementary acoustic signals of very short effective durations, whose amplitude can be divided equally into quanta in the sense of information theory. However, these elementary signals constitute sinusoidal functions having a Gaussian "bell" curve as an envelope. But one can pretty well represent these signals of Gabor's by sine waves of short duration with an approximately rectangular envelope.
2. The choice of the logarithmic scale and of the base between 2 and 3 is made in order to establish our ideas. In any case, it corresponds to the results of research in experimental music made by the author, e.g., *Diamorphoses*.

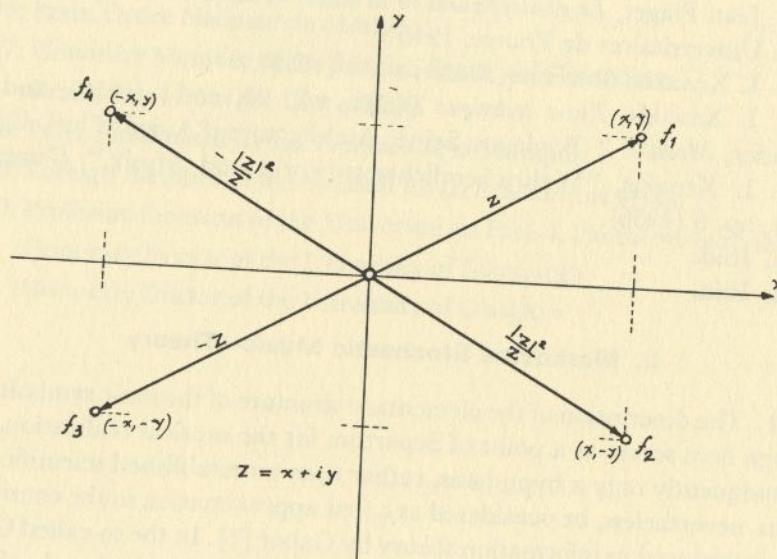
V. Free Stochastic Music by Computer

1. See *Gravesaner Blätter*, nos. 11/12 (Mainz: Ars Viva Verlag, 1957).
2. $(V3)e^R$ must be equal to the upper limit, e.g., to 150 sounds/sec. in the case of a large orchestra.

VI. Symbolic Music

1. A second-degree acoustic and musical experience makes it necessary to abandon the Fourier analysis, and therefore the predominance of frequency in sound construction. But this problem will be treated in Chapter IX.

2. From previous edition of *Formalized Music*, another way to map these same four forms:



$$z = x + yi$$

$$f_1 = Z = x + yi = Z = f_1(Z) = \text{original form}$$

$$f_2 = x - yi = |Z|^2/Z = f_2(Z) = \text{inversion}$$

$$f_3 = -x - yi = -Z = f_3(Z) = \text{inverted retrogradation}$$

$$f_4 = -x + yi = -(|Z|^2/Z) = f_4(Z) = \text{retrogradation}$$

VII. Towards a Metamusic

1. Cf. I. Xenakis, *Gravesaner Blätter*, no. 29 (Gravesano, Tessin, Switzerland, 1965).
2. Cf. I. Xenakis, *Gravesaner Blätter*, nos. 1, 6; the scores of *Metastasis* and *Pithoprakta* (London: Boosey and Hawkes, 1954 and 1956); and the recording by Le Chant du Monde, L.D.X. A-8368 or Vanguard.
3. I do not mention here the fact that some present-day music uses quarter-tones or sixth-tones because they really do not escape from the tonal diatonic field.
4. Cf. Chap. VI.
5. Johannis Tinctoris, *Terminorum Musicae Diffinitorum* (Paris: Richard-Masse, 1951).
6. Jacques Chailley, "Le mythe des modes grecs," *Acta Musicologica*, vol. XXVIII, fasc. IV (Basel Bärenreiter-Verlag, 1956).
7. R. Westphal, *Aristoxenos von Tarent, Melik und Rhythmis* (Leipzig: Verlag von Ambr. Abel (Arthur Meiner), 1893), introduction in German, Greek text.
8. G. Th. Guilbaud, *Mathématiques*, Tome I (Paris: Presses Universitaires de France, 1963).
9. Aristidou Kointiliano, *Peri Mousikes Proton* (Leipzig: Teubner, 1963), at Librairie des Méridiens, Paris.
10. The Aristoxenean scale seems to be one of the experimental versions of the ancient diatonic, but does not conform to the theoretical versions of either the Pythagoreans or the Aristoxeneans, $X(9/8)(9/8) = 4/3$ and $6 + 12 + 12 = 30$ segments, respectively. Archytas' version, $X(7/8)(9/8) = 4/3$, or Euclid's are significant. On the other hand, the so-called Zarlino scale is nothing but the so-called Aristoxenean scale, which, in reality, only dates back to Ptolemy and Didymos.
11. Avraam Euthymiadis, *Στοιχειώδη Μαθήματα Βυζαντινῆς Μουσικῆς* (Thessaloniki: O.X.A. Apostoliki Diakonia, 1948).
12. In Quintilian and Ptolemy the perfect fourth was divided into 60 equal tempered segments.
13. See Westphal, pp. XLVIIff. for the displacement of the tetrachord mentioned by Ptolemy: lichanos (16/15) mese (9/8) paramese (10/9) trite (Harmonics 2.1, p. 49).

14. In Ptolemy the names of the chromatic tetrachords were permuted: the soft chromatic contained the interval 6/9, the hard or syntomon the interval 7/6. Cf. Westphal, p. XXXII.

15. Selidion 1 : a mixture of the syntomon chromatic (22/21, 12/11, 7/6) and toniaion diatonic (28/27, 7/8, 9/8) ; selidion 2 : a mixture of the soft diatonic (21/20, 10/9, 8/7) and the toniaion diatonic (28/27, 8/7, 9/8), etc. Westphal, p. XLVIII.

16. Egon Wellesz, *A History of Byzantine Music and Hymnography* (Oxford: Clarendon Press, 1961), pp. 71ff. On p. 70 he again takes up the myth that the ancient scales descended.

17. The same negligence can be found among the students of ancient Hellenic culture; for example, the classic Louis Laloy in *Aristoxène de Tarente*, 1904, p. 249.

18. Alain Daniélou lived in India for many years and learned to play Indian instruments. Mantle Hood did the same with Indonesian music, and let us not forget Than Van Khé, theoretician and practicing performer and composer of traditional Vietnamese music.

Cf. Wellesz. Also the transcriptions by C. Höeg, another great Byzantinist who neglected the problems of structure.

20. Imagine the bewilderment of the "specialists" when they discovered that the Byzantine musical notation is used today in traditional Romanian folk music! *Rapports Complémentaires du XIIe Congrès international des Etudes byzantines*, Ochrida, Yugoslavia, 1961, p. 76. These experts without doubt ignore the fact that an identical phenomenon exists in Greece.

21. Cf. my text on disc L.D.X. A-8368, issued by Le Chant du Monde. See also *Gravesaner Blätter*, no. 29, and Chap. VI of the present book.

22. Among themselves the elementary displacements are like the integers, that is, they are defined like elements of the same axiomatics.

23. Alain Daniélou, *Northern Indian Music* (Barnet, Hertfordshire: Halcyon Press, 1954), vol. II, p. 72.

24. This perhaps fulfills Edward Varèse's wish for a spiral scale, that is, a cycle of fifths which would not lead to a perfect octave. This information, unfortunately abridged, was given me by Odile Vivier.

25. These last structures were used in *Akrata* (1964) for sixteen winds, and in *Nomos alpha* (1965) for solo cello.

VIII. Towards a Philosophy of Music

1. The "unveiling of the historical tradition" is used here in E. Husserl's sense; cf. *Husserliana*, VI. "Die Krisis der Europäischen Wissenschaften und die transzendentale Phänomenologie (Eine Einleitung in die phänomenologische Philosophie)", *Pure Geometry* (The Hague: M. Nijhoff, 1954), pp. 21–25, and Appendix III, pp. 379–80.

2. Cf. *Upanishads* and *Bhaga di Gita*, references by Ananda K. Coomaraswamy in *Hinduism and Buddhism* (New York: Philosophical Library, 1943).

3. "Perhaps the oddest thing about modern science is its return to pythagoricism." Bertrand Russell, *The Nation*, 27 September 1924.

4. In this translation I have considered the original Greek text and the translations by John Burnet in *Early Greek Philosophy* (New York: Meridian Books, 1962) and by Jean Beaufret in *Le Poème de Parménide* (Paris: P.U.F., 1955).

1. Elements are always real: (earth, water, air) = (matter, fire) = energy. Their equivalence had ready been foreseen by Heraclitus.

6. Lucretius, *De la Nature*, trans. A. Ernout (Paris, 1924).

7. The term stochastic is used for the first time in this work. Today it is synonymous with probability, aleatory, chance.

8. E. Borel, *Eléments de la théorie des probabilités* (Paris: Albin Michel, 1950), p. 82.

9. Uncertainty, measured by the entropy of information theory, reaches a maximum when the probabilities p and $(1-p)$ are equal.

10. Cf. I. Xenakis, *Gravesaner Blätter*, nos. 1, 6, 11/12 (1955–8).

11. I prepared a new interpretation of Messiaen's "modes of limited transpositions," which was to have been published in a collection in 1966, but which has not yet appeared.

12. Around 1870 A. de Bertha created his "gammes homotones première et seconde," scales of alternating whole and half tones, which would be written in our notation as $(3_n \vee 3_{n+2}, 3_n \vee 3_{n+1})$.

13. In 1895, Loquin, professor at the Bordeaux Conservatory, had already preconceived the equality of the twelve tones of the octave.

14. The following is a new axiomatization of the sieves, more natural than the one in Chaps. VI and VII.

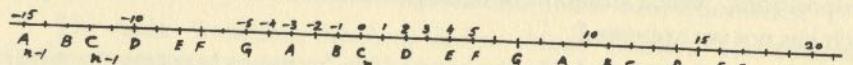
Basic Assumptions. 1. The sensations create *discrete* characteristics, values, stops (pitches, instants, intensities, . . .), which can be represented as points. 2. Sensations plus comparisons of them create *differences* between the above characteristics or points, which can be described as the movement, the displacement, or the step from one discrete characteristic to another, from one point to another. 3. We are able to repeat, iterate, concatenate the above steps. 4. There are two orientations in the iterations—more iterations, fewer iterations.

Formalization. Sets. The basic assumptions above engender three fundamental sets : Ω , Δ , E , respectively. From the first assumption characteristics will belong to various specific domains Ω . From the second, displacements or steps in a specific domain Ω will belong to set Δ , which is independent of Ω . From the third, concatenations or iterations of elements of Δ form a set E . The two orientations in the fourth assumption can be represented by + and -.

Product Sets. a. $\Omega \times \Delta \subseteq \Omega$ (a pitch-point combined with a displacement produces a pitch-point). b. $\Omega \times E \subseteq \Delta$ (a displacement combined with an iteration or a concatenation produces a displacement). We can easily identify E as the set N of natural numbers plus zero. Moreover, the fourth basic assumption leads directly to the definition of the set of integers Z from E .

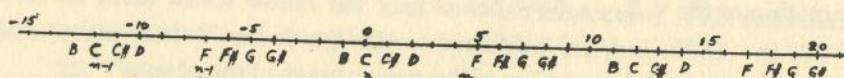
We have thus bypassed the direct use of Peano axiomatics (introduced in Chaps. VI and VII) in order to generate an *Equally Tempered Chromatic Gamut* (defined as an ETCHG sieve). Indeed it is sufficient to choose any displacement ELD belonging to set Δ and form the product $\{ELD\} \times Z$. Set Δ (set of melodic intervals, e.g.), on the other hand, has a group structure.

15. Cf. Olivier Messiaen, *Technique de mon langage musical* (Paris: Durand, 1944).



C scale

Figure 2.



Messiaen Mode No 4

Figure 3.



Messiaen Mode No 4

Figure 4.

16. ". . . therefore tones higher than needed become relaxed [lower], as they should be, by curtailment of movement; conversely those lower than needed become tensed [higher], as they should be, by adjunction of movement. This is why it is necessary to say that tones are constituted of discrete pieces, since it is by adjunction and curtailment that they become as they should be. All things composed of discrete pieces are said to be in numerical ratio to each other. Therefore we must say that tones are also in numerical ratio to each other. But among numbers, some are said to be in multiplicative ratio, others in an *epimorphos* $[1 + 1/x]$, or others in an *epimeris* ratio [an integer plus a fraction having a numerator other than one] ; therefore it is necessary to say that tones are also in these same ratios to each other. . ." Euclid, *Katatomé Kanonos* (12-24), in Henricus Menge, *Phaenomena et Scripta Musica* (Leipzig: B. G. Teubner, 1916). This remarkable text already attempts to establish axiomatically the correspondence between tones and numbers. This is why I bring it in in the context of this article.

17. Cf. my analysis of *Metastasis*, in Corbusier, *Modulor 2* (Boulogne-Seine: Architecture d'Aujourd'hui, 1955).

18. Cf. Score by Boosey and Hawkes, eds., and record by Pathé-Marconi and Angel.

19. *Hibiki-Hana-Ma*, the electro-acoustic composition that I was commissioned to write for the Japanese Steel Federation Pavilion at the 1970 Osaka World Expo, used 800 loudspeakers, scattered in the air and in the ground. They were divided into approximately 150 independent groups. The sounds were designed to traverse these groups according to various kinematic diagrams. After the Philips Pavilion at the 1958 Brussels World's Fair, the Steel Pavilion was the most advanced attempt at placing sounds in space. However, only twelve independent magnetic tracks were available (two synchronized six-track tape recorders).

20. Mario Bois, *Iannis Xenakis: The Man and His Music* (New York: Boosey and Hawkes, 1967).

21. Jean Piaget, *Le développement de la notion de temps chez l'enfant*, and *La représentation de l'espace chez l'enfant* (Paris. Presses Universitaires de France, 1946 and 1948).

X. Concerning Time, Space and Music

1. Shannon C. and Weaver W., *The Mathematical Theory of Communication* (Urbana: University of Illinois Press, 1949).
2. Eddington, *The Nature of the Physical World* (New York: Macmillan, 1929).
3. Prigogine, I., *Physique Temps et Devenir* (Paris: Masson, 1982).
4. Born, Max, *Einstein's Theory of Relativity* (New York: Dover, 1965).
5. Morrison, Philip, "The Overthrow of Parity," *Scientific American*, April, 1957.
6. Gardner, Martin, "Can Time Go Backward," *Scientific American*, Jan. 1967, p.98.
7. Reichenbach, H., *The Philosophy of Space and Time* (New York: Dover, 1958).
8. Linde, A. D., *Physics Letters* (1983), 129B, 177.
9. See also Coveney, Peter V., "The Second Law of Thermodynamics: Entropy, Irreversibility and Dynamics," *Nature* N° 333 (1988).
10. The idea of the Big Bang, a consequence of the shift (expansion of the universe) toward the red, is not accepted by all physicists. See Nikias Stravroulakis, "Solitons et propagation d'actions suivant la relativité générale," *Annales de la Fondation de Broglie* 12 N° 4 (1987).
11. Russell, B., *Introduction à la philosophie mathématique* (Paris: Payot, 1961).
12. Cf. chapter 9 in *Formalized Music*, "New Proposals in Microsound Structure."
13. Cf. *Xenakis*, autori vari (a cura di Enzo Restagno) (Torino: E.D.T., 1988).

XI. Sieves

1. Earlier articles on "sieves" by Iannis Xenakis have appeared in *Preuves*, Nov. 1965, Paris; *La Nef* n° 29, 1967, Paris; *Revue d'Esthétique* vol. xxi, 1968, Paris; *Tempo* no 93, 1970, as well as the previous editions of *Formalized Music*.
2. As for rhythm outside of Western civilization, cf. AROM, Simha, "Du pied à la main: Les fondements métriques des musiques traditionnelles d'Afrique Centrale;" *Analyse Musicale* 1° trimestre, 1988.

3. Let there be (M, I) , with M being a composite of the form:

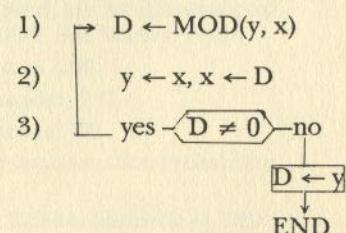
$$M = m^k * n^l * \dots * r^j.$$

It is sometimes necessary and possible to decompose it into :

$$(m^k, I_m) \cap (n^l, I_n) \cap \dots \cap (r^j, I_r) = (M, I).$$

4. Euclid's algorithm. Let y, x be two positive whole numbers. Begin by letting $D = \text{MOD}(y, x)$, then replace y with x and x with D . If D is not equal to 0, then start over. But if $D = 0$, then the last is the largest common denominator. Let us call this last y , D .

take two numbers : y, x



example :

$$y = 30, x = 21$$

$$D \leftarrow \text{MOD}(30, 21) = 9 \rightarrow D \leftarrow \text{MOD}(21, 9) = 3 \rightarrow D \leftarrow \text{MOD}(9, 3) = 0$$

$$y \leftarrow 21, x \leftarrow 9$$

$$y \leftarrow 9, x \leftarrow 3$$

$$y \leftarrow 3, x \leftarrow 0$$

$$D \leftarrow 9 \neq 0$$

$$D \leftarrow 3 \neq 0$$

$$D \leftarrow 0 = 0$$

therefore

$$D \leftarrow y = 3$$

END

5. a modulo b, notated $\text{MOD}(a, b)$, is equal to the residue of the division of a by b : $a / b = e + r / b$ where r is this residue, if a, b, e , and r are elements of N .

6. $\text{MOD}(\xi * C_2, C_1) = 1$ represents the integer equation:
 $\xi * C_2 / C_1 = v + 1 / C_1$.

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